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THE LONGWAVE ASYMPTOTIC FORMS OF SOLUTIONS SOUND DIFFRACTION TASKS BY ELASTIC BODIES

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The review of known results and new tasks solutions of sound waves diffraction by elastic bodies in the inhomogeneous medium are presented in the report. The main new results have obtained in case of inhomogeneous half-space with variable on depth density and speed of sound. Asymptotic functional proportions which connected characteristics of the near-field region and the far field diffraction zone with elastic and spectral characteristics of inhomogeneous half-space have been found. The research makes for a prolate body of revolution in the inhomogeneous medium on base of version spliced of asymptotic expansion (SAE) . In the report reviewed a scattering objects, describable the Dirichlet, Noiman, mixed (impedances) boundary conditions and the equations of motion of an elastic thin-waled shells. The representation of near field for classical boundary conditions have found in the form of recurrent system boundary task for equation's Laplace and Poisson, which solution obtained in explicit form.

1. Stating a problem.

We consider the following model of inhomogeneous medium. In the Cartesian coordinate system (x, y, z) a water layer $H_1 \geq x \geq 0$ has a density varying with the depth $\rho(x)$ and sound speed $c(x)$. At $H_2 \geq x > H_1$ an elastic inhomogeneous layer is situated with variable density $\rho_1(x)$ and Lame parameters $\lambda_1(x)$ and $\mu_1(x)$. At $x > H_2$ there exists an elastic half-space with parameters ρ_2 , λ_2 and μ_2. Let us consider the normal wave, with the real wave number α_n , incident on a prolate shell of revolution S in a water layer

$$p_{ins} = H_0^{(1)}\left(\alpha_n |r_1 - r_1'|\right) p(\alpha_n, x) \tag{1}$$

Here (x, r_1, θ) - origin x_0, coincident with centre S, $|r_1| = \left(y^2 + z^2\right)^{\frac{1}{2}}$, $|r_1'| = \left(y'^2 + z'^2\right)^{\frac{1}{2}}$ - distance between the source of incident wave and centre S; $p(\alpha_n, x)$ - eigen function of cross section , with the wave number α_n, $H_0^{(1)}$ - the Hankel- function of the first kind.

The scattered field satisfies equation

$$\rho dir\left(\frac{1}{\rho} gradp\right) + \frac{\omega^2}{c^2} p = 0,\ x \notin S, \tag{2}$$

and boundary conditions on the mean surface S of the shell of revolution, given by an equation $r = \varepsilon F(z)$, in cylindrical coordinate system (r, φ, z), where $r = \left[(x - x_0)^2 + y^2\right]^{\frac{1}{2}}$, $\varepsilon = \frac{d}{l} << 1$; $F(z) > 0$, ; d - a maximum shell diameter, $z \in \left(-\frac{l}{2}, \frac{l}{2}\right)$; $F\left(\pm\frac{l}{2}\right) = 0$, x_0 - centre of shell .

Let us assume that $F(z)$ is function sufficiently smooth everywhere except $z = \pm\frac{l}{2}$ and frequency is $kl \approx 1$. The time dependence $\exp(-i\omega t)$ is assumed and suppressed throughout the analysis.

. The shell by next parameters: h - half the thickness of the shell walls, ε_1 =2h/d << 1, E - Young's modulus, ρ_p -the density of the shell material, ν - Poisson's ratio, $c_p = \sqrt{E/\rho_p}$ - the velocity of longitudinal waves in a rod of shell material. The mean surface of the shell is given by the differential system of equations of motion

$$\sum_{j=1}^{3}\left(h^2 N_{ij}/3 + L_{ij}\right)u_j = k_p^2 u_i + \gamma p_i, i = 1,2,3,$$
$$\frac{\partial}{\partial n}\left(p + p_{ins}\right) = \rho\omega^2 w(\bar{r}),\ \ \bar{r} \in S \quad ; \tag{3}$$

Here N_{ij}, L_{ij} - differential operators in the partial derivatives on surface S defined by the theory thin-waled shell (in the present paper moment of Love's theory of thin-waled shells is used [1]); u_1, u_2 - the tangential components of the strain vector, $u_3 = w$ - the sag, $\gamma = (2Eh)^{-1}, p_1 = p_2 = 0,\ \ p_3 = p + p_{iins}.,\ \ k_p = \omega / c_p,\ \frac{\partial}{\partial n}$ - the derivative in the direction of the external normal to the mean surface of the shell.

The sound pressure satisfies condition $p + p_{ins} = 0$ at $x = 0$ and condition for nonflow at $x = H_1$. The elastic half-space, as the bottom model, can be defined by the elastic theory equations.

2. Methods of solution.

Let us look for solution (2) in the form

$$p = p^{(r)} + p^{(e)} \tag{4}$$

where $p^{(r)}$ - the scattered field of the rigid surface S; $p^{(e)}$- elastic component of the scattered field of the shell.

To solve this problem the method of spliced asymptotic expansions (SAE) should be used. This method was used for the homogeneous Helmholz equation [2]. The variables don't separate in cylindrical coordinate system (r, φ, z), with the axes revolution S, because there are the variables coefficients in (2) in the case half-space. Then, let us consider the description of SAE in detail.

In cylindrical coordinate system (r, φ, z), with the axes z, directed in the axes revolution body S equation (2) is given by

$$\left(\Delta + \Delta_\rho + \frac{\omega^2}{c^2}\right)p = 0. \tag{5}$$

Here $\Delta = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \varphi^2} + \frac{\partial^2}{\partial z^2}$ - Laplace operator, $\Delta_\rho = \frac{\rho}{r}\frac{\partial}{\partial r}\left(\frac{r}{\rho}\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \varphi}\left(\frac{1}{\rho}\frac{\partial}{\partial \varphi}\right)$. In the case of stratification directed in the axes x there are

$$p = p(r\cos\varphi),\ c = c(r\cos\varphi),$$
$$\Delta_\rho = \frac{\rho'}{\rho}\left(\cos\varphi r\frac{\partial}{\partial r} - \frac{\sin\varphi}{r}\frac{\partial}{\partial \varphi}\right), \tag{6}$$

where ρ' - the derivative of density . According to SAE let us transform the coordinates $\xi = r/\varepsilon$ and represented equations (5), (6) in internal variables for the scattered field of the rigid surface

$$\left(\frac{1}{\varepsilon^2}\Delta_\xi + \frac{1}{\varepsilon}\Delta_{\rho\xi} + \frac{\partial^2}{\partial z^2} + \frac{\omega^2}{c^2(x)}\right)p^{(r)} = 0, \tag{7}$$

where $\Delta_\xi = \frac{1}{\xi}\frac{\partial}{\partial \xi}\left(\xi\frac{\partial}{\partial \xi}\right) + \frac{1}{\xi^2}\frac{\partial^2}{\partial \varphi^2},\ \Delta_{\rho\xi} = \frac{\rho'}{\rho}\left(\cos\varphi\frac{\partial}{\partial \xi} - \frac{\sin\varphi}{\xi}\frac{\partial}{\partial \varphi}\right).$

Let us look for solution (7) in the form

$$p^{(r)}=\sum_{m=1}^{\infty}\varepsilon^m\left[p_{m0}(\xi,\varphi,z)+\ln\varepsilon\ \ p_{m1}(\xi,\varphi,z)\right] \quad (8)$$

After substitution (8) in (7) we define recurrent system Poisson's equations

$$\Delta p_{1i}=0,\ \Delta p_{2i}=-\Delta_{\rho\xi}p_{1i},\ \Delta p_{3i}=-\Delta_{\rho\xi}p_{2i}-(\frac{\partial^2}{\partial z^2}+k^2+\Delta_{\rho\xi})p_{1i},\ \ldots,\ \text{i=0,1.} \quad (9)$$

The determination procedure of the boundary conditions for system (9) depends on the source coordinates. Let us transform the incident wave in the form of system diverging cylindrical waves according to Graf theorem in the case of the near source. Then Bessel function could be expanded in series at ε and transforming to the internal variables. Thus the necessary boundary conditions in the internal variables are found.

This procedure is simply when the far source ($|r_1^{'}|>>l$). The argument is presented as function a small parameter ($|r_1|/|r_1^{'}|<1$). Then $|r-r^{'}|=|r^{'}|+|r_1|\cos(\theta-\theta_0)+O(|r_1|^2/|r_1^{'}|^2)$), where $\theta_0=arctg(y^{'}/z^{'}),\ \theta=arctg(y/z)$.

Angle θ_0 is the analog sliding angle for the cylindrical wave in direction the axes z. Taking into account the Hankel- function asymptotic the incident wave and the derivative in the direction of the external normal are presented in internal variables and the boundary conditions are determined for all members of series of internal expansion. It given us infinite recurrent succession of the boundary tasks for the Poisson's equations system (9).

3. Basic results.

The main asymptotic term we will find in the form of potentials simple and double layers distributed on the segment [-l/2,l/2]. The simple layer potentials asymptotic is given by

$$\Pi=\frac{1}{2i}\sum_{\alpha_m\in R^+}\sqrt{\frac{2}{\pi\alpha_m|r|}}p(\alpha_m,x)p(\alpha_m,x_0)\int_{-l/2}^{l/2}\exp[i(\alpha_m|r|-\alpha_m z'\cos\theta-\pi/4]\nu(z')dz',$$

Where R^+ is set of wave numbers from the upper half-plane , $p(\alpha_m,x)$ - corresponding to the wave number half-space eigenfunctions , $\nu(z)$ - the unknown potentials density . The unknown potentials density of simple layer we can obtain by the splicing procedure of external and internal expansions in the form

$\nu=2\pi FAi\alpha_n p(\alpha_n,x)(F'\cos\theta_0-\frac{i\alpha_n}{2}F\sin^2\theta_0)$. The double layers potentials density is given by corresponding formula.

The main asymptotic term of rigid component scattered field is given by in the next form

$$p^{(r)}=\frac{1}{\pi}\sum_{\alpha_m\in R^+}\left(\alpha_m\alpha_n|r|\,|r^{'}|\right)^{-1/2}e^{i\left(\alpha_m|r_1|+\alpha_n|r^{'}_1|\right)}p(\alpha_m,x)$$

$$\int_{-l/2}^{l/2}\left\{\alpha_n p(\alpha_m,x_0)p(\alpha_n,x_0)\left[\alpha_m S(z^{'})\sin\theta\sin\theta_0-\alpha_n S(z^{'})\sin^2\theta_0-iS^{'}(z^{'})\cos\theta_0\right]\right\}$$

$\exp\left[i\left(a_n z^{'}\cos\theta_0-a_m z^{'}\cos\theta\right)\right]dz^{'}$, where $S=\pi\varepsilon^2F^2(z)$ - the cross sections area of the shell .

The elastic component scattered field was defined by corresponding the spliceing procedure asymptotic expansions. In result, the main asymptotic term were obtained

$$p_1^{(e)} = \rho\omega^2 \sum_{\alpha_m \in R^+} \sqrt{\frac{2\pi}{\alpha_m |r|}} p(\alpha_m, x_0) p(\alpha_m, x) e^{i(\alpha_m |r_1| - \pi/4)} \int_{-l/2}^{l/2} AFw_{\circ\circ}(z) e^{-i\alpha_m z' \cos\theta} dz'.$$

Here $w_{\circ\circ}(z')$ - the boundary task solution for operator elastic shell theory (3), where exiting surface force was given by normal to surface force proportional to the expression $\sqrt{\frac{2}{\pi\alpha_n |r_1'|}} e^{i\left(\alpha_n |r_1'| + \alpha_n z' \cos\theta_\circ - 3\pi/4\right)}$, what gives the closed boundary task for shell equation motion more simply then tasks in papers [3,4].

The dispersion properties propagating normal modes have shown to observe in the space resonances scattered field in the directions determined by angles.

$$\theta_p = \arccos\left(\frac{\alpha_n}{\alpha_m \cos\theta_0}\right), \quad |\alpha_n| \le |\alpha_m|.$$

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