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TO CALCULATION of DIFFRACTION OF NORMAL WAVES IN LAYERED STRUCTURES WITH SEMI-INFINITE ELEMENTS FACTORIZATION METHOD

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On the basis of a factorization method VHF [1, 2] the mixed regional problem about diffraction of normal waves in layered structure from an elastic plate between two semi-infinite layers of a liquid is solved. Factorization it is applied twice, in the decision of two systems of the functional not predetermined equations. The offered updating of a method can appear effective for all this sort of class of regional problems. Solved the diffracting problem about Lamb's waves in a plate half shipped in a liquid, is actual in the theory of ultrasonic level gauges.

1. Introduction. About a considered class diffracting mechanics and electrodynamics problems. The problem of diffraction of Lamb's waves in a boundless plate between two liquid semi-infinite layers – a plate in a vessel is solved reference, typical for some class of problems; from here the mixed boundary conditions. And in general, boundary conditions in such problems (fig. 1-2 see) are mixed and test not only jump, but also cross-section shift carry concerning a line

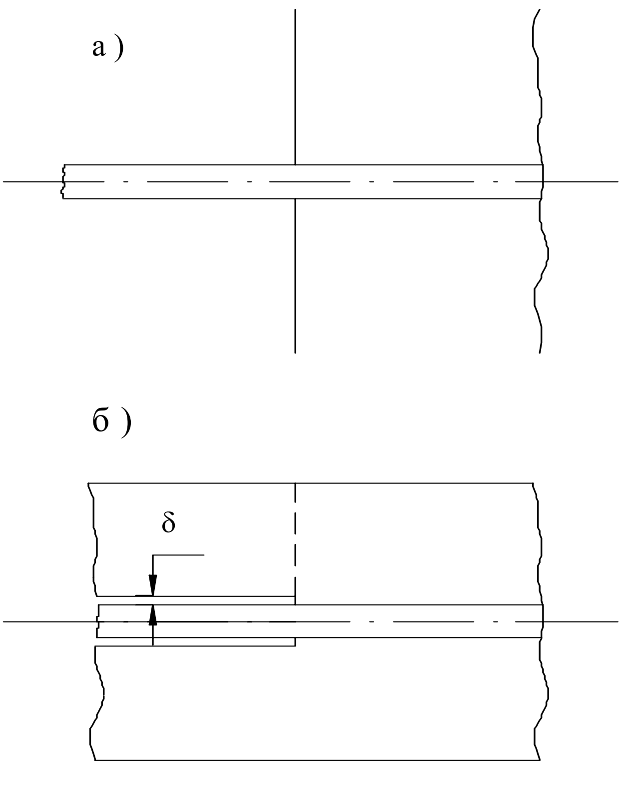

Fig. 1.

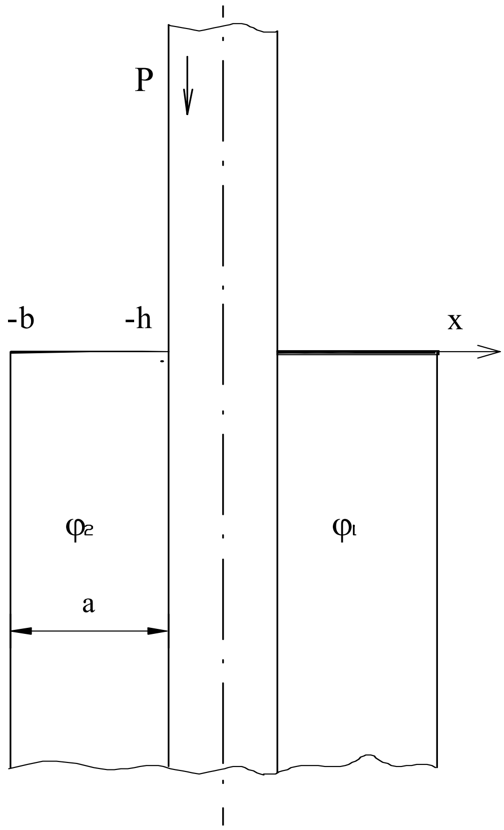

Fig. 2.

of movement of normal waves [2, § 5.4; 3 (1966г.); 4, p. 197]. And consequently all analytical methods of the decision and even a factorization method Wiener – Hopf - Fok [1, 2] (**VHF**), appear inefficient, modern reviews [5] see. In a close class of problems of diffraction of waves of Lemba on thin stratifications it was possible to use successfully method VHF [6], and falling at an angle already its updating [7]. In a considered case (–fig. 1a and 2) this method in a classical kind "does not work". And for construction of the trial decision in [8] it was necessary is artificial to enter extremely thin backlashes (fig. 1б).

In the present work the analytical decision managed to be constructed only by means of original transformations and not trivial, consecutive double factorization. The related electrodynamic problem with a semi-infinite step has been independently solved in [9]. Obviously, considered elastic both electromagnetic processes and problems will be if are not completely identical, are equivalent or complementary methodically, depending on identity of boundary conditions and the wave difference equations. We will notice that the general statement of considered acoustic problems Lev Grigorevich Merkulov (3.8.1927 – 8.1.1973), untimely deceased, outstanding soviet acoustic, the professor of chairs of electroacoustics LETI (Leningrad, to 1971) and TRTI (initiated still in 1960/70th Taganrog, 1971 – 73).

2. Statement and the integrated decision of a problem. Problem statement is typical – fig. 2 and, for example, [5 – 8, 10 – 12] see. Potentials falling (index *p*) and diffracting fields describe a vector of displacement of particles in a plate and liquid layers: $u = grad\varphi + rot\psi$, $u = grad\varphi_{1,2}$, $\psi = \psi_y$. Integration Gelmgolts`s equations and boundary conditions gives for amplitudes Fure`s transformant, $\tilde{\varphi}(k;x) = C(k)sh\gamma x + D(k)ch\gamma x$, $\tilde{\psi}(k;x) = E(k)sh\beta x + F(k)ch\beta x$, system functional equations:

$$-i\cdot 2k\gamma(Cch\,\gamma h + Dsh\,\gamma h) + (\beta^2 + k^2)(Esh\,\beta h + Fch\,\beta h) = 0;$$
$$-i\cdot 2k\gamma(Cch\,\gamma h - Dsh\,\gamma h) + (\beta^2 + k^2)(-Esh\,\beta h + Fch\,\beta h) = 0;$$
$$(\beta^2 + k^2)(Csh\,\gamma h + Dch\,\gamma h) + i\cdot 2k\beta(Ech\,\beta h + Fsh\,\beta h) = \tilde{\varphi}_1^+(k;h)k_\tau^2\rho_0/\rho;$$
$$(\beta^2 + k^2)(-Csh\,\gamma h + Dch\,\gamma h) + i\cdot 2k\beta(Ech\,\beta h - Fsh\,\beta h) = \tilde{\varphi}_2^+(k;-h)k_\tau^2\rho_0/\rho; \qquad (1)$$
$$\tilde{U}_x^+(k;h) + \tilde{U}_{xp}^+(k;h) = \frac{\partial\tilde{\varphi}_1^+(k;h)}{\partial x};$$
$$\tilde{U}_x^+(k;-h) + \tilde{U}_{xp}^+(k;-h) = \frac{\partial\tilde{\varphi}_2^+(k;-h)}{\partial x},$$

Where $\gamma^2 = k^2 - k_\ell^2$, $\beta^2 = k^2 - k_\tau^2$, k_ℓ, k_τ, k_0 – the wave numbers of volume longitudinal and shift waves in the firm environment and in a liquid, ρ, ρ_0 – densities, accordingly;

$\tilde{\varphi}_j^+(k;x) = (2\pi)^{-1/2}\int_0^\infty \varphi_j(x;z)\exp(ikz)dz$, $j = 1, 2$ $\tilde{U}_x^+$ and $\tilde{U}_{xp}^+$ are entered similarly. The badge plus (and further and a minus) over transformant specifies that they are regular, analytic, in top (or bottom) a complex semiplane $k = \sigma + i\tau$; k_n, $n = 1, 2, \ldots$, – required wave numbers diffracting fashions.

The developed chain of transformations, unfortunately, cannot be stated within the given theses. Therefore we will allocate only focal points. And we will consider further antisymmetric waves and boundary conditions of Dirihle for liquid potentials (then they are equal to zero – soft, thin walls of a vessel). Three other problems, for symmetric fluctuations and rigid walls (Neumann's condition), are completely identical to the first. Thus we assume that owing to symmetry waveguide systems, at falling of an antisymmetric wave (an index *a*) will be are raised only *a* - fashions. And on the contrary, at symmetric (– s) falling – only *s* - fashions. It twice simplifies systems of the equations. This position is proved mathematical in item 5.

3. Consecutive double factorization. 1st factorization is applied to equation:

$$\left[\tilde{\varphi}_1^+(k;h) - \tilde{\varphi}_2^+(k;-h)\right] - \left[\tilde{\varphi}_1^+(-k;h) - \tilde{\varphi}_2^+(-k;-h)\right] = (-thqa/q)\left[S^+(k) + S_p^+(k) - S^+(-k) - S_p^+(-k)\right];$$

Where $S^+(k) = \tilde{U}_x^+(k;h) + \tilde{U}_x^+(k;-h)$, and S_p^+ it is entered similarly. It is as a result had: $\tilde{\varphi}_1^+(k;h) - \tilde{\varphi}_2^+(k;-h) = -q^{-1}tg\,qa\left[S^+(k) + S_P^+(k)\right] + F_S(k)$; where zero $chqa$:

$$\alpha_m = \sqrt{k_0^2 - a^{-2}(\pi(m+0{,}5))^2}\,; \quad F_S = \sum_{m=0}^{\infty}\frac{S^+(\alpha_m) + S_P^+(\alpha_m)}{a\,\alpha_m}\left(\frac{1}{k-\alpha_m} - \frac{1}{k+\alpha_m}\right).$$

Again factorization it is applied to a following functional equation (also *not predetermined:* one equation – two unknown functions): $W_a S^+(k) + S^-(k) + (W_a - 1)\,(S_p^+(k) - F_S\, q\, cth\, qa) = 0$ (where $W_a = \delta_a / \Delta_a = 1 + k_\tau^4 \rho_0 thqa \cdot ch\gamma h \cdot ch\beta h / \rho \Delta_a q$ – meromorfycal function, see item 4). Its decision:

$$S^+(k) = -S_P^+(k) + \frac{S_P^+(k)}{W_a^+(k_P) W_a^+(k)} + F_S qcthqa \quad -$$

$$- \frac{1}{W_a^+(k)} \left(\frac{\pi}{2} \right)^2 \sum_{n=1}^{\infty} \frac{n^2 F_S(L_n)}{aL_n} \left[\frac{W_a^+(L_n)}{L_n - k} + \frac{1}{W_a^+(L_n)(L_n + k)} \right]$$

This decision gives also system, $\sum_{n=0}^{\infty} \left[S^+(\alpha_n) + S_P^+(\alpha_n) \right] B_{n,m} = S_P^+(\alpha_m)$, for the unknown persons $S^+(\alpha_m)$, entering in F_S. Elements $B_{n,m}$ here are not resulted in view of bulkiness.

Thus, the method factorization twice, was consistently applied to the functional not predetermined equations: on 2 unknown persons on one equation. And not predetermined equations make an essence of method VHF. A problem of our equations also that the functions giving the decisions, depend on these decisions. And in it an originality of the given type of problems and the updating of method VHF offered by us.

4. Factorization dispersive functions and the analysis of roots of the dispersive equations. Factorization meromorfycal functions $W_a = W_a^-(k) W_a^+(k)$ (in *product*, in distinction from the sum – in item 3) it is carried out by known reception [1, 2], with the account of roots lying in the top semiplane corresponding Lamb's dispersive equations (*l*) and a three-layer wave-guide (*T*):

$$\Delta_a(k_{an}^T) \equiv \delta_a(k_{an}^{Л}) \equiv 0 \; ; \; W_a^-(k) = \sqrt{W_a(0)} \prod_{n=0}^{\infty} \frac{1 - k / k_{an}^T}{(1 - k/\alpha_n)(1 - k / k_{an}^{Л})}$$

The analysis and calculation of roots a separate independent problem; including here multiple roots are important also, up to 10 order (for Lamb's – 4th), and back-wave fashions [7, 10, 16].

5. A correctness of statement and the decision of a regional problem. Matrix methods [14, 15] prove the important problem of a correctness of a regional problem on amplitudes diffracting fields – a correctness on own functions [12, 16, etc.]. Together with on conditions of radiation of running fashions [16].

6. Calculation of potentials diffracting fields. Calculation of potentials in liquid layers is original: $\varphi_j(x; z) = \frac{1}{\sqrt{2\pi}} \int_{ic-\infty}^{ic+\infty} \varphi_j^+(k; x) \exp(-ikz) dk$. Without change of value of integrals, instead of subintegral functions, we take $\tilde{\varphi}_j^+(k; x) - \tilde{\varphi}_j^+(-k; x)$. Thus, we have:

$$\frac{1}{A} \varphi_{1,2}(x, z) = \sum_{n=0}^{\infty} \left[T_n \frac{\rho}{k_\tau^2 \rho_0} \frac{\Delta_a(k_n) \gamma_n shq_n (x \mp b)}{(\beta_n^2 + k_n^2) ch \beta_n h shq_n a} \exp ikz \right]_{aT}$$

Here T_n – factor of passage of *n*-th fashion (expression is bulky enough); pluses and a minus concern an index 2 and 1; the index *aT* specifies assignment: $k = k_{an}^T$.

Decision for a rigid vessel are under construction similarly, since sine replacement on косинус in the formula for $\tilde{\varphi}_j^+(k; x)$ (in item 2). At falling of a symmetric wave there are spectra diffracting, only symmetric fashions, and formulas received thus also are similar to the stated.

7. The conclusion. To the physical analysis of diffraction. Except applied value of the given problems for the theory of the ultrasonic level gauge [17], the received decisions are interesting and in the physical relation. Calculation of amplitudes and reflexion and passage factors – a numerical problem. 4 spectra diffracting fashions are physically interesting. It, first of all, running, direct, and also seldom meeting, but unique return waves ([7, 10, 16], etc.). We will notice that in experiment [10] return of Lamb`s fashions were rather effectively reflected from contact of a plate to a liquid. 3rd type of waves, more likely, fluctuations – not extending non-uniform fashions-fluctuations, i.e. it is sharp, exponential decreasing (not on dissipating, and to the cinematic mechanism) from edge of contact to a liquid ($z=0$), and corresponding to purely imaginary roots. And 4 type, quasi-standing fashions: $exp(-\tau_n\,|z|)\,sin(\sigma_n z+\theta_n)$ – corresponding quadru-poluses in a complex plane k, the четвёркам four complex-interfaced and opposite roots of the dispersive equations.

Actually also research of attenuation of running fashions in force dissipating losses, and, in case of semispace, and in connection with radiation by “following waves”. That also assumes calculation and the analysis of roots of the dispersive equations.

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