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CALCULATION OF SOUND ATTENUATION WHEN EXCITING A STANDING WAVE IN A STRAIGHT ARBITRARY CROSS-SECTION SHAPE TUBE

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The Kirchhoff formula is used for the calculation of the spatial attenuation coefficient of the zero-order acoustic wave travelling in the infinite tube with solid walls. With the generation of the standing wave in the tube the mechanism of the acoustic power dissipation changes because the boundary layer becomes turbulent. The paper gives the calculation of the spatial attenuation coefficient of the standing wave generated in the tube with solid walls. The standing wave in liquid filling the tube is formed by the zero-order travelling waves moving in the opposite directions.

For the first time the problem on the propagation and attenuation of a running acoustic zero-order wave in the infinite tube was examined by G. Kirchhoff in 1868. The problem conditions were laid down as follows. A straight infinite tube with perfectly rigid smooth heat-conducting walls has a cross-section with constant form and area along the full length. The tube cross-section has no sharp bends. The tube is filled with gas (liquid) where the characteristic zero-order wave propagates. The wave front is perpendicular to the tube walls. Sound attenuation in gas volume filling the tube was not taken into consideration.

The problem solution has shown that when the running wave interacts with the surface of the tube walls there appear nonuniform viscosity and heat waves that form the acoustic boundary layer (ABL). It causes acoustic energy dissipation and, therefore, progressive wave attenuation in the tube. According to Kirchhoff, the progressive acoustic zero-order wave spatial attenuation coefficient in the R-radius round tube can be calculated using this formula

$$\alpha_R = \left(\sqrt{\frac{\omega \nu}{2c^2}} + (\gamma - 1)\sqrt{\frac{\omega a}{2c^2}} \right) / R = D_K / R, \qquad (1)$$

where $\omega = 2\pi f$, f is the wave frequency, c – the sound velocity, $\nu = \eta / \rho$ – the kinematic viscosity, η – the dynamic viscosity, ρ – the density of a gas, $a = \chi / C_P \rho$ – the thermal exchange, χ – the thermal conductivity of gases. For gases $\gamma = C_P / C_V$ – the Poisson ratio, where C_P and C_V – the medium heat capacity at constant pressure and volume. For liquids $\gamma = \Gamma$ – the nonlinear parameter of a liquid, D_K – the acoustic absorption coefficient in the laminar ABL.

From the expression (1) it is easy to see that the attenuation coefficient value is completely determined by the physical parameters of a gas filling the tube, the cross-section radius value of the tube and does not depend on the intensity of a wave. The Kirchhoff formula (1) was repeatedly verified experimentally. Thus, it has been shown that when it is possible to provide the mode of a travelling wave in the tubes, the results of measurements prove to be close enough to calculated values. These values for tubes with smooth internal walls are especially close. In every case the results of measurements of the spatial attenuation coefficient always exceed the values obtained using the formula (1).

Let us consider a case where two zero-order acoustic waves propagate in the tube towards each other. Their interaction obviously gives rise to a standing acoustic wave in the tube. With the interaction of the standing acoustic wave with the tube walls there occur acoustic Schlighting streams in the wall gas layer [2,3] whose occurrence results in the formation of the turbulent acoustic boundary layer (TABL). Hence, at the interaction of the standing acoustic wave with the solid surface the mechanism of acoustic energy dissipation changes. The problem on sound absorption in the TABL of a plane infinite surface has been solved in our work [4] which shows that in this case the density of a thermal flow can be calculated using the expression

$$\tilde{q}_T = \frac{0.17(\gamma - 1)C_P}{c^2} \cdot \frac{T_0 J_0}{\ln(y / y_0) + 2\nu / a} = \tilde{D}_T J_0, \qquad (2)$$

where y is the characteristic value of turbulent area, y_0 - the thickness of a viscous sublayer, $J_0 = 0.5\rho c U_0^2$ – the intensity, U_0 - the vibrational velocity amplitude of a travelling wave, $y_0 = 5.10\nu / U_0$ – the thickness of a viscous sublayer.

According to the design procedure stated in the book [5], in the case under consideration the characteristic size of the tube cross-section should be taken as the characteristic size of turbulent area y. For cylindric tubes $y = R$. If the tube has smooth internal walls ($y_0 \geq h$, h – the average height of the surface roughnesses), the thickness of turbulent area will be equal to $y_0 + 1.9\delta$, where $\delta = \sqrt{2\nu / \omega}$ - the ABL thickness, and the expression (2) gets the following form

$$\tilde{q}_T = \frac{0.17(\gamma - 1)C_P}{c^2} \cdot \frac{T_0 J_0}{\ln\left(\frac{R}{y_0 + 1.9\delta}\right) + 2\nu / a} = \tilde{D}_{TR} J_0, \qquad (3)$$

For the tube with rough internal walls ($y_0 < h$), the formula (2) is as follows

$$\tilde{q}_T = \frac{0.17(\gamma - 1)C_P}{c^2} \cdot \frac{T_0 J_0}{\ln\left(\frac{R}{h + 1.9\delta}\right) + 2\nu / a} = \tilde{D}_{TRh} J_0 , \qquad (4)$$

If in the formulae (2) and (3) the thickness of a viscous sublayer or the average height of roughnesses in comparison with the tube radius, with decreasing the frequency the value 1.9δ increases and at the frequency $f = f_{кр}$ (at $y_0 << R$ and $h << R$) the equality will hold true

$$1.9\sqrt{2\nu / \omega_{кр}} = R . \qquad (5)$$

The density of the thermal flow going into heating the tube internal walls becomes maximum and frequency-independent and its value at the frequencies $f \leq f_{кр}$ is determined using the expression

$$\tilde{q}_T = \frac{0.17(\gamma - 1)C_P}{c^2} \cdot \frac{T_0 J_0}{2\nu / a}, \qquad (6)$$

Physically meeting the condition (5) means that at the frequencies $f \leq f_{кр}$ Schlighting eddies and, hence, the turbulent boundary layer fill the tube volume completely. The frequency $f_{кр}$ value can be found from the equation (5). Solving (5) with regard to frequency, we have

$$f_{кр} = 1.15\nu / R^2 . \qquad (7)$$

Taking into consideration the real values of the coefficients of kinematic viscosity of liquids and gases makes it possible to claim that in most practically important cases the frequency $f_{кр}$ value falls into the area of infrasonic frequencies.

If at the fixed frequency the vibrational velocity amplitude of a travelling wave U_0 decreases, the thickness of a viscous sublayer y_0 increases and there comes a moment when the equality $R = y_0 + 1.9\delta$ will hold true. From this equality it is easy to find the critical value of the wave vibrational velocity amplitude

$$U_{0кр} = 5.10\nu / \left(R - 1.9\sqrt{2\nu / \omega}\right), \qquad (8)$$

at which the formula (3) is transformed into the expression (6). Hence, in this case the turbulent boundary layer also fills the tube volume completely. With the further reduction in the value U_0 in comparison with the value $U_{0кр}$ the turbulent ABL excitation in the tube becomes impossible. For this reason the value $U_{0кр}$ can be considered as the threshold one whose excess guarantees the turbulent ABL excitation in the tube.

From the formulae (3) or (4) it is easy to notice that with increasing the tube radius the $\tilde{q}_T$ values decrease and in the limiting case at $R \to \infty$ the $\tilde{q}_T$ value → 0. It corresponds to the limiting case of the acoustic wave distribution in the free space. In the other limiting case when the frequency $\omega \to \infty$, $1.9\delta = 1.9\sqrt{2\nu/\omega} \to 0$ in the formulae (3) and (4). They are transformed as the following expressions, respectively:

$$\tilde{q}_{TR} = \frac{0.17(\gamma-1)C_P}{c^2} \cdot \frac{T_0 J_0}{\ln(R/y_0) + 2\nu/a} = \tilde{D}_{TR} J_0, \qquad (9)$$

$$\tilde{q}_{TRh} = \frac{0.17(\gamma-1)C_P}{c^2} \cdot \frac{T_0 J_0}{\ln(R/h) + 2\nu/a} = \tilde{D}_{TRh} J_0. \qquad (10)$$

From the formulae (9) and (10) it can be seen that at high frequencies in the tubes the dissipated acoustic energy value in the TABL becomes frequency-independent. The values of critical frequencies, at which the absorption coefficients $\tilde{D}_{TR}$ and $\tilde{D}_{TRh}$ become constant, can be found from the expressions for smooth and rough tube walls, respectively:

$$f_{ky} = 115\nu/y_0^2, \qquad f_{kh} = 115\nu/h^2. \qquad (11)$$

Let us calculate the spatial attenuation coefficient value of the standing acoustic wave excited in a straight infinite tube of uniform cross-section. The spatial attenuation coefficient of the acoustic wave excited in the tube is equal to the energy dissipated per unit of time on the surface of the walls of the tube length unit divided by the double energy flow passing over a time unit through its cross-section. Using this definition and the formulae (3) and (4) for a round tube with smooth and rough walls we get as follows, respectively:

$$\alpha_{Rt} = \tilde{D}_{TR}/2R, \qquad \alpha_{Rth} = \tilde{D}_{TRh}/2R. \qquad (12)$$

The analysis of the formulae (12) shows that generally the value of spatial attenuation coefficients of a standing acoustic wave a рис. 1 excited in the tube does not depends only on the tube radius but on the wave vibrational velocity amplitude. Besides, at high frequencies the coefficients α_{Rt} and α_{Rth} values become frequency-independent while in the case of a travelling wave such a dependence takes place.

Let us compare the results obtained (12) with the Kirchhoff formula (1). As an example we will take a tube filled with air and having the radius R = 100 mm. The air in the tube is at static pressure P_0 = 1.0 atm and temperatureepатype T_0 = 273 K. The intensity of travelling waves forming the standing wave is J_0 = 3.5 Wt/m^2. The average height of roughnesses of the tube internal surface h = 700 μm. Applying the formulae obtained above, we will obtain the parameter values characterizing sound absorption in the TABL of the tube given: U_0 = 13.0 cm/s, y_0 = 593 μm, $f_{кр}$ = 1.74·10^{-4} Hz, $U_{0кр}$ = 7.70·10^{-4} m/s, J_{0min} = 1.23·10^{-4} Wt/m^2, f_{ky} = 4.94 kHz, f_{kh} = 3.54 kHz.

Fig. 1 shows the frequency dependence of the spatial attenuation coefficient (1) caused by the acoustic energy dissipation of the travelling acoustic wave in the ABL.

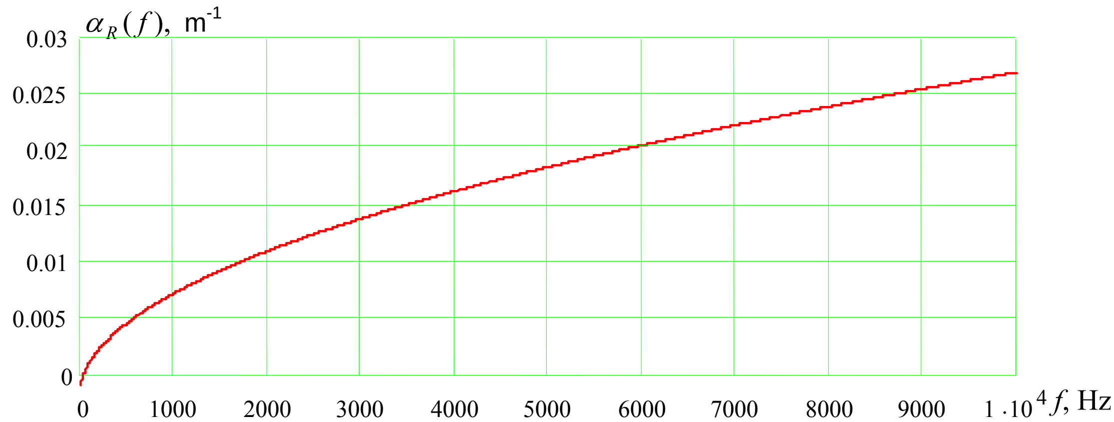

Fig. 1

Fig. 2 shows frequency dependences of spatial attenuation coefficients of a standing acoustic wave (12) : 1 - attenuation in the tube with smooth walls, 2 - attenuation in the tube with rough internal walls. The comparison of the results of calculations shows that at low frequencies sound absorption in the TABL more than two orders higher the dissipation of acoustic energy in the laminar

ABL. The presence of roughnesses on the tube internal surface results in increasing the sound absorption in the TABL.

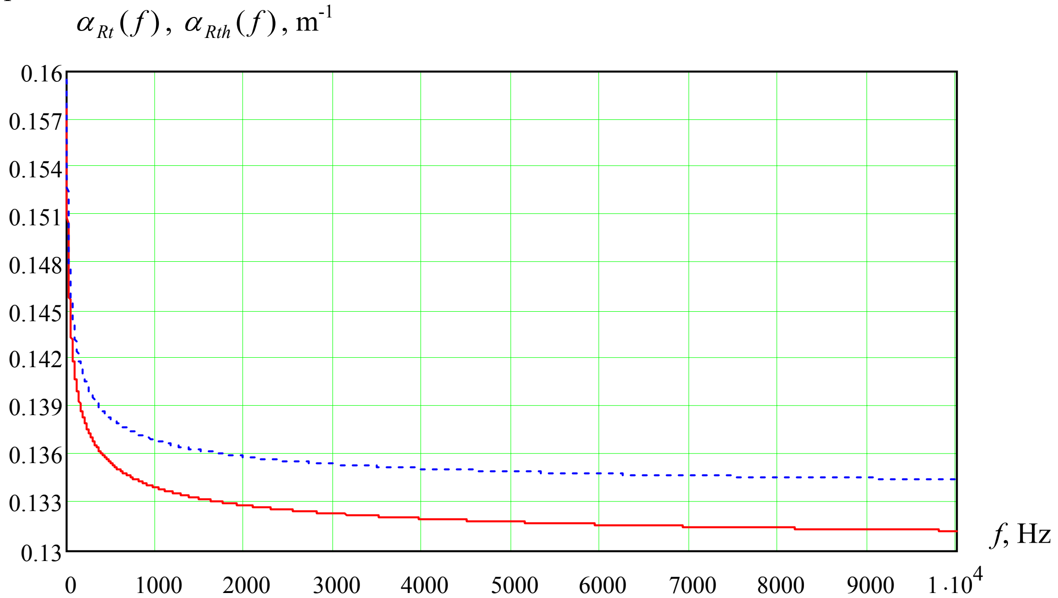

Fig. 2

The expressions obtained during the work are easy to generalize for a straight tube with an arbitrary cross-section. The acoustic field in the tube is normally the superposition of steady and unsteady fields. In this case the Kirchoff formula (1) and the expressions (12) should be used for calculating losses. It is obvious that the work results are to be verified experimentally. From our point of view the realization of such an experiment is quite feasible.

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