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MULTIPOLAR HUIGENCE SOURCES

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The thtree dimensional miniature Huigence source (HS) is investigated analytically. This source is characterized not only by anisotropy of radiated power (as usually) but by almost only omnidirectional radiation. HS considered is combined from two coaxial linear (in geometrical sense) multipolar sources of arbitrary order, phased in accordance with the distance between their centers. We estimate the dimensions of the space domain of HS's near field and its relative level as a function of dimensions of monopoles, distances between them and multipolarity order. Maximum cross section of incident wave absorption has been obtained for multipolar HS of arbitrary order. Estimates of radiation pressure and supporting forces, necessary for HS's elements (monopoles) are represented too. It is shown, that in the regime of absorption of incident plane wave multipolar HS of enough high order is equivalent of "black body"(BB). The last scatters (unlike multipole) only forward and not back. The equivalent BB is characterized by large absorption cross section and narrow directional pattern of scattering and absorption.

INTRODUCTION

Multipolar emitter of sound waves and many other waves (electromagnetic waves, water surface waves, bending waves in plates…) described by linear wave equation, presents classical object of investigation. Absorption characteristics of multipole we investigated too [1-6] as a special interaction of radiation, scattering and absorption effects of antennas. Huigence source based on multipoles [7] has't been considered yet, however it presents some significant aspects relating to multipoles in the radiation and absorption problems.

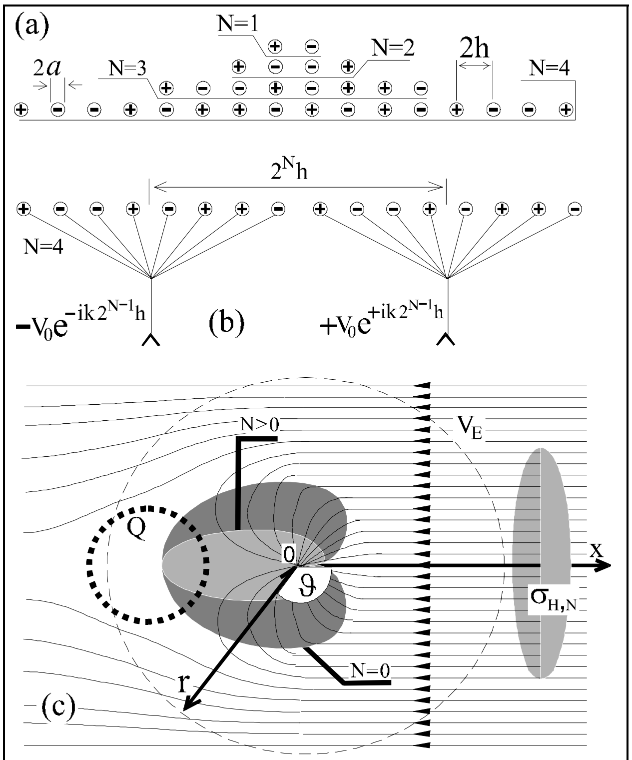

Fig. 1. The structure development of (a) multipole with growth of its order N. The structure of multipole Huigence source (b). The conversion of the field of power flux lines (c) of the plane incident wave caused by the resonant absorbing Huigence source (centered in the point $x = 0$) with radiation directivity patterns corresponding to $N = 0$ and $N > 0$. More absorption cross section causes more dimension of the area Q of relative "silence".

Linear multipole. As the multipolar HS consists of two phased linear multipoles let's consider at first characteristics of the only linear multipole to compare them later with cvoresponding characteristics of multipolar HS. Linear multipole of N-th order (Fig. 1-a) and length $L_M = 2(2^N - 1)h$ presents 2^N monopoles (pulsing spheres of radius), spaced on axis "x" with space period $2h$. Center of polar coordinates (r, ϑ) is in the center of multipole and axis "x" presents multipole axis. We will consider sound field on the sphere of radius r with center in the point $x = 0$ (Fig. 1-c). Sound field of linear multipole is combined from fields of pressure and radial velocity, produced by monopoles ($N = 0$) or the spheres of radius a, pulsing with complex amplitude V_0 of velocity and with the following form in far zone

$p_0 \approx +[i\omega\rho a^2 V_0 / r]\exp(-ikr)$, $\upsilon_0 \approx -[ika^2 V_0 / r]\exp(-ikr)$ (1)

For the monopole spaced in the point $x = 0$ and under condition $a << h << \lambda$, where $\lambda = 2\pi / k$ - sound wavelength, a k-wave number. In addition we assume, that complex amplitudes V_0 of pulsing monopoles velocities (forming the multipole) have the same module and do not depend of outside sound field. The sound pressure field of the N-th order multipole is equal to the difference of fields of two multipoles of ($N-1$)-th order in accordance with the relation

$$p_N(\vec{r}) = p_{N-1}(\vec{r} - 2^{N-1} h\vec{x}_0) - p_{N-1}(\vec{r} + 2^{N-1} h\vec{x}_0), \quad (2)$$

($\vec{x}_0$ unit vector along axis "x") which is satisfied in any observation point $\vec{r}$. Minus before the second term in (2) means the inversion of amplitudes of elements of corresponding ($N-1$)-th order multipole. Vector $\pm 2^{N-1} h\vec{x}_0$ means the distance from point $x = 0$ to the centers of ($N-1$)-th order multipoles (Fig.1-a). In far zone $r > r_N = 2^{2N-1}\pi^{-1}kh^2$ of multipole relation like (2) is satisfied for the radial particle velocity too. Complex oscillatory velocity amplitude presents some binary $\pm V_0$ distribution of complex amplitudes of pulsation velocities of monopoles with one complex parameter V_0. Signs of multipole's velocity amplitudes correspond to N-th finite difference model of derivative $(\partial^N / \partial x^N) \sim (ik)^N$ along axis "x". In far zone using polar coordinates distance r from center and polar angle ϑ with axis "x" we obtain

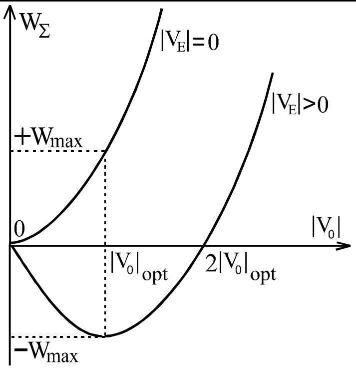

Fig. 2. Qualitative dependence radiated ($W_\Sigma > 0$) or absorbed ($W_\Sigma < 0$) by multipole or HS (or by sphere of radius r) power from the module $|V_0|$ of pulsation velocity of their elements at optimum pulsation phases $\arg(V_0)$ in the absence ($|V_E| = 0$) and in the presence ($|V_E| > 0$) of incident wave.

$p_{M,N}(r,\vartheta) = (ikh\cos\vartheta)^N 2^{\frac{N(N+3)}{2}} p_0(r)$, $\upsilon_{M,N}(r,\vartheta) = (ikh\cos\vartheta)^N 2^{\frac{N(N+3)}{2}} \upsilon_0(r)$. So the radiation power flux density $\Pi_{M,N}(r,\vartheta) = (1/2)\,\mathrm{Re}[p^*_{M,N}(r,\vartheta)\upsilon_{M,N}(r,\vartheta)]$ on the surface of sphere of radius r (Fig. 1-c) in the absence of incident wave is equal to $\Pi_{M,N}(r,\vartheta) = 2^{-1}\rho c(kh\cos\vartheta)^{2N} 2^{N(N+3)}|V_0|^2$ with radiation resistance $\mathrm{Re}\,Z_{M,N} = 4\pi a^4 k\omega\rho(kh)^{2N} 2^{N(N+3)}(2N+1)^{-1}$ and total radiation power $\overline{W}_{M,N} = 2^{-1}\rho c|V_0|^2\,\mathrm{Re}\,Z_{M,N}$. To ensure at the distance r from multipole the sound pressure (or velocity) the same as by one pulsing sphere at the same distance one needs pulsation amplitude in $|\upsilon_0|/|\upsilon_{M,N}| = (kh)^{-N} 2^{-N(N+3)/2}$ times larger than multipole amplitude. This determines the limitations of multipole dynamical range.

Now let's consider plane incident wave propagating along the axis "x" with velocity amplitude V_E and fields of pressure $p_E = \rho c V_E \exp(+ikr\cos\vartheta)$ and normal to sphere's surface velocity $\upsilon_E = V_E\cos\vartheta\;\exp(+ikr\cos\vartheta)$. Total sound power $W_{MN} = (1/2)\,\mathrm{Re}\oiint_S (\upsilon_{M,N} + \upsilon_E)^*(p_{M,N} + p_E)dS$ outcoming from sphere, embracing HS, is equal $W_{M,N} = \overline{W}_{M,N} + \overline{\overline{W}}_{M,N}$, where the term $\overline{\overline{W}}_{M,N}$ caused by interaction between HS and incident wave is equal

$$\overline{\overline{W}}_{M,N} = +\pi r a^2 \omega\rho(kh)^N 2^{\frac{N(N+3)}{2}}\,\mathrm{Re}(+q_{M,N} I_{M,N} - q^*_{M,N}\bar{I}_{M,N}), \quad (3)$$

where $q_{M,N} = V_E^* V_0\; i^{N+1}\exp(-ikr)$,

$$I_{M,N} = \int_0^\pi \exp(-ikr\cos\vartheta)\;(\cos\vartheta)^N \sin\vartheta d\vartheta, \quad \bar{I}_{M,N} = \int_0^\pi \exp(+ikr\cos\vartheta)\;(\cos\vartheta)^{N+1}\sin\vartheta d\vartheta.$$

From (3) we obtain that maximum absorption cross section $\sigma_{M,N} = W_{M,N} / S_E$ (where $S_E = \rho c |V_E|^2 / 2$ -the power flux density in the incident wave) of linear multipole is equal to

$$\sigma_{M,N} = (2N+1)(\lambda^2 / 4\pi), \tag{4}$$

where $\lambda^2 / 4\pi$ - resonant absorption cross section of lonely monopole in free space, and $\sigma_{M,N}$ is achieved at pulsation velocity module $|V_{0,M}| = (2N+1)a^{-2}h^{-N}k^{-N-2}2^{-\left(1+\frac{3}{2}N+\frac{1}{2}N^2\right)}|V_E|$ and velocity phase satisfying the condition $\arg(V_E) - \arg(V_{0,M}) = N(\pi/2)$, т.е. $V_0 = (V_0)_{opt}$.

Total power flux $\overline{W}_{M,N} = W_{BM,N} + W_{FM,N}$ of scattering (radiation) of multipole consists of integrated power flux $\overline{W}_{BM,N}$ into the back half-space and integrated power flux $W_{FM,N}$ into forward half-space. So we have $\overline{W}_{BM,N} = \overline{W}_{FM,N}$ and $\overline{W}_{FM,N} = \overline{W}_{BM,N} = W_\Sigma / 2$. Fig. 2 presents the dependencies of power flux $W_\Sigma = W_{M,N}$, outcoming some sphere, which embraces HS, in the absence and in the presence of incident wave. It is shown, that it the regime of maximum absorption at $V_0 = (V_0)_{opt}$ power flux absorbed is equal to power flux scattered.

Multipole HS. Now we consider HS, formed from two linear multipoles (Fig. 1-b) of length $L_H = 2L_M + 2h$. In th e case of HS the number N means the order of multipoles forming HS. So the simplest two element HS has the order $N = 0$. The sound pressure and radial particle velocity produced by HS in far zone have the following form:

$$p_{H,N} = (2i)\sin[2^N kh(1-\cos\vartheta)]p_{M,N}, \qquad \upsilon_{H,N} = (2i)\sin[2^N kh(1-\cos\vartheta)]\upsilon_{M,N}.$$

At $2^N kh << 1$ we get distribution $\Pi_{H,N}(r,\vartheta) = (1/2)\mathrm{Re}[p^*_{H,N}(r,\vartheta)\upsilon_{H,N}(r,\vartheta)]$ of power flux density in the form $\Pi_{H,N}(r,\vartheta) = 2(kh)^{2(N+1)}2^{N(N+5)}a^4\omega\rho k r^{-2}(1-\cos\vartheta)^2\cos^{2N}\vartheta|V_0|^2$. Radiation power of HS in the absence of incident waves is equal to $\overline{W}_{H,N} = (1/2)\mathrm{Re}\oiint_S (p^*_H \upsilon_H)dS$ or $\overline{W}_{H,N} = 2^5\pi(kh)^{2(N+1)}2^{N(N+5)}a^4\omega\rho k(N+1)(2N+1)^{-1}((2N+3)^{-1}|V_0|^2$ with HS's radiation resistance $\mathrm{Re}\,Z_{H,N} = 2\overline{W}_{H,N}\rho^{-1}c^{-1}|V_0|^{-2}$. The case of one side radiating HS we are interested in the value of radiation pressure (constant in time) too:

$$F_x = \frac{2\pi r^2}{c}\left[\int_0^{\pi/2}\Pi_{H,N}(r,\vartheta)\cos\vartheta\sin\vartheta d\vartheta - \int_{\pi/2}^{\pi}\Pi_{H,N}(r,\vartheta)\cos\vartheta\sin\vartheta d\vartheta\right](r\to\infty),$$

with module $|F_x| = (16\pi/c)(kh)^{2(N+1)}2^{N(N+5)}(2N+2)^{-1}(2N+3)^{-1}(2N+4)^{-1}a^4\omega\rho k|V_0|^2$.

Now we consider plane incident wave propagating along HS's axis (Fig. 1-c). Total sound power outcoming the sphere which embrace HS, is iqual to $W_{H,N} = \overline{W}_{H,N} + \overline{\overline{W}}_{H,N}$, where $\overline{\overline{W}}_{H,N}$ caused by interaction between HS and incident wave, and has the form

$$\overline{\overline{W}}_{H,N} = 2\pi r(kh)^{N+1}2^{\frac{N(N+5)}{2}}a^2\omega\rho\,\mathrm{Re}\left[+q_{H,N}I_{H,N} - q^*_{H,N}\bar{I}_{H,N}\right], \tag{5}$$

where $I_{H,N} = \int_0^\pi \exp(+ikr\cos\vartheta)\ (\cos\vartheta)^N(1-\cos\vartheta)\sin\vartheta d\vartheta$,

$\bar{I}_{H,N} = \int_0^\pi \exp(-ikr\cos\vartheta)\ (\cos\vartheta)^{N+1}(1-\cos\vartheta)\sin\vartheta d\vartheta$, $q_{H,N} = V^*_E V_0\ i^{N+2}\exp(-ikr)$.

Maximum absorption cross section of linear multipolar HS is equal to

$$\sigma_{HN} = (2N+1)(2N+3)(N+1)^{-1} \ (\lambda^2 / 4\pi) \quad (6)$$

and can be achieved at velocity module

$$|V_{0H}| = (2N+1)(2N+3)(N+1)^{-1} a^{-2} h^{-N-1} k^{-N-3} 2^{-\left(3+\frac{5}{2}N+\frac{1}{2}N^2\right)} |V_E|$$

and velocity phase satisfying the condition

$\arg(V_E) - \arg(V_{0H}) = (-1)^N (\pi/2)$, т.е. $V_0 = (V_0)_{opt}$.

Total power flux $\overline{W}_{H,N} = W_{BH,N} + W_{FH,N}$ of scattering (radiation) of HS consists of integrated power flux $W_{BH,N} = 2\pi r^2 \int_0^{\pi/2} \Pi_{H,N}(r,\vartheta)\sin\vartheta d\vartheta$ into the back half-space and integrated power flux $W_{FH,N} = 2\pi r^2 \int_{\pi/2}^{\pi} \Pi_{H,N}(r,\vartheta)\sin\vartheta d\vartheta$ into forward half-space, which are related to each other as $W_{H,F} / W_{H,B} = 8N^2 + 16N + 7$ at the multipole HS. Fig. 2 presents the dependencies of power flux $W_\Sigma = W_{M,N}$, outcoming the sphere embracing HS, in the absence and in the presence of incident wave. It is shown, that it the regime of maximum absorption at $V_0 = (V_0)_{opt}$ power flux absorbed is equal to power flux scattered and the last all is scattered forward, i.e. $\overline{W}_{FH,N} = W_\Sigma$ at $N >> 1$.

CONCLUSIONS

One can see from (4) and (6), that growth of multipolarity order N leads the increasing of absorption cross-section proportionally N. Note that unlike multipole HS has't backscattering. So such a HS can be interpreted as some acoustical black body [8] of finite dimensions $\sim N\lambda$, despite its small wave dimension $kL_H << 1$.

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