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THE SELF-REFRACTION OF SHOCK WAVES IN THE INHOMOGENEOUS MEDIUM

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In work the method of the stretched characteristics is applied to the description of intensive acoustic waves in weak inhomogeneous media. This method allows finding asymptotic representation for ray trajectories in the inhomogeneous medium for waves with the arbitrary initial form of wave front. The given method allows calculating also simultaneous influence of medium inhomogeneity and effects of a nonlinear self-refraction on evolution of shock waves. It is found the asymptotic solution for an intensive wave propagating in the inhomogeneous medium with an arbitrary initial time structure and evolution of the perturbations containing shock fronts is described. In particular, distribution focused Gaussian beam is considered and shown, that medium inhomogeneity and a self-refraction can essentially change localization of focal area in comparison with the focus set by initial focusing. Especially a bright example of influence of medium inhomogeneity on spatial structure of an intensive wave is the opportunity of formation of secondary maxima behind focus of initially focused beam.

Dependence of speed of acoustic shock waves on their amplitude leads to the curvature of initial wave front of the beam – to the beam self-refraction [1,2,3]. The medium inhomogeneity concerned with distinction of local sound speed from its average value also distorts wave front. Change of the wave front form leads to occurrence of focusing and defocusing areas, i.e. to change of wave amplitude characteristics and occurrence of areas of a high pressure. Analytical solutions for the focused beam amplitude on its axis in a homogeneous medium being based on formulated in [2] equations are received in works [3,4].

In the given work joint influence of the self-refraction effects and the refraction on inhomogeneities on the evolution of intensive acoustic waves with discontinuities is investigated. We use system of the equations of nonlinear geometrical acoustics for axessymmetrical beams where the wave self-refraction [2] and medium inhomogeneity [5] are taking into account:

$$\frac{\partial\alpha}{\partial z}+\alpha\frac{\partial\alpha}{\partial r}=-\frac{\mu}{2}\frac{\partial A}{\partial r}+\varsigma(r,z)\quad\text{(a)},\quad \frac{\partial p}{\partial z}-\gamma\left(p-\frac{A}{2}\right)\frac{\partial p}{\partial T}+\alpha\frac{\partial p}{\partial r}+\frac{p}{2}\left(\frac{\partial\alpha}{\partial r}+\frac{\alpha}{r}\right)=0\quad\text{(b)}. \qquad (1)$$

The eikonal equation (1a) is written for function of the ray inclination that is the derivative of eikonal on the transversal coordinate [5], function $\varsigma(r,z)$ is connected with a deviation of local speed of a sound from average value, p – acoustic pressure and A – its amplitude. All variables in (1) are normalized on the its characteristic values. The parameter γ is equal to the attitude of a focal length to nonlinear length. The parameter μ – to the attitude of a square of a focal length to nonlinear and diffractive lengths and in the most interesting cases from the point of view of practice appears in small size [3].

The eikonal equation (1) describes a curvature of the ray trajectories due to a self-refraction and inhomogeneous media. Without these factors the ray trajectory is a straight line with a constant initial inclination. At relative weakness of perturbation effects the ray trajectories will be close to non perturbed ones. However even small deviations of trajectories lead to restriction of amplitude in focus. At small value of the right hand of (1a) it is possible to search for the solution by the stretched characteristics method in the form of asymptotic series [3,4]

$$\alpha=\alpha_0(\xi)+\mu\alpha^{(1)}+\dots,\ r=r^{(0)}(z,\xi)+\mu r^{(1)}(z,\xi)+\dots, \qquad (2)$$

where $\alpha_0(\xi)$ – the implicit solution of non perturbed equation and ξ – the unknown characteristic of the eikonal equation (1a). In the zero order of the perturbation theory the characteristic ξ of non perturbed equation is defined by the following expression $r^{(0)}=\xi+\alpha_0(\xi)z$. In the first order one can receive:

$$\frac{\partial \alpha^{(1)}}{\partial z} = -\frac{\mu}{2}\frac{1}{r_\xi}\frac{\partial A(\xi,z)}{\partial \xi} + \mu\varsigma\left(r^{(0)}(z,\xi),z\right),\ \frac{dr^{(1)}}{dz} = \alpha^{(1)}, \quad (3)$$

$r_\xi = \partial r/\partial \xi$ – Jacobian or convergence of rays. Series (2) represent the parametrical form of the solution, and the magnitude ξ is the parameter of equation and is equal to initial transversal coordinate of the ray. The solution of the eikonal equation with accuracy of the order μ looks like:

$$\alpha = \alpha_0(\xi) - \frac{\mu}{2}\int_0^z \frac{1}{r_\xi}\frac{\partial A(\xi,\eta')}{\partial \xi}d\eta' + \mu\int_0^z \varsigma(\xi + \alpha_0(\xi)\eta', \eta')d\eta', \quad (4)$$

$$r = \xi + \alpha_0(\xi)z - \frac{\mu}{2}\int_0^z d\eta''\int_0^{\eta''} \frac{1}{r_\xi}\frac{\partial A(\xi,\eta')}{\partial \xi}d\eta' + \mu\int_0^z d\eta''\int_0^{\eta'} \varsigma(\xi + \alpha_0(\xi)\eta', \eta')d\eta'. \quad (5)$$

Expression (5) defines the ray trajectory, i.e. transversal coordinate r on distance z of the ray which has left a point $r(z=0) = \xi$. The ray moves due to initial modulation of wave front, the self-refraction and inhomogeneity. In last items in (4) and (5) it is visible, that values of media inhomogeneity compute along non perturbed ray trajectory beam. The transport equation (1b) in the ray coordinates z,ξ looks like:

$$\frac{\partial p}{\partial z} - \gamma\left(p - \frac{A}{2}\right)\frac{\partial p}{\partial T} + \frac{p}{2}\left(\frac{1}{r_\xi}\frac{\partial \alpha}{\partial \xi} + \frac{\alpha}{r}\right) = 0,\ \ \frac{1}{r_\xi}\frac{\partial \alpha}{\partial \xi} + \frac{\alpha}{r} = \frac{d}{dz}\ln r_\xi + \frac{d}{dz}\ln r = \frac{d}{dz}\ln\frac{r r_\xi}{\xi} \equiv \frac{d}{dz}\ln\Delta, \quad (6)$$

$$\Delta = \left(1 + \frac{\alpha_0}{\xi}z - \frac{\mu}{2}\int_0^z d\eta''\int_0^{\eta''}\frac{1}{\xi}\frac{\partial A}{\partial \xi}\frac{d\eta'}{r_\xi} + \mu\int_0^z d\eta''\int_0^{\eta'}\frac{\varsigma}{\xi}d\eta'\right)\left(1 + \alpha_0' z - \frac{\mu}{2}\int_0^z d\eta''\int_0^{\eta''}\frac{\partial^2 A}{\partial \xi^2}\frac{d\eta'}{r_\xi} + \mu\int_0^z d\eta''\int_0^{\eta'}\frac{\partial \varsigma}{\partial \xi}d\eta'\right). \quad (7)$$

Here characteristic width of a ray tube [5] Δ is normalized on ξ so that $\Delta(z=0) = 1$.

The peak amplitude of a single N-wave (a boundary condition $p(z=0) = R(r)p_0(\tau)$, $p_0(\tau) = -\tau$ at $|\tau| < 1$ and $p_0(\tau) = 0$ at $|\tau| > 1$), containing shock front, is equal:

$$A = \frac{R(\xi)}{\sqrt{\Delta}}\left(1 + \gamma R(\xi)\int_0^z \frac{d\eta}{\sqrt{\Delta}}\right)^{-1/2}. \quad (8)$$

The solution is given by the expressions connected with each other for wave amplitude (8) and the ray tube width (7) for the fixed ray, and also the ray trajectory (5). To find the explicit solution we set the initial gaussian focused beam with the initial ray inclination $\alpha_0 = -\xi$ and the transversal form $R = \exp(-\xi^2)$. The greatest interest represents dependence of the wave amplitude on the beam axis. The certain difficulty in the this approach consists in correlation of the cartesian r and ray ξ coordinates from expression (5). As it is shown from (5), in a homogeneous media the ray leaving the beam center with coordinate $r(z=0) = \xi = 0$ remains on the beam axis. Presence of media inhomogeneity can change this result, and on various distances the initial beam axis will be crossed with various rays. This is the one of the major influences of heterogeneity – the wave field can extend not along an initial direction of propagation, but to be displaced in a transversal direction, for example, the focal area can essentially move in any direction in comparison with position in a homogeneous medium. Nevertheless, the joint solution of the equations (7) and (8) allows to construct ray trajectories and to calculate amplitude along each of them, i.e. to construct a full structure of a field.

Let's consider a field along an initial axial ray $\xi = 0$. In a homogeneous medium we get $r/\xi = r_\xi$ on the beam axes, but inhomogeneity breaks this symmetry. Notice, that $\varsigma/\xi = 0$ at $\xi \to 0$ because it is always possible to choose the sound speed on an beam axis at $z = 0$ as average sound speed. The equation for ray convergence on an axis $r_\xi \equiv Q$, with account only items of the order μ and expression $r/\xi = Q + \mu\varphi$, $\varphi = \int_0^z d\eta''\int_0^{\eta'} \varsigma/\xi\, d\eta' - \int_0^z d\eta''\int_0^{\eta'} \partial\varsigma/\partial\xi\, d\eta'$, looks like:

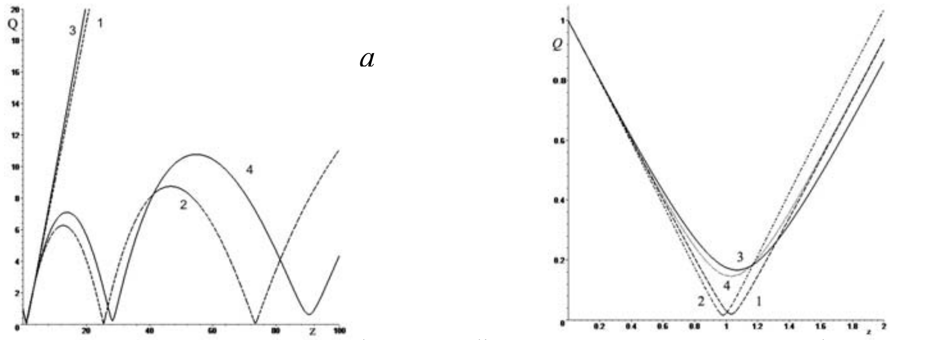

Fig. 1. Dependence of the ray convergence beams on distance. $\mu = 0{,}01$ – curves 1 and 2, $\mu = 0{,}1$ – curves 3 and 4. The homogeneous medium – curves 1 and 3, the inhomogeneous medium – curves 2 and 4. *a* – large interval of distances, *b* – near the geometrical focus of the initial focused wave

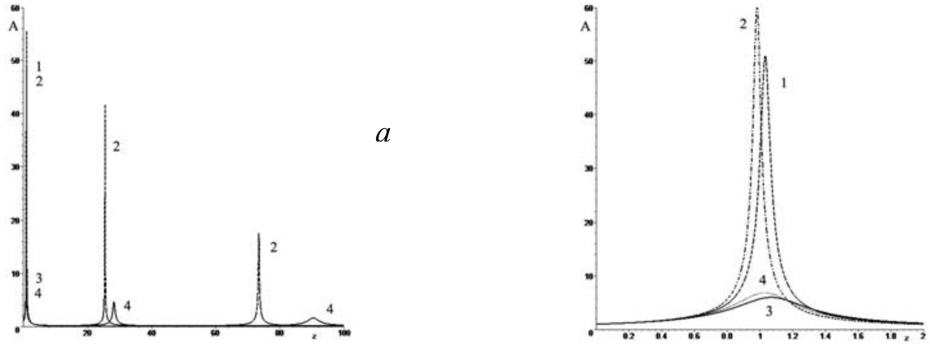

Fig.2.Dependence of peak amplitude on distance. Notation corresponds to Fig. 1

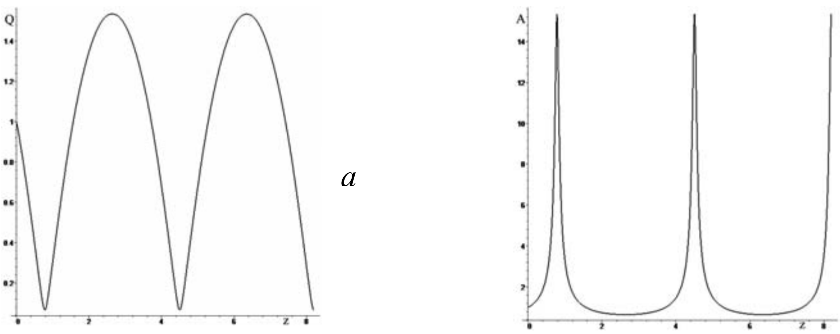

Fig. 3. Dependence on distance of the wave characteristics in a limit of infinitely slowly decreasing inhomogeneity: *a* – ray convergence, *b* – peak amplitude

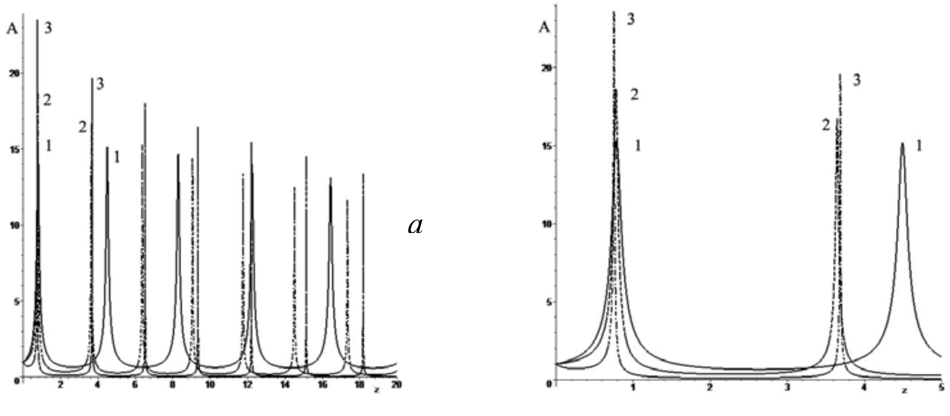

Fig. 4. Dependence of peak amplitude on distance for various values of parameter $\gamma = 0;1;10$ (curves 1-3 accordingly). *a* – large interval of distances, *b* – near the geometrical focus of the initial focused wave

$$\frac{d^2Q}{dz^2} = \frac{\mu}{2Q^2\sqrt{1+\mu\varphi/Q}}\left(\frac{1}{\sqrt{1+\gamma s}} + \frac{1}{(1+\gamma s)^{3/2}}\right) + \mu\frac{\partial\varsigma}{\partial\xi}, \quad Q\big|_{z=0} = 1, \quad \frac{dQ}{dz}\bigg|_{z=0} = -1. \quad s = \int_0^z \frac{dz'}{Q(z')}. \quad (9)$$

Let's set the function ς corresponding to the inhomogeneity with parabolic transversal distribution $\varsigma = r\nu(z)$ (so $\varphi = 0$) which leads to additional focusing or defocusing and to change of the focus position, but leaves it on an initial beam axis. Let $\nu(z) = -\left(1 + z^2/z_0^2\right)^{-1}$ – media inhomogeneity decreasing with distance. The sign "minus" means, that heterogeneity leads to focusing influence in a whole. In the beginning let's consider a case $\gamma \to 0$, i.e. we neglect the nonlinear attenuation of a wave. On fig. 1 dependences of ray convergence Q on distance, and on fig. 2 – dependences of peak amplitude are resulted. Curves 1 and 2 correspond to parameter $\mu = 0.01$, 3 and 4 – $\mu = 0.1$, curves 1 and 3 – a homogeneous environment, 2 and 4 – inhomogeneous. In a homogeneous media the parameter μ influences mainly "acuteness" and size of the ray convergence. The self-refraction leads to shift of position of a amplitude maximum afar from focus, after focus the amplitude quickly enough decreases. Presence of inhomogeneity essentially changes this behavior. In case of slowly enough decreasing inhomogeneity (the big parameter z_0), after passage of the basic focus rays start to divergate as in the usual focused beams. However then focusing inhomogeneity can cause formation of secondary maxima of amplitude, and amplitude in these maxima, their quantity and an arrangement are defined substantially by a kind of the inhomogeneity. Comparison of dependences for various parameters of a self-refraction μ shows, that reduction of this parameter leads to approach of secondary maxima and increase in their amplitude.

The equations (9) in a limit of infinitely slowly decreasing inhomogeneity ($z_0 \to \infty$) are solved precisely. The first integral is equal $dQ/dz = \pm\sqrt{-2Q - 2\mu/Q + 3 + 2\mu}$, and the solution consists of two branches. Feature of the given solution is the presence of two turning points at $dQ/dz = 0$: $Q_{пов} = 3/4 + \mu/2 \pm \sqrt{(2\mu - 1)^2 + 8}/4$. It means, that the ray convergence oscillates between the minimal and maximal values, and, accordingly, the peak structure consists of infinite number of secondary maxima of equal amplitude. The obtained solution is illustrated on fig. 3.

The Fig. 4 illustrates change of peak amplitude at increase in parameter $\gamma = 0;1;10$ (curves 1-3 accordingly), connected with nonlinear attenuation. It is visible, that amplitudes of secondary maxima gradually decrease, and at greater γ is faster, than at small. At the same time the amplitude of maxima at greater γ appears essentially more, than at small, and secondary maxima are located more close. These effects may be explained to that the given inhomogeneity has focusing character, and at the same time the self-refraction aspires to spread focal area and to reduce peak amplitude. The increase in parameter γ reduces a role of a self-refraction and promotes focusing on heterogeneity.

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