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ESTIMATIONS OF POSSIBLE EFFECT OF INTRINSIC VELOCITY DISPERSION ON THE RESULTS OF MATERIALS ELASTIC MODULI DETERMINATION

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Kjartansson constant Q model was used to show that intrinsic dispersion of seismic velocities in absorbing media is one of the main reasons leading to discrepancies of the rock elastic parameters measured by dynamic and static methods. Predicted by this model dispersion of Young's modulus is in a good agreement with the experimental data obtained for the model material (polyvinyl chloride plastic).

It is experimentally determined that for many rocks dynamic elastic moduli are larger than static ones, the difference can reach hundreds percent [1, 2]. These differences mostly are explained as effect of porosity and fracturing that lead to nonlinear behavior of rocks already under small stresses, as a result the local values of elastic moduli for the different stress ranges do not coincide. The influence of other factor, namely intrinsic dispersion of the seismic velocities in the absorbing media, on the results of elastic properties determination by static and dynamic methods is analyzed below.

The inevitability of intrinsic velocity dispersion in the absorbing medium (and it is well known that all rocks absorb energy of elastic waves to a greater or lesser extent) is determined by the causality principle [3]. Numerous published experimental data indicate that the attenuation decrements θ of seismic body waves for the most of crystalline rocks practically do not change with frequency variation from parts of hertz to a few megahertz [4]. The data about intrinsic dispersion of seismic velocities [2, 5] show that it has anomalous character, i.e., increase of the frequency leads to nonlinear velocity growth with gradually decreasing gradient.

Such character of dispersion is described well by many phenomenological models of attenuation, for example, by the models of Kolsky, Lomnitz, Futterman and others [6]. In this work, the analysis is based on the model of the frequency-independent attenuation of Kjartansson [7] which is characterized by the properties of both linearity and causality. In the framework of this model with the frequency independent attenuation decrement θ and under the assumption of its smallness ($\theta \ll \pi$) expression for the velocity dispersion can be written as

$$c(f) \approx c_0 \left(\frac{f}{f_0} \right)^{\frac{\theta}{\pi^2}}, \tag{1}$$

where $c(f)$ is the phase velocity of elastic wave depending on the frequency f; c_0 is the velocity at the arbitrarily reference frequency f_0.

Under the assumption that the velocity dispersion of body waves in the medium is described by the function (1), it is possible using the well-known from the elasticity theory expressions, connecting elastic moduli with the velocities of longitudinal *P*- and shear *S*-waves, to obtain expressions for the relation of elastic moduli at two frequencies. For the shear modulus μ this relation is

$$\frac{\mu(f)}{\mu(f_0)} = \left(\frac{f}{f_0} \right)^{\frac{2\theta_s}{\pi^2}}, \tag{2}$$

where θ_s is the attenuation decrement of *S*-waves.

For the real rocks the exponent of power function in (2) is lower than unity, therefore the shear modulus μ decreases with the increasing gradient when frequency f is reduced. At the very small frequencies, which correspond to the quasi-static conditions of deformation, arbitrarily small values of μ can be obtained in the framework of Kjartansson model. For the elastic medium ($\theta_s = 0$), as it follows from (2), the modulus μ becomes frequency-independent.

Analogous relations for Young's modulus E and bulk modulus K at two frequencies are described by the more complex expressions (they are not given here), which depend on the attenuation decrements of *P*- and *S*-waves θ_p and θ_s and on the ratio of the velocities of these waves c_{0s}/c_{0p} at the reference frequency f_0. Nevertheless, the analysis of these expressions showed that according to the model of Kjartansson these moduli also tend to zero with the increasing gradient in the low-frequency limit, and in the elastic medium they become frequency-independent. In the special case of equal attenuation decrements of *P*- and *S*-waves ($\theta_p = \theta_s = \theta$) expressions for E and K take on the same form as expression (2):

$$\frac{E(f)}{E(f_0)} = \left(\frac{f}{f_0}\right)^{\frac{2\theta}{\pi^2}}, \tag{3}$$

$$\frac{K(f)}{K(f_0)} = \left(\frac{f}{f_0}\right)^{\frac{2\theta}{\pi^2}}. \tag{4}$$

Fig. 1*a* illustrates the character of elastic moduli change with frequency for the attenuation decrements typical for crystalline rocks. Since expressions (2)-(4) have an identical form, parameter M in the figure is equivalent to any of three moduli μ, E or K (for the last two moduli - in the case of equal attenuation decrements of *P*- and *S*-waves).

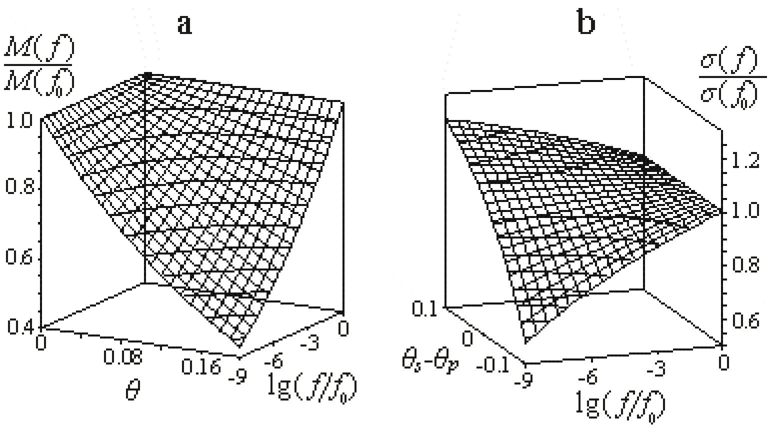

Fig. 1. Dependence of elastic moduli (a) and Poisson ratio (b) on frequency and on attenuation decrements for Kjartansson model.

Since the reference frequency f_0 in the model of Kjartansson can be chosen arbitrarily, it is possible to consider it as corresponding to the frequency of dynamic moduli measurement. Then a frequency reduction by nine orders of magnitude (see Fig. 1*a*) corresponds to transfer from the megahertz range, in which velocities of body waves are measured usually under laboratory conditions, to the millihertz (quasi-static) frequencies. As one can see from Fig. 1*a*, for this frequency changing the model of Kjartansson predicts reduction of dynamic elastic moduli almost to 60% for the materials with the attenuation decrements about 0.2.

Let us examine how intrinsic dispersion affects on one more elastic parameter - Poisson ratio σ. After substitution expression (1) into well-known from the theory of elasticity expression connecting Poisson ratio with velocities of *P*- and *S*-waves we will get

$$\frac{\sigma(f)}{\sigma(f_0)} = \frac{\left(1 - 2\gamma_0^2 \xi^{\frac{2\Delta}{\pi^2}}\right)\left(1-\gamma_0^2\right)}{\left(1 - \gamma_0^2 \xi^{\frac{2\Delta}{\pi^2}}\right)\left(1-2\gamma_0^2\right)}, \tag{5}$$

where $\xi = f/f_0$; $\Delta = \theta_s - \theta_p$; $\gamma_0 = c_{0s}/c_{0p}$ is the ratio of velocities *P*- and *S*-waves at the reference frequency f_0.

As it follows from expression (5), in contrast to the elastic moduli μ, E and K, Poisson ratio becomes frequency-independent for the equal attenuation decrements *P*- and *S*-waves ($\Delta = 0$), as in the ideal elasticity. Fig. 1*b* illustrates character of the change σ with the frequency for $\gamma_0 = 0.5$ and different values $\Delta = \theta_s - \theta_p$. As one can see from the figure, a change in the frequency by nine orders of magnitude also can lead, according to the model of Kjartansson, to a change in Poisson ratio on few tens percent for the difference of decrements of *P*- the *S*-waves about 0.1.

A principal difference between dependence in this figure and dependences for the elastic moduli consists in the fact that frequency reduction may lead both to decreasing and to increasing of dynamic Poisson ratio, depending on $\theta_p > \theta_s$ or vice versa. This feature agrees with the experimental data [2], according to which, in contrast to the elastic moduli, static Poisson ratio for different rocks may be both smaller and greater then dynamic one.

The experimental estimation of possible effect of intrinsic velocity dispersion on the results of elastic moduli measurements will be obtained below using data about discrepancy of dynamic and static Young's moduli of polyvinyl chloride plastic (we will call it plastic) [8] and about extensional velocity dispersion in the same material [9].

Static Young's modulus of plastic was determined from diagrams stress-strain $\tau(\varepsilon)$ obtained as a result of the sample tensile tests on the press. Diagrams $\tau(\varepsilon)$ were determined by standard method, i.e., under the assumption about the uniaxial stress state. The magnitude of static Young's modulus of plastic $E_s = 2.6 \pm 0.09$ GPa was found in accordance with Hooke's law in the linear range (bellow elastic limit $\tau_e \approx 8$ MPa) as average for several diagrams $\tau(\varepsilon)$.

Dynamic Young's modulus E_d of plastic was calculated using its density $\rho = 1.5 \pm 0.07$ g/cm^3 and measured at the frequency 20 kHz extensional velocity $c_b = 1746 \pm 2$ m/s according to known from the elasticity theory expression

$$E_d = \rho c_b^2, \qquad (6)$$

its value is $E_d = 4.57 \pm 0.23$ GPa. Thus, the ratio of Young's moduli of plastic measured by static and dynamic methods is equal $E_s / E_d = 0.57$.

The experimental data about intrinsic dispersion of the extensional wave in the thin bar that was made from the same plastic sheet as model for the tensile tests on the press are given in the paper [9]. Note that wavelengths in the investigated frequency range exceeded the bar diameter as minimum by the order, so the geometric dispersion practically was absent. The experimental results showed that attenuation decrement of the extensional wave in the plastic thin bar practically does not depend on the frequency (average value of decrement in the range 8-23 kHz $\theta_b = 0.154$), and extensional velocity dispersion has anomalous character, i.e., velocity decreases with reduction of frequency with the gradually increasing gradient, which is in good agreement with the model of Kjartansson.

Fig. 2*a* shows the experimental dependence on the frequency of Young's modulus of plastic, obtained according to expression (6) with the using of data about the phase extensional velocity at the different frequencies from the work [9]. Solid line approximates the experimental dependence $E(f)$ by the least squares method according to valid for the special case $\theta_p = \theta_s$ function (3). Such approximation is completely correct, since the attenuation decrements of different types of waves in this plastic differ insignificantly [9].

Fig. 2*b* illustrates the relationship of experimental static Young's modulus with calculated from experimental extensional velocity dynamic Young's modulus in the ultrasonic frequencies and with approximating them according to the model of Kjartansson curve. Square markers on this curve note the values of Young's modulus for frequencies 0.2 and 0.002 Hz. The ratio of quasi-static moduli at these frequencies to dynamic modulus, measured at the frequency $f_0 = 20$ kHz, is equal $E(f)/E(f_0) = 0.7$ and $E(f)/E(f_0) = 0.6$, respectively. To the observed value $E_s / E_d = 0.57$, according to (3), the frequency about $f = 0.0003$ Hz corresponds for $f_0 = 20$ kHz.

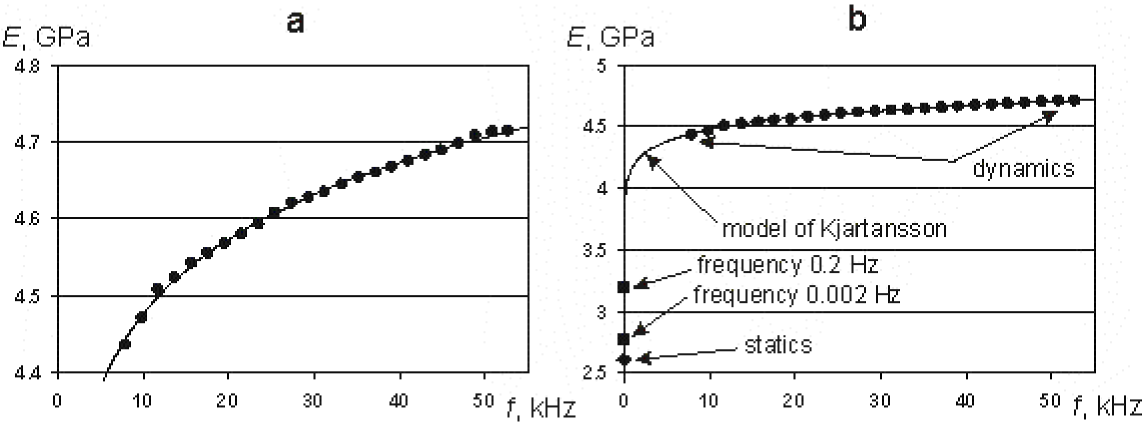

Fig. 2. Experimental dependence on the frequency (circles) of Young's modulus for polyvinyl chloride plastic and its approximation (solid line) according to the constant *Q* model of Kjartansson (a). The same data given on other scale (b) in the comparison with experimentally determined static Young's modulus; square markers noted the points of the approximating curve for two frequencies.

Thus, the represented results show that intrinsic velocities dispersion, connected with the energy absorption of elastic waves, is one (although not only) of the main reasons for experimentally observed differences of elastic parameters, measured by static and dynamic methods.

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