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SPECTRAL ANALYSIS OF INTERNAL WAVES COASTAL OCEAN

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Abstract. This paper studies the internal wave band of temperature fluctuation spectra in the coastal zone of the Japan Sea, in the $\Omega << \omega << N_*$ range, where N_* is the representative buoyancy frequency and Ω is the inertial frequency, the rate tends to $\sim \omega^{-3}$, and in the high-frequency band of temperature spectra the spectral exponent tends to $\sim \omega^{-1}$. These features of spectra are simulated by the model spectrum of nonlinear internal waves in the shallow water. Interaction of high-frequency internal waves with an internal wave of semidiurnal frequency is considered. It is shown that as a result of the interaction the spectrum of high-frequency internal waves take the universal form and the spectral exponent tends to ~ ω^{-1}.

Introduction. Internal gravity waves play a significant role in the dynamics of an oceanic coastal zone [5]. The spectrum represents one of the major characteristics of internal waves. It is used as a representative statistical description of the internal wave field in studies acoustic propagation [1]. As a result of the analysis of several time series of temperature and currents belonging to different areas of the World Ocean, Garrett and Munk (from here, *GM*) [4] have constructed the generalized spectrum of internal waves for the open ocean. It has permeated the literature and is used in ways that sometimes exceed its applicability. The analysis of some publications [2, 3, 7, 8] showed that the construction of a universal spectrum for internal waves on a shelf, similar to the *GM* spectrum, was not possible. Such spectrum should depend on fast disintegration of the main internal fluctuation (for example, semidiurnal) and redistributions of its energy to fluctuations of high frequencies. The shape also depends on the choice of measurement site on the shelf (i.e. on the location of thermocline depth relative to the surface and bottom) as well as other factors. Because it is impossible to construct a universal spectrum of internal waves in the coastal zone, which would be similar to the GM spectrum, some researchers have engaged in studies of processes that account for these spectral exponents and attempted to model them. This is, to some extent, the objective of our work.

The model. A spectral model of nonlinear internal gravity waves is developed. It is assumed that $H/\lambda << 1$, and $H/a << 1$, where H is the water depth, λ is a characteristic wave-length, a is a representative wave amplitude, and wave frequence meets the condition $\omega << N_*$, where N_* is a representative buoyancy frequency. The basic component of this theory is the simple wave equation. For the first most powerful mode of the small-amplitude internal waves the simple wave equation is written as

$$\partial\eta/\partial x - \beta\eta\,\partial\eta/\partial\theta = 0\,, \qquad (1)$$

where $\eta(t,x)$ is the vertical displacement of the pycnocline, x is a horizontal coordinate and $\theta = t - x/c$. The parameters β and c are coefficients of nonlinearity and the phase speed of long internal waves, respectively. Parameter of nonlinearity is determined as:

$$\beta = \left(3\int_{-H}^{0}\varphi_z^3 dz\right)\cdot\left(2c\int_{-H}^{0}\varphi_z^2 dz\right)^{-1}, \qquad (2)$$

where z is a vertical coordinate. The phase speed of a linear long wave c and the amplitude function of the wave mode $\varphi(z)$ are determined from the solution of the eigenvalue problem

$$d^2\varphi/dz^2 + c^{-2}N^2(z)\,\varphi = 0\,, \qquad (3)$$

$$\varphi(-H) = \varphi(0) = 0\,, \qquad (4)$$

and with the normalization

$$\varphi_{\max} = 1\,. \qquad (5)$$

Let us consider transformation of the intensity spectrum of shallow water internal wave's vertical displacements or the spectrum of those internal waves. We assume that the wave field $\eta(t,0)=$ $\eta(t)$ at the boundary $x = 0$ is statistically homogeneous and is described by the Gaussian statistics with a zero mean value and covariant function $B_0(\tau)=\sigma_0^2 R(\tau)$, where $\sigma_0^2 = B(0) = \overline{\eta}_0^2$ is the root-meansquare value of vertical displacements and R is the correlation coefficient.

We confine our analysis to the initial stage of waves evolution, which is characterized by the condition $x < x_*$, where $x_* = (\beta c^2 H / a\omega)H$ is the distance on which shock waves appear. Since the field at the boundary is statistically homogeneous, we obtain the following relation for the spectra of vertical displacements at the arbitrary distance $x < x_*$ following [2, 5]:

$$Sp_\eta(\omega;x) = (2\pi\beta\omega x)^{-2}\exp[-(\sigma_0\beta\omega x)^2]\int_{-\infty}^{\infty}\{\exp[B_0(\tau)(\beta\omega x)^2]-1\}\exp(i\omega\tau)\cdot d\tau \ . \qquad (6)$$

Let us consider the behavior of the spectrum as frequency increases. In this case, we can use the saddle-point method to estimate integral (6) and we get

$$Sp_\eta(\omega;x) \sim \omega^{-3}(\beta x)^{-3}\sqrt{2\pi/\nu}\,\exp[-(\sigma_0\beta\omega x)^2/2] \ . \qquad (7)$$

It follows from the obtained expression (7) that the spectrum of nonlinear shallow water internal waves for $\sigma_0\beta x << 1$ with $\omega\to\infty$ decreases according to a power law $Sp_\eta(\omega;x)\sim\omega^{-3}$. Thus, equation (6) makes it possible to conclude that the quadratic nonlinearity produces the spectral exponent ~ ω^{-3} (physically caused by energy transfer to higher frequencies). Such is indeed the case in the middle-frequency band of the spectrum at the Figure 3.

Tidal and/or inertial internal waves with small but finite amplitude and linear random waves (RW) with a typical frequency $\omega_* \sim N_*/2$ are a common feature of coastal areas. Let us consider interaction nonlinear internal wave with frequence Ω (tidal or inertial) and linear RW using asymptotic theory of evolution of the spectrum of internal waves described in terms of the simple wave equation (1). Let the vertical displacement of the pycnocline $\eta(x,t)$ at the boundary of the coastal area $x=0$ be the superposition of internal wave with frequence Ω and background linear RW with a typical frequency $\omega_* << \Omega$ and the variance $\sigma^2 = <\xi_0^2>$

$$\eta_0(t) = A\cos(\Omega t+\psi)+\xi_0(t)\,, \qquad (8)$$

where ψ - is a random phase with uniform distribution in the interval $[-\pi,+\pi]$.

The solution of equation (1) that satisfies the initial condition at $x=0$, is

$$\eta = \eta_0(\theta+\beta x\eta)\,. \qquad (9)$$

We confine our analysis to the wave evolution stage, which is characterized by conditions $x < x_T$, where $x_T = (\beta A\Omega)^{-1}$ and $x < x_W$, where $x_W = (\beta\sigma\omega_*)^{-1}$. In this stage the shock internal wave appears and it is not accompanied by generation of internal solitons. We introduce parameters $d_T = x/x_T$ and $d_W = x/x_W$, which determine the similarity between an internal shock-wave and internal wave and consider the case $d_T < 1$, $d_W < 1$. For this case spectral amplitude wave field η is

$$a(\omega,x) = (2\pi i\omega\beta x)^{-1}\int_{-\infty}^{+\infty}\{\exp[-i\omega\beta xA\cos(\Omega t+\psi)-i\omega\beta x\xi_0(t)]-1\}e^{i\omega t}dt\,. \qquad (10)$$

We shall consider the wave-tide interaction using formula (10). Consider the case $d_W << 1$, i.e., when the steepening of the waves is weak. Taking into account that $< a(\omega)a^*(\omega') >= S(\omega)\delta(\omega-\omega')$, $<\xi_0(t+\tau)\xi_0(t)>= B(\tau)$ we obtain the following relation for the spectra of vertical displacements at the arbitrary distance $x < x_*$

$$Sp_\eta(\omega,x) = \sum_{\substack{n=-\infty\\ n\neq 0}}^{\infty}\frac{J_n^2(XAn\Omega)\exp\{-(X\sigma n\Omega)^2\}}{(Xn\Omega)^2}\times\delta(\omega-n\Omega)+$$

$$+\sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{J_n^2(\omega XA)}{2\pi(\omega X)^2} \exp[-(\omega X\sigma)^2] \times \int_{-\infty}^{+\infty} \{\exp[B_0(s)(\omega X)^2]-1\} e^{i(\omega-n\Omega)s} ds\,, \qquad (11)$$

where $X = \beta x$

The first series in (11) is a superposition of tidal harmonics distorted by the interaction with the waves. The second series in the (11) is the spectrum of the linear random waves Sp_ξ distorted by the interaction with the tide. We will determine the spectrum of waves Sp_ξ, when $d_{TW} << 1$. For that, following [3] we obtain

$$Sp_\xi(\omega, x) \sim \frac{1}{(\pi\omega\beta x A)} \frac{1}{\Omega} \int_{-\infty}^{\infty} S_0(\omega)\, d\omega \qquad (12)$$

Based on this formula, we can observe that the high-frequency wave spectrum is non-homogeneous and has the asymptotic form $Sp_\xi \sim \omega^{-1}$. Parameters of the spectrum are full energy waves $\sim \sigma^2$, amplitude - A and frequency - Ω for low-frequency waves (for example, semidiurnal or inertial frequency).

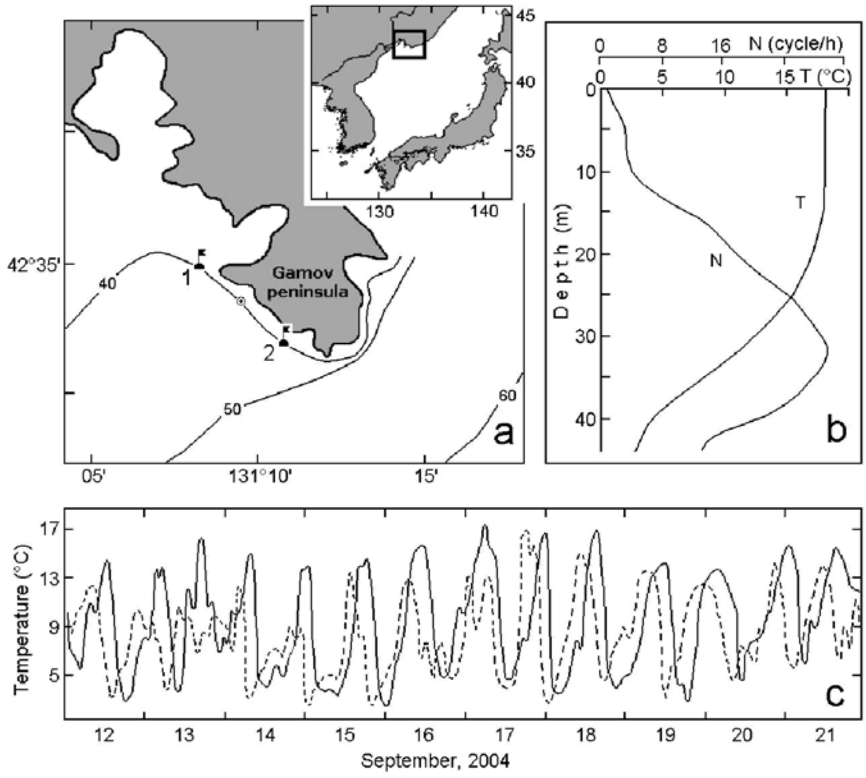

Fig. 1. (a) Study area on the Japan/East ocean shelf, September 2004. The mooring location is shown in Arabic. The circle indicates the location of the moored vessel, from which hourly casts were conducted on September, 20. (b) Daily mean vertical profiles of temperature *T* and buoyancy frequency *N*. (c) Temperature variations from the two moorings: buoy 1, continuous line; buoy 2, dashed line.

The model has been validated with the values observed on a shelf of the Japan Sea. In the first region, measurements were performed during 12 days starting September 12, 2004, near the Gamov Peninsula on the Japan Sea coast of Russia (Figure 1a). Time series of temperature were collected from two moorings which were deployed along the coastline at a distance of 800 m from it, approximately at 40 m depth and separated by a distance of 5.5 km from each other. The first of them was deployed at 28 m depth and the second one, at 35 m depth (below the surface). Each mooring was equipped with digital thermographs made by a Russian manufacturer. The devices had a measuring precision of 0.05°C for temperature. The sampling rate was 1 minute. Temperature and salinity vertical profiles were performed on September, 20-21 from a vessel anchored between the moorings with Canadian Guideline CTD profiler, whose errors were no larger than 0.01°C in temperature and 0.02 psu in salinity. In total, 25 hourly casts were made.

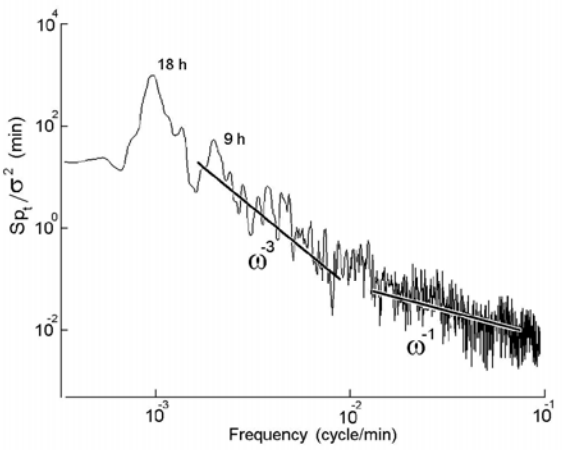

Fig. 3. Normalized spectra for the temperature fluctuations of the 35 m level (buoy 2, near the Gamov peninsula). The lines show dependences of the spectrum slope with growth of frequency.

Results and discussion. On Fig.1b mean profiles of temperature and buoyancy frequency in the survey area are shown. The time series of temperature fluctuations on moorings 1 and 2, smoothed by a Tukey window (with a 1-hour width) are shown in Figure 1c. It is clear that fluctuations of temperature had a quasi-periodic mode with an average period of about 18 hours. Cross spectral analysis of these time series has shown that a phase difference between fluctuations of inertial frequency was about 4.1 hours, correspon-ding to the mean velocity propagation between the moorings (close to 0.38 m/s). The phase velocity c_1 of the first mode of the internal Kelvin wave with frequency 1/18 cycles/h is close to 0.4 m/c. This value was calculated by means of the solution of the eigenvalue problem (3) and (4), considering the natural profile of the buoyancy frequency. The velocity was close to 0.38 m/s. We conclude that internal Kelvin waves propagated in the coastal zone of the Japan Sea. These waves had a large height and were caused by an intensive typhoon that had passed near the survey area.

The spectrum of temperature fluctuations normalized by variance on mooring 1 is shown in Fig.3. In the low-frequency part of the spectrum, there are peaks of spectral density at 18- and 9-hour periods, which correspond to internal waves with inertial period and first overtone. The figure shows good conformity between the inclination of a spectral exponent and frequency dependence $\sim\omega^{-1}$ in a range of $0.01 < \omega < 0.1$ cycle/min and the same conformity between character of the spectral exponent and frequency dependence □ ω^{-3} in a range of $0.001 < \omega < 0.01$ cycle/min. Features of spectra of internal waves has been emphasized in numerous works [3, 7, 8], in which the spectral exponents in different bands of internal waves have asymptotic values of ω^{-3} or ω^{-1}.

As a conclusion, in the ocean shelf in the presence of intensive internal tidal or inertial waves, the modeled spectrum of temperature variations in a high-frequency band is non-uniform and has slope$\sim\omega^{-1}$, as demonstrated in Fig. 3. Hence, the influence of nonlinearity can render essential influence on the formation of a universal spectrum of internal waves in a coastal zone of the ocean.

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