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**ESTIMATION OF RELIABILITY OF UNDERWATER NOISE MEASUREMENTS**

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*A method for estimation of reliability of signal amplitude estimation performed using least squares estimation (LSE) in the framework of linear regressive model under a given allowable relative error and with an allowable wrong decision probability is considered; the method uses straightforwardly the results of measurement to perform a prognosis of the estimate error dispersion. This general scheme is applied to the problem of measurement of underwater noise level of a moving source with the use of the method of energetic matched processing whose essence is similar to LSE. It has been shown that a bias of the dispersion estimate for the measured quantity does not play an important role when determining a threshold and can be corrected; a refinement of the threshold constant by means of stochastic simulation does not lead to its sufficient improvement in comparison with its preliminary estimate in the framework of Gaussian approximation. Thus, one can estimate the reliability of measurements of weak noise levels, which are lower than the level of ambient interference.*

**1. General problem formulation.** In many applications related to measurements, the procedure of signal processing comes down to the problem of linear regression, i.e., estimation of $L$ amplitudes $\theta_l$ of "useful" signals based on the $N\times 1$ vector of snapshots of the observed signal $\mathbf{x}$:

$$\mathbf{x} = \sum_{l=1}^{L} \theta_l \mathbf{a}_l + \boldsymbol{\xi}, \qquad (1)$$

where the vectors $\mathbf{a}_l$ characterize the signal shapes, and $\boldsymbol{\xi}$ is a random vector with zero mean (interference, measurement errors, etc.). The estimates $\theta_l$ can be obtained using least squares estimation (LSE) [1,2]: $\hat{\boldsymbol{\theta}} = (\mathbf{A}^H\mathbf{A})^{-1}\mathbf{A}^H\mathbf{x}$, where $\hat{\boldsymbol{\theta}}$ is the $L\times 1$ vector of estimates of the amplitudes of "useful" signals, and the columns of the $N\times L$ matrix $\mathbf{A}$ consist of the vectors $\mathbf{a}_l$, $H$ denotes the Hermitian transpose. At the same time, it can turn out that the amplitude of the $l$-th signal is small, and there arises a problem of estimation of its reliability, since the relative error can be inadmissibly large.

If there are reliable data about the interference, the reliability can be checked by means of comparison of the realization of the estimate $|\hat{\theta}_l|$ and the threshold value $c_0\sigma_l$, where $c_0$ is some coefficient depending on the allowed relative error, confidence probability, etc., and $\sigma_l^2 = C_{ll}$ is the diagonal element of the covariance matrix of error of the estimates $\Delta\boldsymbol{\theta} = \hat{\boldsymbol{\theta}} - \boldsymbol{\theta}$:

$$\mathbf{C} = \mathrm{E}\{\Delta\boldsymbol{\theta}\Delta\boldsymbol{\theta}^H\} = (\mathbf{A}^H\mathbf{A})^{-1}\mathbf{A}^H\mathbf{K}\mathbf{A}(\mathbf{A}^H\mathbf{A})^{-1}, \qquad (2)$$

where $\mathrm{E}\{.\}$ denotes the expectation, and $\mathbf{K} = \mathrm{E}\{\boldsymbol{\xi}\boldsymbol{\xi}^H\}$ is the interference covariance matrix. In practice, it is often impossible to perform separate measurements of interference[1], and the decision whether the result is reliable or not has to be made based on the analysis of the vector $\mathbf{x}$. This approach is developed in the present work. It should be noted that, in spite of the multiplicity of works devoted to methods of linear regression, it is hard to point out works with such a problem statement.

An estimate of the error of determining the amplitudes $\hat{\theta}_l$ can be performed by, first, obtaining some estimate of $\mathbf{K}$ and then using (2). For that, it is necessary to perform a preliminary estimation of the interference $\boldsymbol{\xi}$; a natural form of such an estimate is

$$\hat{\boldsymbol{\xi}} = \mathbf{x} - \mathbf{A}\hat{\boldsymbol{\theta}} = (\mathbf{I} - \mathbf{P})\boldsymbol{\xi}, \quad \mathbf{P} = \mathbf{A}(\mathbf{A}^H\mathbf{A})^{-1}\mathbf{A}^H, \qquad (3)$$

where $\mathbf{I}$ is the identity matrix. It is easy to show that $\mathbf{P}$ is a projector of rank $L$, i.e., the estimate (3) is "defective" since the components of interference "similar" to the useful signals are lost. Thus, when the structure of $\mathbf{K}$ is arbitrary, the task has no solution: it is easy to show that substituting $\mathrm{E}\{\hat{\boldsymbol{\xi}}\hat{\boldsymbol{\xi}}^H\}$ instead of $\mathbf{K}$ to (2) leads to the identity $\mathbf{C} = 0$. It is possible to obtain a practical result if *a priori* infor-

[1] For example, interference may vary fast enough, so its preliminary measurements make no sense.

mation about the interference structure is used, for example, if it is supposed to be stationary. In this case, $\mathbf{K}$ becomes a Toeplitz matrix: $\mathbf{K} = \|K(n-m)\|$, and the estimate of correlation sequence $K(n),\, n = 0,\dots, N-1$ can be expressed as

$$\hat{K}(n) = N^{-1} \sum_{j=n+1}^{N} \hat{\xi}_j \hat{\xi}^*_{j-n} \,. \tag{4}$$

Note that the estimate (4), in general case, is biased and, subsequently, the estimate of the error dispersion $\sigma_l^2$ obtained after substituting (4) into (2) is also biased. Finally, the criterion of reliability of the estimate of the amplitude of the *l*-th "useful" signal can be formulated as

$$|\,\hat{\theta}_l\,| \geq c_0 \hat{\sigma}_l \,, \tag{5}$$

where $\hat{\sigma}_l^2 = \hat{C}_{ll}$ is the dispersion estimate obtained after substituting (4) into (2). The constant $c_0$ can be chosen based on the two given numerical parameters: the allowable relative error of determining of the *l*-th amplitude $\delta_0$ and the allowable probability $p_F^{(0)}$ of making a wrong decision in (5). Define the probability of making a wrong decision $p_F$ as

$$p_F = \mathrm{P}[\,|\,\hat{\theta}_l - \theta_l\,| > \delta_0\,|\,\theta_l\,| \cap |\,\hat{\theta}_l\,| > c_0 \hat{\sigma}_l\,] \,, \tag{6}$$

where $\mathrm{P}[A \cap B]$ is the probability of simultaneous realization of intersecting events *A* and *B*. An exact calculation of (6) is somewhat complicated even in the case of the normal distribution of $\boldsymbol{\xi}$ because of the random nature of the estimate $\hat{\sigma}_l$, but it is possible to find an approximate representation for $p_F$ assuming $\hat{\sigma}_l \approx \sigma_l$. In this approximation,

$$p_F \approx \begin{cases} \left[1 - \mathrm{erf}\left((c_0 - q)/\sqrt{2}\right)\right]/2 \,, & q < c_0/(1+\delta_0) \\ \left[1 - \mathrm{erf}\left(q\delta_0/\sqrt{2}\right)\right]/2 \,, & c_0/(1-\delta_0) \leq q \leq c_0/(1+\delta_0) \\ \left[1 - \mathrm{erf}\left((c_0 - q)/\sqrt{2}\right) - 2\,\mathrm{erf}\left(q\delta_0/\sqrt{2}\right)\right]/2 \,, & q > c_0/(1+\delta_0) \end{cases}, \tag{7}$$

where $\mathrm{erf}(x) = (2/\sqrt{\pi})\int_0^x \exp(-t^2)dt$ is the probability integral, and $q = |\,\theta_l\,|/\sigma_l$. A graph of the function (7) is shown in Fig. 1; it has one global maximum whose value is monotone decreasing when increasing $c_0$, and this constant is approximately found from the condition $\max_q\{p_F\} = p_F^{(0)}$; its refinement, taking into account the random nature of $\hat{\sigma}_l$ and possible bias of (4), can be performed using the method of stochastic modeling.

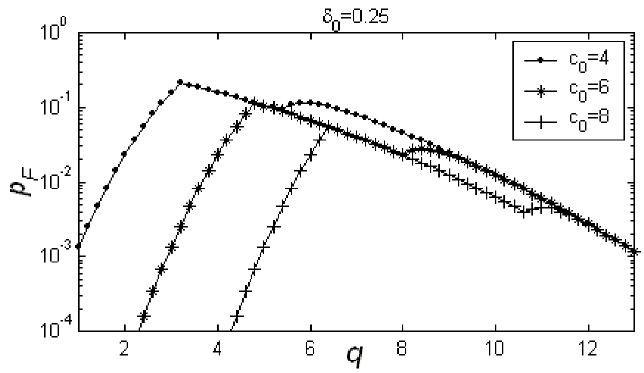


***Fig. 1.*** Dependence of the wrong decision probability on the ratio $q = |\,\theta_l\,|/\sigma_l$ for various threshold constants $c_0$.

**2. Reliability of measurements of underwater noise levels.** Consider the described above general scheme in application to measurements of underwater noise levels [3,4] be the method of energetic matched processing (EMP) [5]. In the framework of this method, current snapshots of squared sound pressure in the frequency band $\Delta f$ averaged over time $T_i$ are accumulated while a noise source moves with respect to a hydrophone with the velocity *V* with the distance of closest approach *d*, and then, using LSE, an estimate of source noise power $\theta_2$ is found in the framework of the model

$$x_n = \theta_1 + \theta_2 a_{2,n} + \xi_n, \qquad (8)$$

where $\theta_1$ is the power of ambient interference and $a_{2,n}$ describe the way the source noise power changes while it moves in respect to a hydrophone: $a_{2,n} = [1 + ((t_n - \tau)/T_V)^2]^{-\alpha}$; here, $t_n$ are the instants of snapshots of the noise power, $\tau$ is the time of closest approach, $T_V = d/V$, $\alpha$ can posses the discrete values 0.5, 1 or 2. In practice, the parameters $\tau$ and $\alpha$ are usually unknown and are found based on maximum correlation between the model and the data [2]. Besides, at measurements, the ambient interference often slightly varies; denoting these variances as $\Delta\theta_1(t)$, it is possible to introduce the correlation function $K_\Delta(t' - t'') = \mathrm{E}\{\Delta\theta_1(t')\Delta\theta_1(t'')\}$ of these variances. Taking into account what is noted above, the elements of **K** can be expressed as

$$K_{nm} = K_\Delta(t_n - t_m) + (\Delta f T_i)^{-1}[(\theta_1 + \theta_2 a_{2,n})^2 + K_\Delta(0)]\delta_{nm}, \qquad (9)$$

where $\delta_{nm}$ is the Chronicler's symbol. Note that, in the representation (8), the interference $\xi_n$, strictly speaking, is distributed by the law $\chi^2$ with $2\Delta f T_i$ degrees of freedom; however, this is inessential when $\Delta f T_i \gg 1$.

A bias of the estimate $\hat{\sigma}_2$, being a part of (5), has been investigated for the correlation function of variances of ambient interference of the form $K_\Delta(t) = \sigma_\Delta^2 \exp(-|t|/\tau_{\mathrm{cor}})$. This bias can be characterized by the ratio of standard deviations (SD) $\sqrt{E\{\hat{\sigma}_2^2\}}/\sigma_2$. It is obvious that the bias itself (i.e., the difference of the ratio of SD from one) does not play an important role because it always can be taken into account by means of the constant $c_0$ in (5); it is important that this bias should not depend on the parameters characterizing the measurement conditions: $\theta_2/\theta_1, \sigma_\Delta, \tau_{\mathrm{cor}}, \Delta f T_i$, etc. As an example, the dependences of the ratio of SD on $\sigma_\Delta/\theta_1$ are shown in Fig. 2 for various time scales of fluctuations (ratios $\tau_{\mathrm{cor}}/T_V$ in the range 0.6…24). At the same figure, dependences of SD of the estimate $\sigma_2$ on the same parameters are given. For simulations, it was assumed $\Delta f T_i$=23 and $\theta_2/\theta_1$=0.2. As follows from the graph, the ratio of SD characterizing the estimate bias varies in the range 0.75...0.85, which is inessential from the practical point of view, whereas SD of the noise power estimate may change by several times.

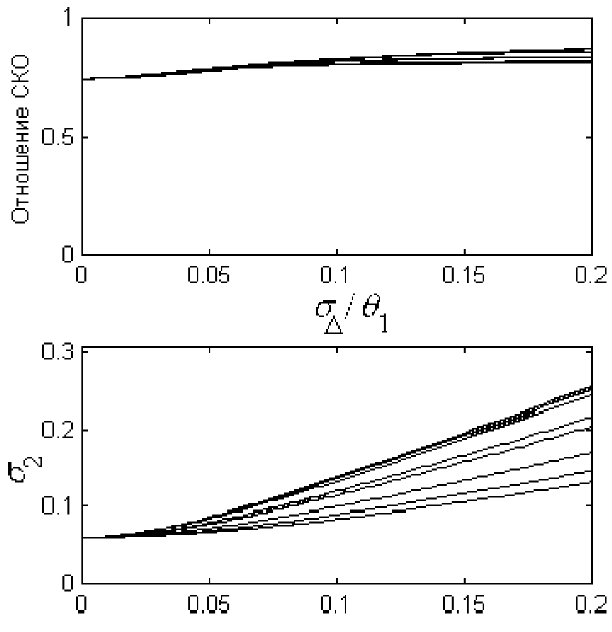


***Fig. 2.*** Dependence of the ratio of SD (at the top) and SD of the estimate $\theta_2$ (at the bottom) on SD of ambient interference fluctuations for various correlation times of fluctuations.

Similar calculations have been performed for other parameters characterizing the measurement conditions; as a result, the correction factors eliminating the bias have been refined. The value of the constant $c_0$ have been refined by methods of stochastic modeling taking into account all the peculiarities of real measurements such as the random character of the estimate $\hat{\sigma}_2$ in (5), non-Gaussian distribution of the interference $\xi$, a necessity for additional estimation of the time of closest approach $\tau$ and the parameter $\alpha$, etc. It has been shown that taking into account these peculiarities leads to the increase of the constant by ~1.3 times (for example, for the allowable relative measurement error $\delta_0$ = 0.25 and wrong decision probability 0.05, the threshold constant $c_0$ = 10.5).

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