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ABOUT ENVELOPE OF ECHO PULSE SET IN MONOCRYSTALS WITH HETEROGENEOUS MECHANICAL STRESSES DURING ACOUSTICAL DIAGNOSTICS OF STRESSED STATE

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In the article is viewed influence of heterogeneous (ramp) mechanical stresses in monocrystals with special physical characteristics on form of envelope of echo pulse set and echo pulse number in string of reverberations during ultrasonic transverse wave propagation perpendicularly to plane of mechanical stresses and polarized along stresses. With increasing of mechanical stresses heterogeneity (gradient) exponential envelope of echo pulse set becomes practically a straight line and number of echo pulses in reverberations set is reduced. There are give equations reflecting dependence of amplitude of ultrasonic echo pulse with arbitrary number and echo pulses number in set of multiple reverberations from ultrasonic wave speed gradient. Making of the given equations is based on assumption of refraction of elastic wave on heterogeneous mechanical stresses.

Lets view influence of single-axis compressing ramp mechanical stresses (MS) on the form of echo signals set envelope during ultrasonic (US) pulses propagation in elastically isotropic non stressed monocrystal (MC). MC sample is a rectangular parallelepiped. AA' is directed along OY axis. Stresses σ proceed in plane XOY which is parallel to $ABB'A'$ plane (fig. 1). Receiving-emissive transducer of transverse US-waves 2 with width $2a$ and length $2b$ is placed on $AA'B'B$ surface of the sample; US-wave is polarized in XOZ plane parallel to $ABCD$ plane and propagates along OZ axis. OZ axis is perpendicular to the plane of ramp MS σ. When compressing MS decrease US-wave speed decreases proportionally to MS value.

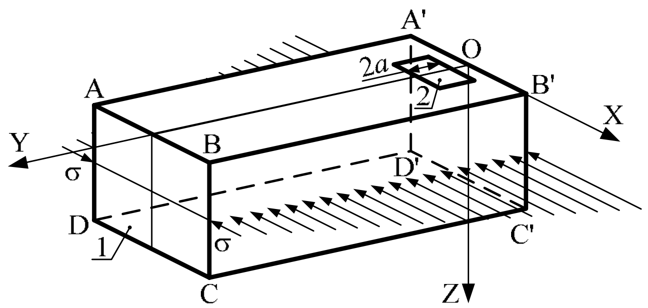


Fig. 1. Sample 1 with US-transducer 2

Therefore when stresses change linearly along the sample length speed propagation of transverse US-wave along the sample length is also linear

$$V_s = V_{s1}\left[1 - \frac{G_\upsilon}{V_{s0}}(y - y_1)\right],$$

where V_s – propagation speed of transverse US-wave in point y; V_{s1} – maximum speed of US-wave in the sample; V_{s0} – speed of US-wave in MC with no residual MS; G_υ – speed gradient of transverse US-wave; y_1 – sample point where speed of transverse US-wave propagation is maximum.

Speed gradient G_υ is obtained according to equation

$$G_\upsilon = \frac{V_{s1} - V_{s2}}{y_2 - y_1}, \quad (1)$$

where V_{s2} – minimum speed of US-wave in the sample between y_1 and y_2; y_2 – point of the sample, where US-wave speed changes according to linear dependence $V_s(y)$ and is minimum.

We obtain the envelope of echo pulse set determining US-transducer response on each US-beam falling on the transducer during its propagation between two plane-parallel edges with multiple reverberations from them [1].

Lets use geometrical (ray) theory of US-waves propagation [2], which results coincide with wave theory if heterogeneity parameter change is minor on the interval of wave length, i. e. the following in equation is true

$$\frac{kV_{s0}}{G_\upsilon} >> 1,$$

where k – wave number.

It is known that in stratified heterogeneous medium which is characterized by constant US-wave speed gradient the ray path as a result of refraction is a circle with radius R:

$$R = \frac{V_{s0}}{G_\upsilon}, \qquad (2)$$

and path is bending towards decreasing US-wave speed [3].

In the fig. 2, a is shown US-wave speed distribution along length l of sample 1; in the fig. 2, b are shown US-ray propagation paths: *AEBFCHD* – in sample 1 and *AEB'C'D'* – in infinite medium. Position 2 is integrated emissive and receiving US-transducer. Center of curvature of the ray path is point O with coordinate y_0, in which US-wave propagation speed is equal to zero.

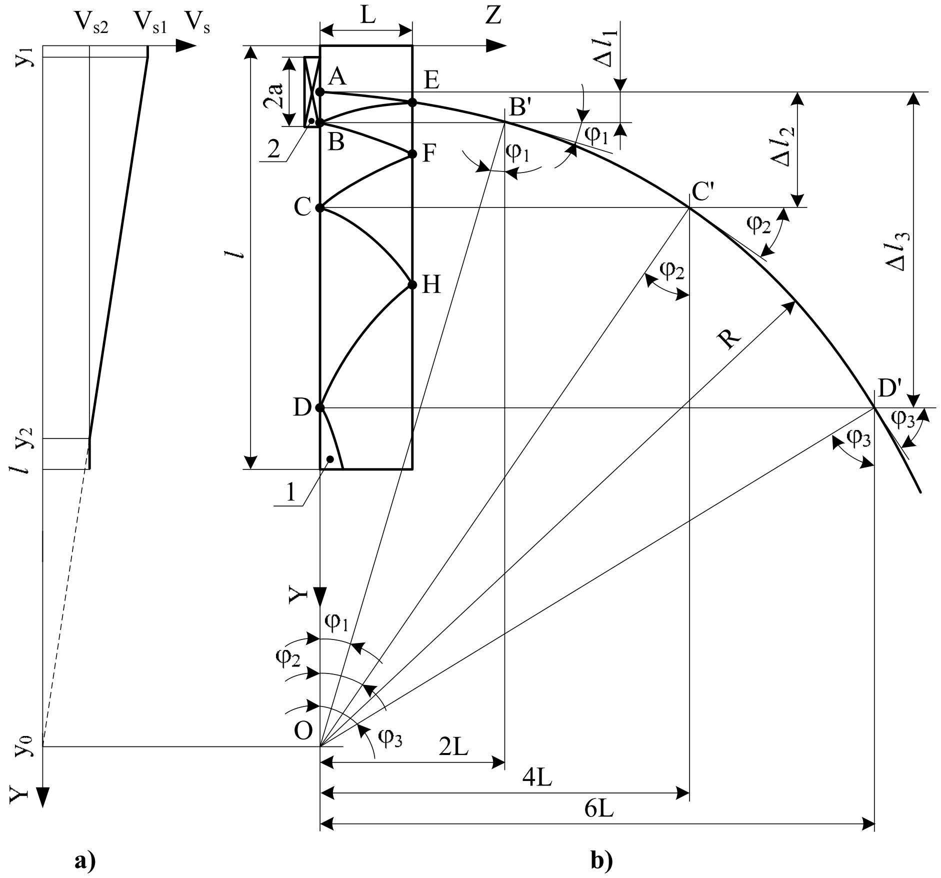


Fig. 2. US-ray propagation in single axis compressed sample with ramp MS

As US-transducer "feels" phase of incident wave and transducer response is result of integration all along transducer so variations of wave phase along transducer surface have an influence on transducer response. During US-wave propagation in the sample over the circle arc US-beam is falling on opposite surface of the sample and is reflecting at an angle not equal to zero (fig. 2, b). After the beam passing through the sample in direct and reverse directions we can obtain US-beam dip angle on the transducer φ_1 from triangle *OBB'*

$$\varphi_1 = \arcsin\frac{2L}{R}.$$

US-beam dip angle on the transducer after n double passing through the sample and taking into account (2) is the following:

$$\varphi_n = \arcsin\frac{2nLG_\upsilon}{V_{s0}} . \quad (3)$$

As angle φ_n is also equal to angle between plane wave front and the transducer, phase change $\Delta\varphi_n$, which is “felt” by transducer, depending on distance *y* from transducer center is obtained from equation

$$\Delta\varphi_n = ky\sin\varphi_n . \quad (4)$$

Solving jointly (3) and (4) and taking into account that $\sin\varphi_n \approx \varphi_n$ when φ is small we obtain:

$$\Delta\varphi_n = k\frac{2LG_\upsilon n}{V_{s0}} y .$$

If we write initial amplitude of offset on transducer as

$$u(0,t) = u_0 e^{i\omega t} ,$$

then offset after *n* reverberations will be

$$u(2Ln,t) = u_0 e^{i(\omega t + 2kLn + \Delta\varphi_n)} . \quad (5)$$

Average offset on transducer can be obtained according to equation

$$\bar{u} = \frac{1}{4ab}\int_S u ds , \quad (6)$$

where S – transducer square, on which US-beam is falling; *ds* – area element during integration.

Here during integration is necessary to take into account the fact that when US-beam is propagating along circle arc the beam is moving towards decreasing of speed of US-wave propagation. In consequence of this moving only some part of beam cross-section falls on US-transducer. Beam moving during one double passing through the sample Δl_1 can be obtained from equation

$$\Delta l_1 = R - \sqrt{R^2 - (2L)^2} ,$$

and beam moving for *n* double passing (fig. 2, b) with proportion (2) can be written as

$$\Delta l_n = \frac{V_{s0}}{G_\upsilon} - \sqrt{\frac{V_{s0}^2}{G_\upsilon^2} - 4L^2 n^2} . \quad (7)$$

Decomposing radicand into binomial series and taking only first two expansion terms, lets rearrange equation (7) and obtain

$$\Delta l_n = \frac{2L^2 G_\upsilon n^2}{V_{s0}} . \quad (8)$$

We take *u* from (5) and put it into (6). Then taking into consideration US-beam moving Δl_n during calculation limits of integral, we obtain for the case of rectangular transducer:

$$\bar{u} = \frac{u_0}{2a} e^{i(\omega t + 2kLn)} \int_{-a+\Delta l_n}^{a} e^{ik\frac{2LG_\upsilon n}{V_{s0}} y} dy = \frac{V_{s0} e^{i(\omega t + 2kLn)}}{4ikLG_\upsilon an}\left(e^{ik\frac{2LG_\upsilon an}{V_{s0}}} - e^{-ik\frac{2LG_\upsilon an(a-\Delta l_n)}{V_{s0}}} \right) . \quad (9)$$

Selecting real part of equation (9) and transforming it we obtain transducer area average amplitude of US-wave:

$$\bar{u} = \frac{u_0 V_{s0}}{2kLG_\upsilon an}\sin\left(k\frac{LG_\upsilon(2a - \Delta l_n)}{V_{s0}} n \right)\cos\left(k\frac{LG_\upsilon \Delta l_n}{V_{s0}} n \right)\sin(\omega t + 2kLn) ,$$

or

$$\bar{u} = \frac{u_0 V_{s0}}{4kLG_\upsilon an}\left(\sin\frac{2kLG_\upsilon an}{V_{s0}} + \sin\frac{2kLG_\upsilon(a - \Delta l_n)n}{V_{s0}} \right)\sin(\omega t + 2kLn) .$$

Envelope of multiple reverberations set will be modulated according to:

$$\frac{V_{s0}}{4kLG_\upsilon an}\left[\sin\frac{2kLG_\upsilon an}{V_{s0}} + \sin\frac{2kLG_\upsilon(a - \Delta l_n)n}{V_{s0}} \right] ,$$

and equation for obtaining amplitude of echo pulse with number *n* in the set will be the following

$$A_n = A_n^0 \frac{V_{s0}}{4kLG_\upsilon an}\left[\sin\frac{2kLG_\upsilon an}{V_{s0}} + \sin\frac{2kLG_\upsilon (a-\Delta l_n)n}{V_{s0}}\right], \qquad (10)$$

where A_n^0 – amplitude of echo pulse with number n in the set of multiple reverberations for the sample with no residual MS. It is known that in the sample with no residual MS in the set of multiple reverberations amplitude changes according to exponential law:

$$A_n^0 = A_0 e^{-2\alpha Ln}, \qquad (11)$$

where α – attenuation coefficient of US-wave.

Lets substitute equation (11) into equation (10) and we obtain expression which describes echo pulses amplitude in multiple reverberations set:

$$A_n = \frac{A_0 V_{s0} e^{-2aLn}}{4kLG_\upsilon an}\left[\sin\frac{2kLG_\upsilon an}{V_{s0}} + \sin\frac{2kLG_\upsilon (a-\Delta l_n)n}{V_{s0}}\right]. \qquad (12)$$

Lets define form of envelope of echo pulse set and its variations depending on speed gradient. Using equation (12) and setting $V_{s0} = 3575{,}82$ ms^{-1}, $k = 17{,}6\cdot 10^3$ m^{-1}, $L = 73\cdot 10^{-3}$ m, $a = 4\cdot 10^{-3}$ m, $\alpha = 2\cdot 10^{-3}$ dB/mkS lets calculate multiple reverberations sets for G_υ from 0 to 40 s^{-1} in 5 s^{-1} and further to 70 s^{-1} in 10 s^{-1}.

Pattern of change of echo pulse amplitude in these sets for specified gradient speed values is given in fig. 3.

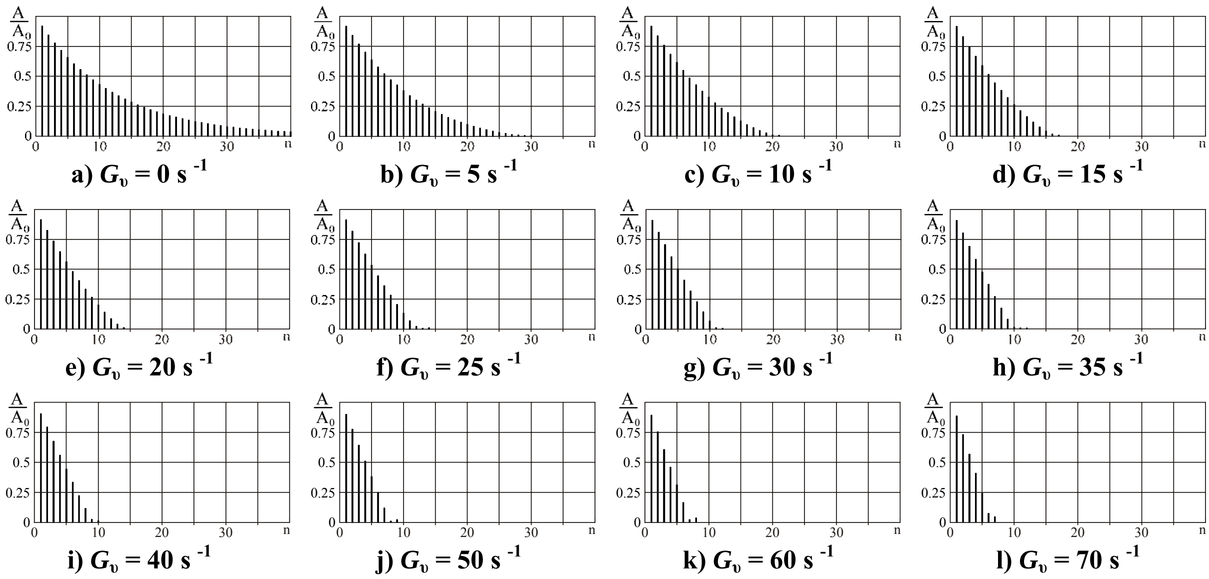


Fig. 3. US-pulse reverberations sets for various speed gradient values

Lets compare reverberations set obtained for various G_υ values and also compare these sets with set obtained for $G_\upsilon = 0$. It is obvious that sets envelopes given in fig. 3, b – 3, l, differ from exponent (fig. 3, a), and difference grows as G_υ grows. When $G_\upsilon \geq 25$ s^{-1} (fig. 3, f) envelope becomes almost a line.

Fig. 3 also shows that for various G_υ echo pulse number N in reverberations set is different. When G_υ grows this number is decreasing.

In this case echo pulse number in multiple reverberations set is number of double passing of US beam through the sample when beam moving is equal to US-transducer width $2a$.

Lets substitute into (8) $2a$ instead of Δl_n and solving it relative to echo pulse number we obtain

$$N = \frac{1}{L}\sqrt{\frac{aV_{s0}}{G_\upsilon}}. \qquad (13)$$

As echo pulse number in reverberations set can be calculated easily lets use it to define G_υ

$$G_\upsilon = \frac{V_{s0}a}{L^2 N^2}\ . \tag{14}$$

Dependence of echo pulse number in multiple reverberations set from US-wave speed gradient is given in fig. 4. It is clear that when G_υ grows, in our case from $G_\upsilon = 50\ s^{-1}$, echo pulse number in set changes slightly. Speed gradient values G_υ, when amplitude of echo pulse with number n in reverberations set (n changes from 10 to 1) turns into zero, are given in table 1.

From table 1 we can see that reverberations set consisting of five pulses, for example, corresponds to speed gradient interval from 74,6 s^{-1} to 108 s^{-1}. Amplitude of the fifth pulse is maximum when $G_\upsilon = 74{,}6\ s^{-1}$, and then is decreasing when G_υ is growing and becomes equal to zero when $G_\upsilon = 108\ s^{-1}$. There is only one echo pulse in the reverberations set when $G_\upsilon = 671\ s^{-1}$. Its amplitude turns into zero when $G_\upsilon = 2684\ s^{-1}$.

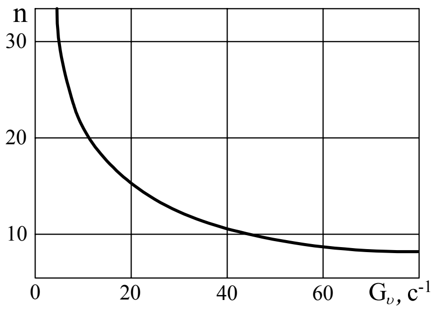


Fig. 4. Dependence of echo pulse number n in multiple reverberations set from US-wave speed gradient G_υ

US-wave speed change $V_{s1} - V_{s2}$ is a result of compressing MS in MC and this change is slight. So distance $y_2 - y_1$ where speed changes has the main influence upon G_υ.

For experimental acknowledgement of the fact that US-waves refraction on heterogeneous MS in MC exists there is suggested model of necessary stress condition realization. Gallium gadolinium garnet MC ingot is elastically isotropic in non-stressed condition. The ingot has form of straight circular cylinder with length $L = 20$ mm and diameter $D = 76$ mm and is pressed to diameter in hydraulic press. As a result of such ingot loading in its center is created stresses condition, which is plane heteronymous biaxial with main MS $\sigma_1 = -\ 6F/\pi LD$ and $\sigma_2 = 2F/\pi LD$, where F – is external stress which can be defined using hydraulic press manometer and known square of press piston; sign “minus” in the equation for σ_1 means that stress σ_1 is compression stress [4].

Table 1. Values of speed gradient G_υ, for which amplitude of echo pulse with number n in multiple reverberations set is equal to zero

n	10	9	8	7	6	5	4	3	2	1
G_υ, c^{-1}	26,8	33,1	41,9	54,8	74,6	108	168	298	671	2684

When distance from MC ingot center along direction σ_2 is growing (σ_1 and σ_2 are mutually perpendicular, direction σ_1 coincides with direction of compression force F) MS area within transducer cross-section is not plane. It is heterogeneous and within transducer width σ_2 may be considered to be ramp.

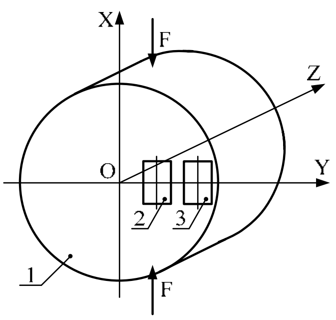


Fig. 5. Monocrystal 1 with US-transducer 2 end 3

First on plane surface of cylindrical MC-ingot 1 is set only one integrated emissive and receiving shearing US-transducer 2 as shown in fig. 5. Plane of polarization of US-wave emanating from transducer 2 is parallel to XOZ plane.

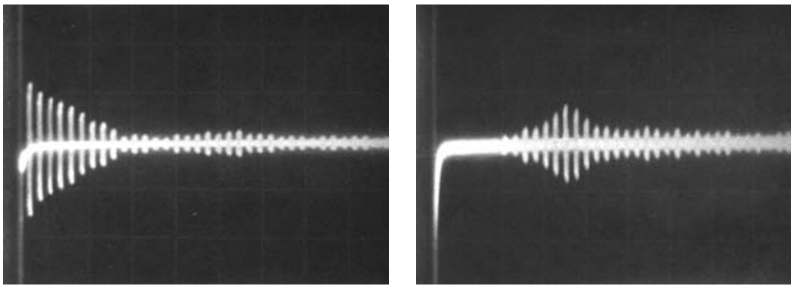
In fig. 6, a is given photogram of echo pulse set obtained when US-transducer 2 was functioning as oscillator and receiver.

In the photogram we can see that echo pulse set envelope differs from exponential and close to the straight line.

Then on plane surface of MC-ingot 1 near US-transducer 2 was set US-transducer 3. Distance between transducers centers was 12 mm. Transducers centers were on axis line of plane surface (on axis OY). Waves directions of both transducers are parallel. In this case transducer 2 was used only as oscillator.

Emitted US-pulse during propagation between opposite plane-parallel faces of MC-ingot with multiple reverberations from them are to move towards decreasing of US-wave speed, i.e. towards US-transducer 3, used for receiving echo pulses.

In fig. 6, b is given photogram of echo pulse set received by transducer 3. In this photogram we can see that echo pulses appear in some time after US-pulse emanating into MC-ingot. Amplitude of echo pulses in the set firstly increases and then linearly decreases. Regularity of echo pulse amplitude variation in photograms, given in fig. 6, and rise and dying-away of echo pulses received by US-transducer 3 in a time, when US-beam is moving from emissive transducer 2 to receiving 3, confirm that refraction of US-waves on heterogeneous MS exists.



a) **b)**

Fig. 6. Photograms of echo pulse sets in plane-parallel sample with single-axis compressing ramp MS

Alternatively as US-transducers 2 and 3 were used two identical shear US-transducers (quartz piezo-plate of YX-section with pyramid ceramic damper) with equal base resonance frequency 40 MHz. Epoxy resin without hardening agent was used for acoustical contact layer. Requisite quality of acoustical contact layer and its properties stability were achieved owing to special rotary-pressure device which provided constant pressing force of US-transducer to using MC-ingot.

Characteristic change of amplitude and echo pulse number in multiple reverberations set depending on US-wave speed gradient may be used for identification residual MS during acoustical diagnostics of monocrystal stress condition by echo pulse method.

R E F E R E N C E S

1. Truell R., Elbaum C., Chick B. Ultrasonic Methods in Solid State Physics. – N-Y.: Academic Press, 1969. – 307 c.
2. L.M. Brekhovskih. Waves in stratified mediums. – M.: Science, 1973. – 244 p.
3. A.P. Stashkevitch. Sea acoustics. – L.: Shipbuilding, 1966. – 255 p.
4. S.P. Timoshenko. Theory of elasticity. – L.-M., 1934. – 451 p.