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A MODEL OF STATIONARY FLOW FORMATION NEARBY THE OSCILLATING SYMMETRICAL BODIES

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This work is continuation of the work [1], which describes common properties of stationary flows nearby oscillating bodies or flows, formed by means of powerful acoustic wave. This work present a model of nonlinear forces, which cause flows formation in these cases. At the moment the theory of acoustic streaming or "quartz wind" is well developed, but according to these theories the stationary flows are considered to be secondary phenomena. As it has been shown by the experiment, under appropriate conditions velocities in stationary flows could be comparable with oscillating velocities in the system. The model described within this work provides sensible estimations for stationary flows velocities which are in agreement with experimental data.

Description of the nonlinear forces model

Consider Navier-Stokes equation for incompressible viscous fluid:

$$\begin{cases} \frac{\partial \vec{u}}{\partial t} + (\vec{u}\nabla)\vec{u} = -\frac{1}{\rho}\nabla p + \upsilon\Delta\vec{u} \\ \operatorname{div}\vec{u} = 0 \end{cases} \quad (1)$$

To find approximate solution of these equations we will use an approach analogous to that described in [2]. The solution is assumed to be a sum of oscillatory field component and the field of stationary flows:

$$\vec{u} \approx \vec{u}_{osc} + \vec{u}_s \quad (2)$$

In contrast to method of successive approximations we are not demanding $\vec{u}_{osc}$ to be Small comparative to $\vec{u}_s$. But we will neglect the term $\frac{\partial \vec{u}_{osc}}{\partial t}$.

Substituting (2) into (1) we will get the following equations for different field components:

$$\begin{cases} \frac{\partial \vec{u}_{osc}}{\partial t} + (\vec{u}_{osc}\nabla)\vec{u}_s + (\vec{u}_s\nabla)\vec{u}_{osc} = -\frac{1}{\rho}\nabla p_{osc} + \upsilon\Delta\vec{u}_{osc}, \\ \operatorname{div}\vec{u}_{osc} = 0 \end{cases} \quad (3.1)$$

$$\begin{cases} (\vec{u}_s\nabla)\vec{u}_s = -\frac{1}{\rho}\nabla p_s + \upsilon\Delta\vec{u}_s + F(\vec{r},\vec{u}_{osc}), \\ \operatorname{div}\vec{u}_s = 0 \end{cases} \quad (3.2)$$

The function $F(\vec{r},\vec{u}_{osc})$ in right hand side of equation of movement (3.2) has form:

$$\vec{F}(\vec{u}_{osc},\vec{r}) = -\langle(\vec{u}_{osc}(\vec{r})\nabla)\vec{u}_{osc}(\vec{r})\rangle \quad (4)$$

This expression could be rewritten as follows:

$$\vec{F} = \langle -(\vec{u}_{osc}\nabla)\vec{u}_{osc}\rangle = -\left\langle \frac{1}{2}\nabla\vec{u}_{osc}^{\,2}\right\rangle + \langle \vec{u}_{osc}\times(\nabla\times\vec{u}_{osc})\rangle \quad (5)$$

First item in this expression is fully potential force and it will not cause stationary flows. Below we will take into account second component in (5) only.

Thus, for nonlinear force field computation we have to evaluate the field $\vec{u}_{osc}$ first, which is, in fact, the solution of linearized equations (3.1). This problem could be solved analytically in very few simple cases Let us introduce the following simplifying assumption. Assume that oscillating velocity with satisfying accuracy field could be as follows

$$\vec{u}_{osc} = \beta(\vec{r}) \cdot \vec{U}_0(\vec{r}) \quad (6)$$

where $\vec{U}_0(\vec{r})$ - the velocity field of streaming the considered body by ideal incompressible fluid, and $\beta(\vec{r})$ - is boundary layer modeling function.

Let us assume that thy function $\beta(\vec{r})$ has form either:

$$\beta(\vec{r}) = (1 - e^{-\sqrt{i}\,r/\delta}) \tag{7.1}$$

in case of immovable body streamlined by flow with oscillating velocity, or

$$\beta(\vec{r}) = e^{-\sqrt{i}\,r/\delta} \tag{7.2}$$

in case of oscillating body.

Taking into account the oscillatory field is presented as a product of irrotational field by scalar function, the expression (5) could be significantly simplified:

$$\vec{F} \approx \frac{1}{2}\left\langle \vec{U}_0^2 \right\rangle \nabla(\beta^2) \tag{8}$$

Let us perform qualitative analysis of this field. Consider the edge of a plate of finite thickness, streamlined by vertically oscillating flow (Fig 1).

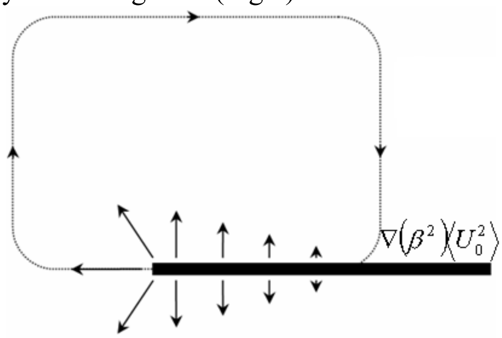


Fig. 1. Qualitative analysis of nonlinear force

The gradient of boundary layer modeling function is always directed normally to the plate. The fluid velocity nearby the plate is always parallel to its surface. Fig 1. qualitatively shows direction of forces expressed by (8). One can see there is a contour (shown on the figure by dotted line) with non-zero circulation of the force (8). Thus, such forces are able to cause stationary flows.

Nonlinear forces analysis

Let us perform an analysis of major properties of force field expressed by (8). It is well known that under the streaming of a body by ideal fluid, velocity fluid nearby edges could significantly exceed stream velocity. Hence first two major properties are as follows:

1. Nonlinear forces are localized in boundary layer,
2. Nearby the edges the nonlinear forces have the largest magnitude.

Below we will compare cases of oscillating body in fluid and streaming of body by oscillating flow. To demonstrate these differences the force field and forces amplitudes were calculated of infinite cylinder (Fig. 2 and 3).

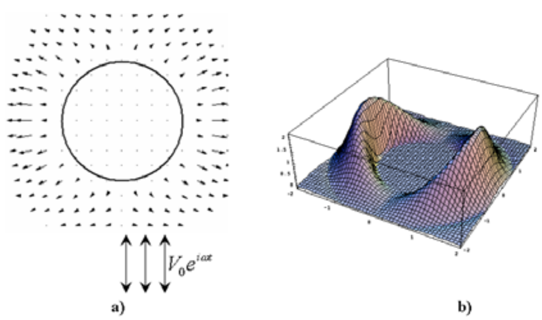
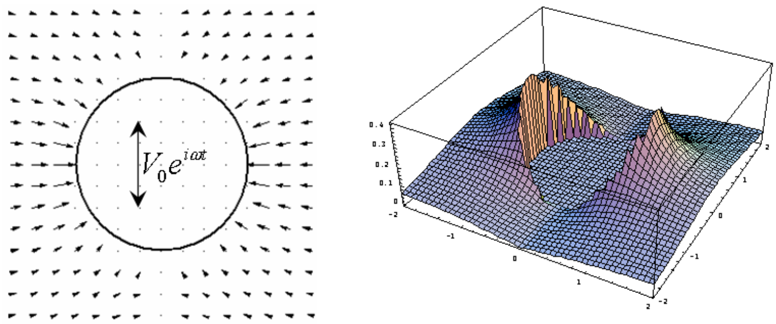


Fig 2. a) nonlinear force field b) magnitude of nonlinear force near by cylinder being streamlined by flow with oscillating velocity



a) b)

Рис 3. a) nonlinear force field b) magnitude of nonlinear force near oscillating cylinder

In both cases the field $\vec{U}_0(\vec{r})$ is the same, the difference in field $\vec{u}_{osc}$ in these cases is caused by difference of boundary layer modeling function. Major differences between case of streamlining by oscillatory flow and oscillation in liquid could be formulated as follows:

1. Opposite forces direction. Indeed, nonlinear force field is described by expression (8). Directions of forces depend on direction of boundary layer modeling function gradient which is opposite in these two cases.

2. Different position of nonlinear force maximum. Magnitude of boundary layer modeling function gradient is the same in both cases. In case of oscillating body, maximum of velocity is achieved on its surface, but in case of streaming by oscillatory flow, velocity on body surface is zero. Maximum of the velocity is achieved on infinity, where boundary layer function is equal to zero.

3. As corollary of p.2, magnitudes of nonlinear forces will be different. In case of oscillating bode the nonlinear force will be greater.

Model application examples

As an example, let us calculate stationary flows near by oscillating plate. For simplicity 2D case is considered. Assume, that the plate is part segment *(-a;a)* of *y*-axis and it oscillates along *x*-axis.

According to [3] velocity field under streaming by ideal incompressible fluid of a plat with width *2a* has form:

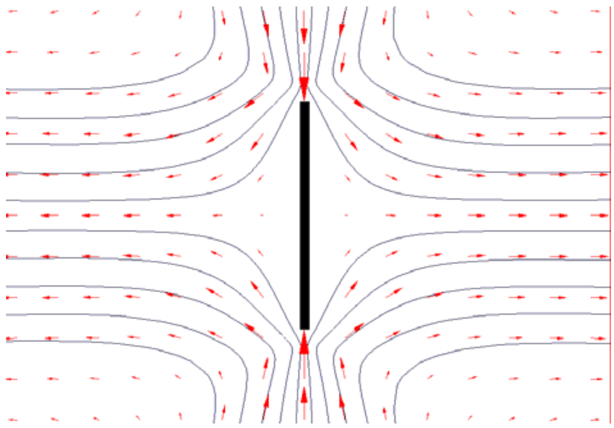


Fig 4. Velocity field and streamlines nearby oscillating plate

$$\begin{cases} U_{0x} = \mathrm{Re}[\dfrac{x + \mathrm{i}\, y}{\sqrt{(x + \mathrm{i}\, y)^2 + a}}] \\ U_{0y} = -\mathrm{Im}[\dfrac{x + \mathrm{i}\, y}{\sqrt{(x + \mathrm{i}\, y)^2 + a}}] \end{cases} \qquad (9)$$

According to (5) in 2D case nonlinear forces component could be expressed as:

$$\begin{cases} F_x = U_{0y}\beta\left(U_{0y}\dfrac{\partial\beta}{\partial x} - U_{0x}\dfrac{\partial\beta}{\partial y}\right) \\ F_y = U_{0y}\beta\left(U_{0x}\dfrac{\partial\beta}{\partial y} - U_{0y}\dfrac{\partial\beta}{\partial x}\right) \end{cases} \quad (10)$$

Be means of Finite Element Modeling the equation (3.2) has been solved with no slip boundary conditions on plate surface. The Fig 4. presents field of stationary flows and streamlines nearby the plate.

In the same way the velocity field and streamlines have been calculated for oscillating infinite cylinder (Fig 5).

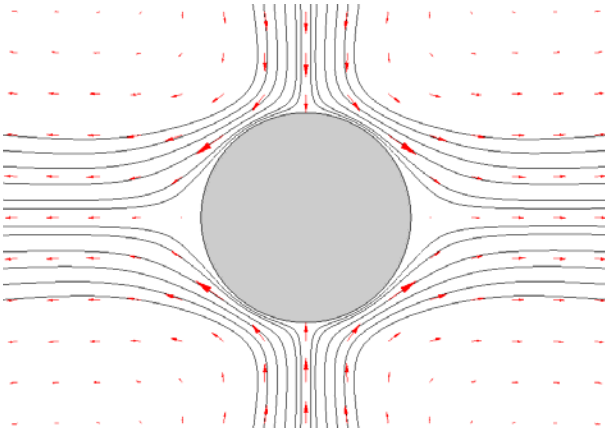


Fig 5. Velocity field and streamlines nearby oscillating cylinder

It was shown that according to proposed model under Stroukhal number $\mathrm{St} \approx 0.1$ and Reynolds number $\mathrm{Re} \approx 100$ velocity of stationary flow nearby oscillating cylinder could achieve 15% of its oscillating velocity.

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