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**WAVEGUIDE PROPERTIES THE ACUT WEDGE PLATE**

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*Waves propagating along the wedge edge are considered. Mathematical model is proposed describing spatially located wave beams which are a form of movement along the edge. There are given theoretical researches. There are of wave fields formed about the edge, which satisfy the movement equations and boundary conditions. Obtained dispersion relation for waveguide modes. Model structure of the wave beam well goes with experimental observations.*

## 1. Introduction

As it is known [1], along the wedge edge a surface wave may propagate, its energy concentrated in narrow range that result's in its using in waveguides.

However, wide using of wedge waveguides is limited because of the lack of knowledge about main feature of wave processes. For example, some experimental results cant be explained withies the framework of the planar wave model. The classic analysis of wave characteristics of these elastice waves [2] results in the conclusion that the wave velocity is unalterable from the wedge angle but it doesn't go with experimental, data that is caused by the fact of no taking into account the main features of propagation of small disturbances in the wedge.

## 2. Formalion

The solution of the problem must satisfy the movement equation, which in terms of wave potentials has a form:

$$\Delta\Phi + k_\ell^2\Phi = 0, \qquad \Delta\Psi + k_\ell^2\Psi = 0, \tag{1}$$

and standard boundary conditions absence of tensions on a free surface.

In the expression (1) $k_\ell$, $k_t$ – wave number longitudinal and transverse waves, Δ – Laplasion operator in cylinder system of coordinates is binded (ρ, θ, z) with axis z along the edge of wedge:

$$\Delta = \frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2}{\partial\theta^2} + \frac{\partial^2}{\partial z^2}$$

For harmonical case and because of linear character of equations (1) it is possible to split the motion of small disturbances for two directions:

to the normal to the edge wedge:

$$\frac{\partial^2\Phi_1}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial\Phi_1}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2\Phi_1}{\partial\theta^2} + k_\ell^2\Phi_1 = 0,$$

$$\frac{\partial^2\Psi_1}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial\Psi_1}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2\Psi_1}{\partial\theta^2} + k_t^2\Psi_2 = 0, \tag{2}$$

and along the edge:

$$\frac{1}{\rho^2}\frac{\partial^2\Phi_2}{\partial\theta^2} + \frac{\partial^2\Phi_2}{\partial z^2} + k_\ell^2\Phi_2 = 0,$$

$$\frac{1}{\rho^2}\frac{\partial^2\Psi_2}{\partial\theta^2} + \frac{\partial^2\Psi_2}{\partial z^2} + k_t^2\Psi_2 = 0, \tag{3}$$

In [3] it is shown propagation of oscillations to the normal to the edge hase some specific features. While approaching the wedge edge the surface wave turns into symmetrical and asymmetrical modes, their velocities vary in different way. The symmetrical mode velocity decreases while affroaching the edge and velocity of the second component increases. The asymmetrical mode gets up to the edge but the definite distance forming a «blocking» which doesn't let longitudinal component move towards the edge.

The impose of incident and reflected waves occurred together with the appearance of stagnant way. It is significant, that the energy of acoustic wave is not transferred.

On fig. 1 for the time moment correspondent with maximum oscillation of stagnant water, dependencies of amplitude of wave replacement from the distance to the wedge edge for angles of the wedge б$_1$ = 5,7° (continuous line) and б$_2$ = 2,9° (dotted line), where $U_r$ – the amplitude of Rayleigh wave are shown.

The solution of the second problem by a classic method [2] results in dispersion relations for asymmetric (m = 1) and symmetric (m = -1)modes of a surface wave:

$$\frac{(k^2+s_2^2)^2}{4s_1s_2k^2}-\left[\frac{\operatorname{th} s_2\rho\theta}{\operatorname{th} s_1\rho\theta}\right]^m=0 \qquad (4)$$

In the expression (1) $s_1$ , $s_2$ – coefficients of amplitude diminution of longitudinal and shear displacements the further-getting from the boundary and are defined as:

$$s_1=\sqrt{k^2-k_\ell^2}\ ,\ s_2=\sqrt{k^2-k_t^2}\ .$$

At big spatial coordinates the second item in [4] tends to one resulting the well known Rayleign equation. Notices that this wave structure remains unalterable while spatial coordinates continue vary.

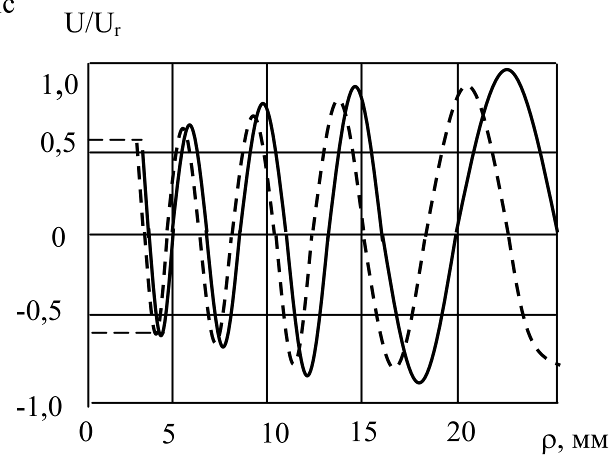


***Fig.1*** Spatial view of deffusion of stagnant wave

At small values of coordinates the wave structure begins to vary sharply and it's reasonable toconsider (4) together with expressions (2). While the wave velocity is constant along the edge the solution is stable at the coordinate ρ when the displacements amplitude becomes corresponding the crest of the wave.

**3. Analyses and conclusion**

Far from the edge solution of dispersion equation (4) form continuous spectrum of a field and near the edge – a discrete one. The wave structure near the edge is of a complex character. Towards the edge displacements amplitude changing is determined by a stagnant wave condition and along the edge becomes thit of a running harmonic wave.

The wave velocity is determined by a coordinate which to the condition of the best wave generation that is the point of maximum displacements amplitude. On a Rayleign wave it occurs when it's velocity is determined by boundary conditions on free surface where displacements amplitude is maximum.

The observed above effect is of the jam physical nature as when Lamb wave of high modes generates [4]. Shiftily the wave is forming as a stagnant one but while frequency increases a complex spatial picture appears: over the plate width the wave has a form of stagnants waves and along the plat – that of running ones.

Symmetric mode of a surface wave is unstable. The further it gets from the wedge edge its velocity decreases resulting in deflection of the depth of media and leaving the channel. Besides the change of wave spectrum structure occurs getting a stagnant Rayleign wave.

Asymmetric mode will approach the edge up to the area where longitudinal oscillations can't move to. The extent of this blocking changes with increase of the wedge angle and the track of the wave will remove from the edge of the wedge. Simultaneously the local wedge thickness increases (e.d. thickness of the wedge in the track of the wave in the crest) that causes the increase of the wave velocity. The wave structure and velocity are determined by the coordinate structure of perturbation. Hear the boundary of the blocking the lowerest mode of the surface wave generates and change of displacements amplitude occurs during one half period. Perturbations of particles of media far from the wedge edge results in increasing of a number of half period and wave velocity. The width of wave channel is determined by the number of these half periods.

Fig. 2 and 3 show dependencies of displacements amplitude of second and the sixth mode on the distance to the wedge edge for the wedge edge ( – the coordinate of the wave generates)

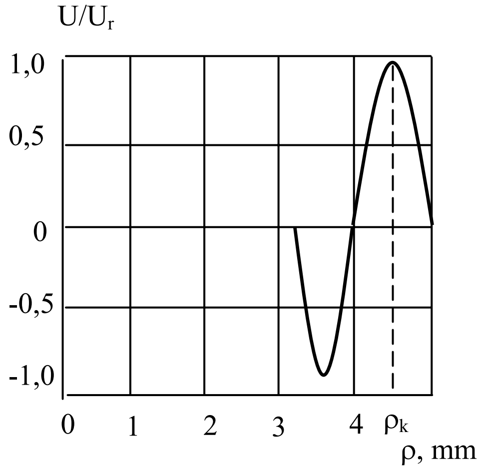


***Fig.2.*** Changes of displacement amplitude of the sixth waterguide mode from spatial coordinate

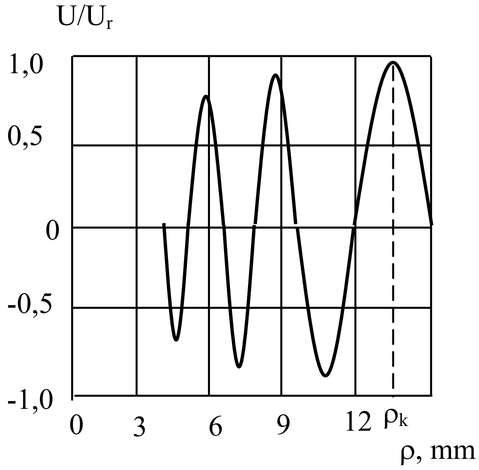


***Fig.3.*** Changes of displacement amplitude of the second waterguide mode from spatial

Let's not that the increase of the wedge edge results in narrowing of the channels and at big angles they practically interflow. This explains difficulties of generating of high modes of edge angle wave.

Fig. 4 shows theoretical dependencies of velocity of mode 2 and 6 of edge wave on wedge angle ($C_r$ – the velocity of Rayleign wave) by continuous lines, tested curves are shown by dotted lines from [5].

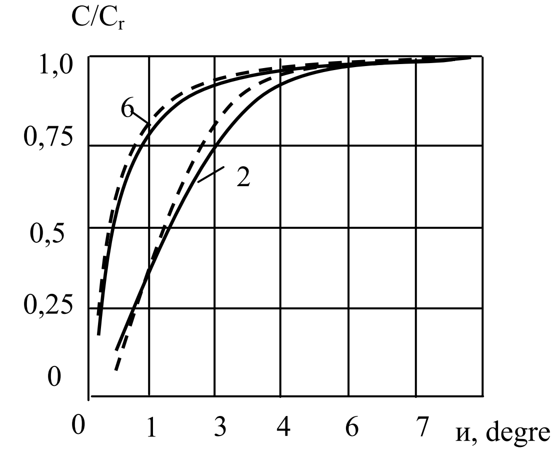


***Fig.4.*** Dependencies of velocites of second and sixth waveguide mode on edge angle

The theoretical research and calculations allow to clear the problem for all its aspects. Technically two kinds of movement appear when vibrations arise: towards the wedge edge ( only at the, beginning till arising of the stagnant wave) and asymmetrical movement along the edge. Regularity of spatial changes of field structure removing from the edge much depends on wedge angle.

The coincidence of theoretical dependencies with experimental curves shows that suggested physical-mathematical model is right.

**R E F E R E N C E S**

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