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RESONANCE OF FREQUENCY SHIFTING, CAUSED BY INTERNAL WAVES ON OCEANIC SHELF

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The results of theoretical analysis of frequency shifting of sound field interference structure, caused internal waves on oceanic shelf are presented in the paper. It is shown, that amplitude of frequency shifting depends upon modal structure and frequency of sound field significantly.

As known [1], due to waveguide dispersion the response of the medium on short signal of source "spreads" and becomes like frequency-modulated signal. That leads to forming of characteristic frequency-time interference structure of sound field. Media perturbation (for example internal waves) causes the variations of dispersion and leads to variations of interference structure of sound field. In the paper [2,3] is offered using of that acoustic effect for reconstruction spectrum of internal waves (IW). In given paper the theoretical analysis of frequency shifting (caused by IW) dependence on modal structure and sound field frequency.

Let us present shallow water oceanic waveguide as water layer limited by free surface $z = 0$ and homogeneous absorbing half-space - bottom $z = H$, with density ρ_1 and square refraction index: $n_1^2(1+i\alpha)$, where $n_1 = c(H)/c_1$, α - is defined by absorbing features of bottom. Square refraction index of water layer is presented as sum of two terms: $n^2(\vec{r},z,t) = n^2(z) + \mu(\vec{r},z,t)$. The first of them $n^2(z)$ corresponds to non-perturbed stratification of waveguide (Figure 1), the second one $\mu(\vec{r},z,t)$ - perturbation, caused by IW. Expression for $\mu(\vec{r},z,t)$ has form:

$$\mu(\vec{r},z,t) = -\frac{2\delta c(\vec{r},z,t)}{c(z)} = 2QN^2(z)\zeta(\vec{r},z,t), \text{ where } N(z) = \left(\frac{g}{\rho}\frac{d\rho}{dz}\right)^{1/2} \quad (1)$$

Here δc - variation of sound speed, caused by displacement of surface of constant density, $N(z)$ - buoyancy frequency, determined by density stratification of water layer, g - gravitational acceleration; $Q \approx 2.4$ s^2/m - constant, ζ - vertical displacements of water layer, caused by IW. In this paper the model IW offered in [4] is used:

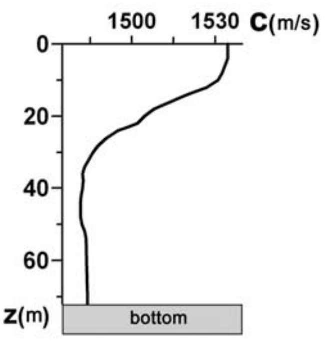

Fig.1 Sound speed profile

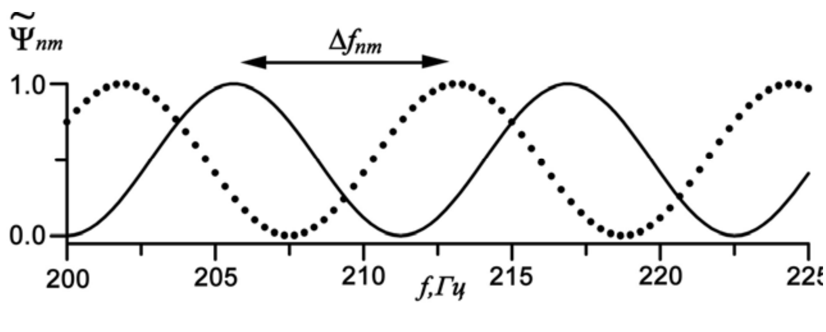

Fig.2 Frequency shifting

Sound field on frequency f, radiated by point source in point with coordinates $(\vec{r}, z)$ is determined by Helmholtz equation:

$$\Delta\Psi(\vec{r}, z, f) + k^2\left(n^2(z) + \mu(\vec{r}, z)\right)\Psi(\vec{r}, z, f) = 0 \qquad (2)$$

Solution of equation (2) is presented in form:

$$\Psi(\vec{r}, z, f) = \sum_n \frac{C_n(r, f)\psi_n(\vec{r}, z, f)}{\sqrt{\xi_n r}} \exp(i\int_0^r \xi_n(r')dr') \qquad (3)$$

where $\psi_n(\vec{r}, z, f)$ - modes, $\xi_n = q_n + i\gamma_n/2$ - corresponding wavenumbers.

Let us consider interference structure of two modes with numbers n and m in receiver point. The maximum of interference structure is on frequency $\bar{f}_{nm}$ in non-perturbed waveguide, and is on frequency f_{nm} in perturbed waveguide. We note Δf_{nm} frequency shifting, caused by perturbation of waveguide (see. Figure 2) :

$$\Delta f_{nm} = f_{nm} - \bar{f}_{nm} \qquad (4)$$

Value of frequency shifting, caused by waveguide perturbation, can be presented in form:

$$\Delta f_{nm} = \left(\frac{\partial\bar{\alpha}_{nm}}{\partial f}\right)^{-1} \tilde{\alpha}_{nm} \qquad (5)$$

Here $\bar{\alpha}_{mn}(f) = \bar{q}_n(f) - \bar{q}_m(f)$, $\alpha_{mn}(f) = q_n(f) - q_m(f)$ - difference of wavenumbers in non-perturbed waveguide and perturbed waveguide correspondently, $\tilde{\alpha}_{mn}(f) = \alpha_{mn}(f) - \bar{\alpha}_{mn}(f)$ is perturbation of value $\bar{\alpha}_{mn}(f)$.

Let us consider within framework of perturbation theory the perturbation $\tilde{q}_n$ of value $\bar{q}_n$ ($\tilde{q}_n(f) = q_n(f) - \bar{q}_n(f)$), caused by perturbation of stratification of water layer:

$$\tilde{q}_n = \frac{k^2}{2q_n}\int_0^H \bar{\psi}_n^2(z, f)\mu(\vec{r}, z)dz \qquad (6)$$

where $\bar{\psi}_n(z)$ – mode with number *n* in non-perturbed waveguide. Let us suppose that perturbation of water layer due to internal waves. In this case from (6) obtain expression:

$$\tilde{q}_n(f) = \bar{q}_n\nu_n(f)\zeta, \text{ где } \nu_n(f) = \frac{Qk^2}{2\bar{q}_n^2}\int_0^H \bar{\psi}_n^2(z, f)N^2(z)\Phi(z)dz \qquad (7)$$

Perturbation of difference of wavenumbers $\alpha_{mn}(f)$, caused by presence of internal waves, is determined by expression:

$$\tilde{\alpha}_{nm} = \tilde{q}_n - \tilde{q}_m = \left(\bar{q}_n\nu_n(f) - \bar{q}_m\nu_m(f)\right)\zeta \qquad (8)$$

It is follows from formula (5), that frequency shifting of interference structure of modes with numbers n and m, caused by internal:

$$\Delta f_{nm} = \left(\frac{\partial\alpha_{nm}}{\partial f}\right)^{-1}\left(\bar{q}_n\nu_n - \bar{q}_m\nu_m\right)\zeta \qquad (9)$$

Let us consider dependence of perturbation of modal refractive index $\nu_n(f)$ from sound field frequency. As known the buoyancy frequency $N(z)$ has significant maximum in thermocline region has and significantly less out of thermocline region. That is why value $\nu_n(f)$ is defined by overlap integral mode n with thermocline region. The dependence of vertical structure of modes with numbers 1-6 from sound field frequency in frequency interval is presented on figure 3. One can see from Figure 3 the vertical structure of mode comes down to bottom while increasing of frequency. It should be noted that maximal value of mode with number 1 is low thermocline region in total frequency region. That is why dependence $\nu_1(f)$ for mode with number 1 is smoothly decreasing curve. For modes with number 2-6 behavior of vertical structure is different from behavior of mode 1. Maximum of mode is above thermocline for low-frequency range. Then maximum of mode passes through thermocline while increasing of frequency. Maximum of mode is below thermocline for high-frequency range.

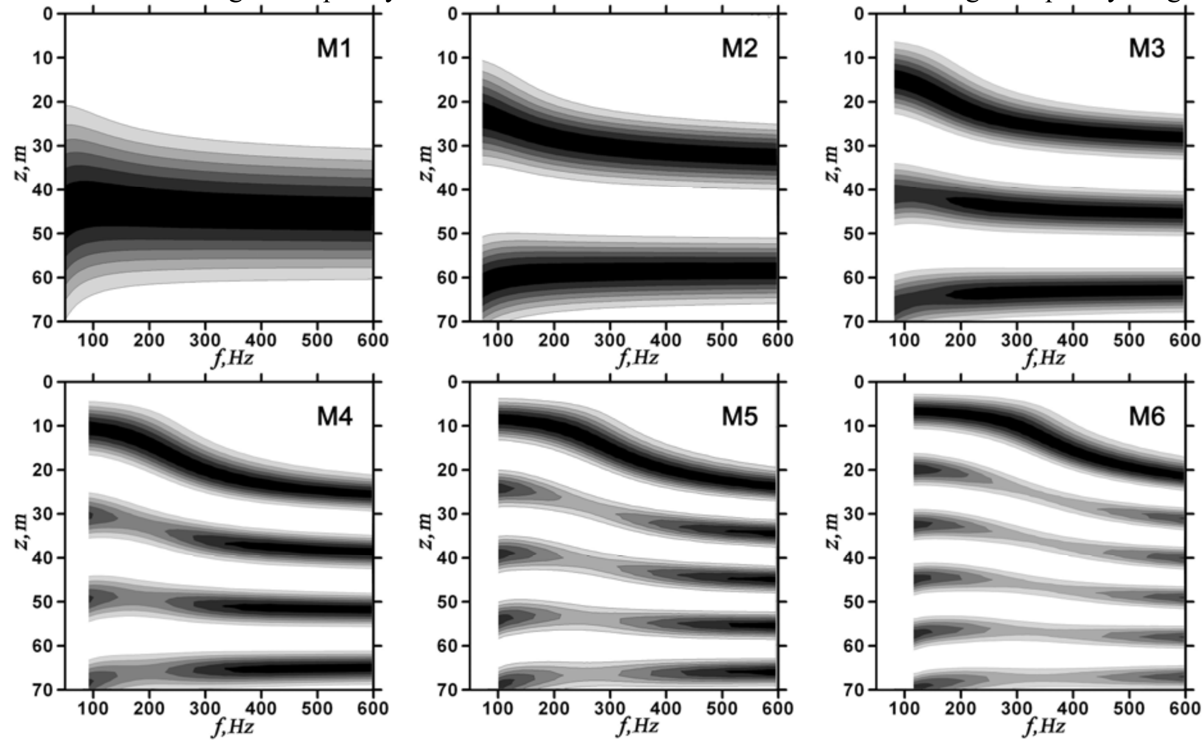

Fig. 3 Frequency dependence of vertical structure of modes $\psi_n(z, f)$ with numbers 1-6

The described above frequency dependence of vertical structure of mode leads to resonance-like frequency dependence of $\nu_n(f)$. Resonance frequency corresponds to location of mode maximum in thermocline region. The different modes have different resonance frequency. The frequency dependence of modal refractive index $\nu_n(f)$ for modes with numbers 1-6 in frequency range 50-600 Hz is presented on Figure 4. Such frequency dependence of $\nu_n(f)$ leads to sharp frequency dependence of difference of modal wavenumbers $(\bar{q}_n\nu_n(f)-\bar{q}_m\nu_m(f))$ So as $\nu_1(f)$ monotonically decreasing value, then difference between perturbation of wavenumbers of modes with numbers 1 and m: $(\bar{q}_1\nu_1(f)-\bar{q}_m\nu_m(f))/f$, divided by sound field frequency, has same features of frequency dependence that curves $\nu_m(f)$ presented on Figure 4.

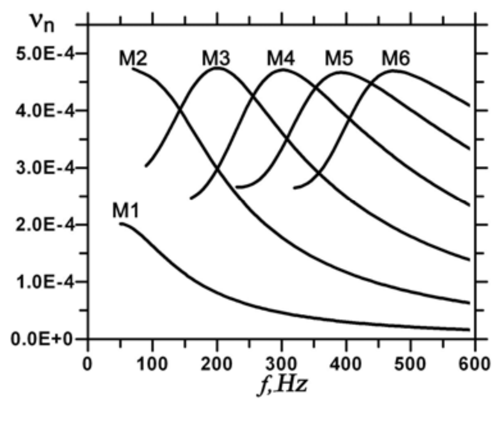

Fig. 4. Frequency dependence of perturbation of modal refractive index.

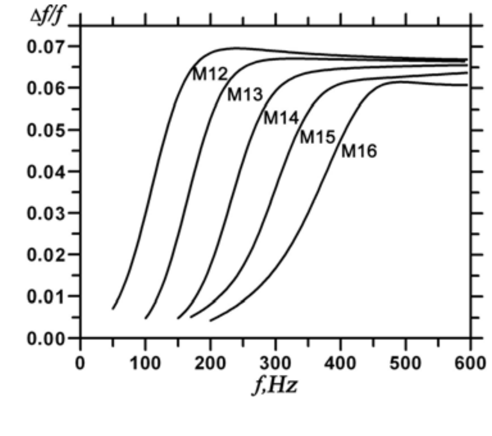

Fig. 5. Dependence of relative frequency shifting as function of sound field frequency.

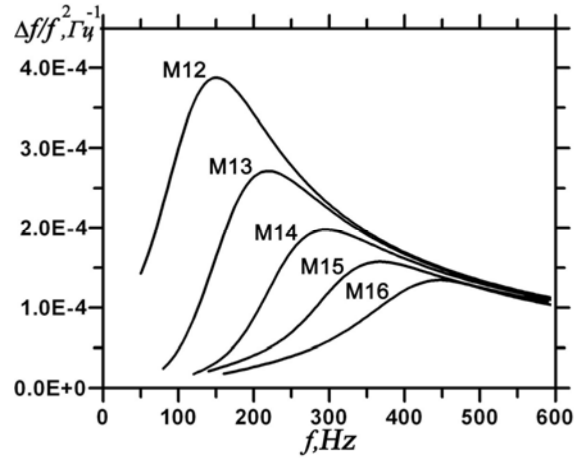

Fig. 6. Dependence of relation of frequency shifting to square of sound field frequency.

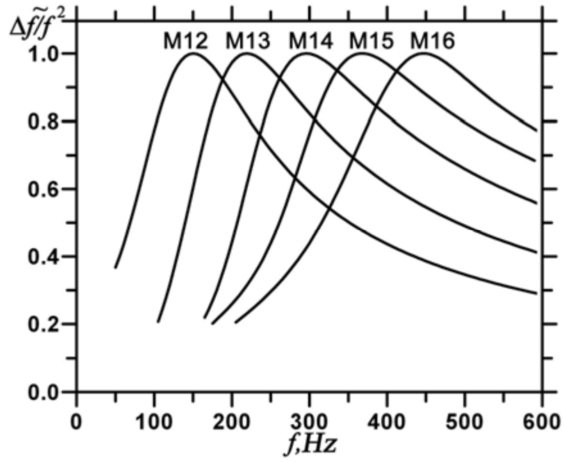

Fig. 7. Normalized dependence of relation of frequency shifting to square of sound field frequency.

The dependence of relative frequency shifting as function of sound field frequency for pair of modes with numbers $n = 1$ and $m = 2,3,4,5,6$ is presented on Figure 5. It is supposed that $\zeta = 1$ m. One can see form Figure 5 for given pair of modes with numbers 1 and m on low frequency, when mode with number m is "bottom-surface" mode, relative frequency shifting $\Delta f / f$ is sufficiently small (less 2%). While frequency increasing the mode with number m comes down to bottom, i.e. pass from group of "bottom-surface" modes in group of "bottom" modes. At that relative frequency shifting $\Delta f / f$ starts to increase and equals to 7%. After mode passing below thermocline i.e. when mode is "bottom" the value $\Delta f / f$ insignificantly decreases and stop changes. Dividing value $\Delta f / f$ on f, we compensate dependence of $\partial\alpha_{nm}/\partial f$ from frequency. In result we obtain value $\Delta f / f^2$, which has frequency dependence like frequency dependence $\nu_m(f)$. The frequency dependence of value $\Delta f / f^2$ for modes with numbers $n = 1$ and $m = 2,3,4,5,6$ is presented on Figure 6. The frequency dependence of value $\Delta f / f^2$ normalized by maximum is presented on Figure 7. One can see from Figure 7 that curves presented on this figure like to curves presented on Figure 4.

Work is supported by "Dynasty" Foundation.

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