B.G. Katsnelson, A.V. Tshoidze

PHASE FRONT PERTURBATION OF SOUND WAVE IN SHALLOW WATER IN PRESENCE OF MOVING INTERNAL SOLITONS

Voronezh State University
Russia, 394006 Voronezh, Universitetskaya sq., 1
E-mail: katz@phys.vsu.ru

Phase front distortion of sound wave, propagating from point source in shallow water in presence of internal waves is studied. It is clear that cylindrical front without perturbations (plain in long distances from source) is distorted in presence of intensive internal waves. At approximation of horizontal rays this is horizontal refraction, i.e. direction of plane wave propagating is changed. In general case form of phase front is distorted. The feature of phase front perturbations effects in shallow water is dependence on frequency and number of vertical modes. A theoretical analysis is carried out for the vertical modes and parabolic equation in horizontal plane. We consider spatial and temporal phase front fluctuations due to train of internal waves moving across acoustic track and analyze ability to measure phase front distortions using horizontal arrays. Computational modeling is carried out for conditions close to continental shelf of East cost of USA [1].

As considered previously both in theory and in experiment, intensive internal waves (IW, internal solitons, IS) cause significant perturbations of low frequency sound field. In works [2, 3] fluctuations of sound filed intensity, resulting from horizontal refraction due to location of acoustic trace approximately parallel to wave front of IWs packet, are analyzed. Organization of experiment for measurements of the intensity fluctuations supposed use of vertical antennas, with which one can make mode filtration. It is of interest to consider fluctuations of direction of sound propagation in horizontal plane, which takes place for mentioned orientation of an acoustic track.

Let's consider signal propagating in shallow water channel in presence of IWs (fig. 1). Shallow water oceanic environment is 3D hydroacoustic waveguide in Cartesian coordinates *(X, Y, Z)*. Waveguide consists of water layer with density $\rho(z)$ and sound field profile $c(x,y,z)=c_0(z)+\delta c(x,y,z)$, where $c_0(z)$ is unperturbed sound speed profile, $\delta c(x,y,z)$ is perturbation corresponding to changing of acoustic properties due to IW. Water layer is limited: $0\le z\le H$.

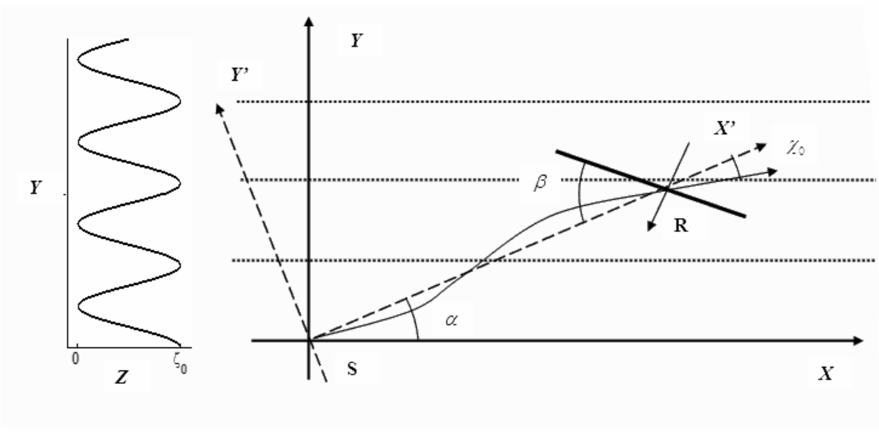

Fig. 1. Scheme of experiment. Cartesian coordinate *X* is parallel to wavefront of IWs, coordinates X' ais directed along acoustic trace, α is angle between track and wavefront of solitons, β is angle between track and antenna, χ is angle of horizontal refraction. Envelope of IS ζ_0 is shown in the left

Let's construct phase on horizontal antenna, lying at the bottom within the framework of theory of horizontal rays and vertical modes [4], i.e. when field amplitude is

$$\Psi(\vec{r},z)=\sum_n \psi_n(H)\psi_n(z)A_n(x,y)\exp[i\theta_n(x,y)], \tag{1}$$

where $A_n(x,y)$, $\theta_n(x,y)$ are amplitude and eikonal of the horizontal ray for the n-th mode. Eikonal satisfies two-dimensional equation

$$(\nabla_\perp \theta_n)^2 = q_n^2 . \quad (2)$$

We can find eikonal using perturbation theory. If addition to wave number due to internal waves is small $\delta q_n << q_n$, then $q_n^2 = (q_n^0)^2 + 2 q_n^0 \delta q_n$,
where

$$\delta q_n = -\frac{\nu_n(\omega)}{q_n^0} \zeta_s(y) , \quad (3)$$

$\zeta_s(y)$ is envelope of IWs, and

$$\nu_n = \frac{2Qk^2}{(q_n^0)^2} \int_0^H \left[\psi_n^0(z)\right]^2 N^2(z)\Phi(z)dz , \quad (4)$$

where $N(z)$ the Brunt-Vaisala frequency, Q~2.4 s^2/m, $\Phi(z)$ is the first gravity mode. So if we suppose that $\theta_n(x,y) = \theta_n^0(x,y) + \delta\theta_n(x,y)$, we take $\theta_n^0 = q_n R - q_n \rho \sin\alpha$.
So in the first order of perturbation theory

$$\delta\theta_n = -\frac{\nu_n(\omega)}{q_n^0} \int_0^{R-\rho\sin\alpha} \zeta_s(y)ds \quad (5)$$

Here we take integral along straight line joining source and point of observation (unperturbed trajectory). Let's suppose for simplicity that length of array is less than wave length of solitons: $\int_0^{R-\rho\sin\alpha} \zeta_s(y)ds \sim \int_0^R \zeta_s(y)ds - \zeta_s(y_R - vT)\rho\sin\alpha$.

Let's measure phase of arrived rays in different points of antenna at different time (or in different phase of solitons) (fig. 2). Rays at different time for different source frequency and mode number arrives with different phases, so phase fluctuations in different points is differ.

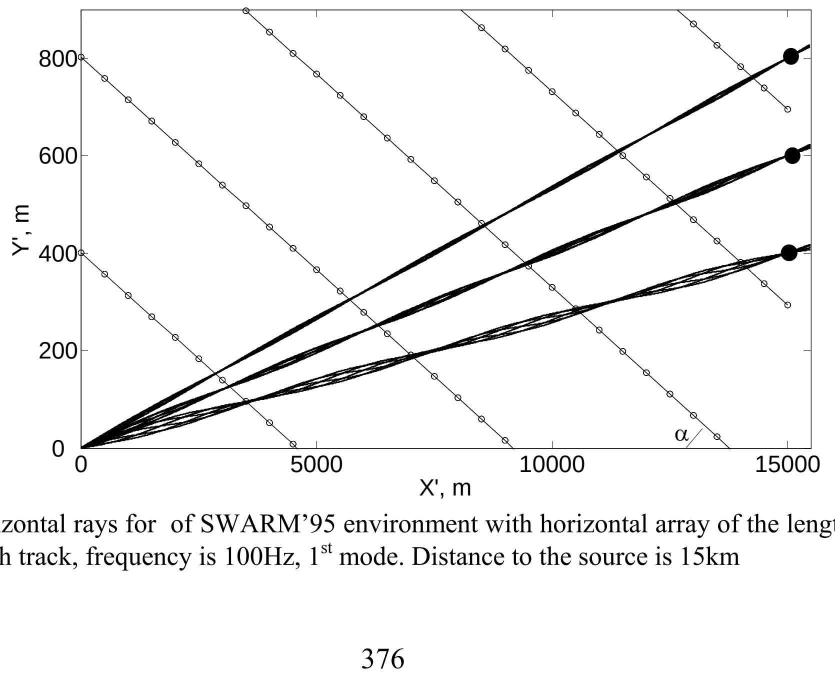

Fig. 2. Horizontal rays for of SWARM'95 environment with horizontal array of the length ~ 400m and angle α=5° of IS with track, frequency is 100Hz, 1st mode. Distance to the source is 15km

Using horizontal array we can estimate shape of wave front along antenna. In the first approximation this front can be supposed to be plane, and in absence of internal waves and wave front revolution corresponds to revolution of direction of array.

In presence of internal waves horizontal refraction also causes revolution of wave front Angle of revolution (angle of refraction) depends on frequency and mode number and changes in time. In other words different modes come to the receiver with different phase and correspondingly different compensation angle is necessary for determination of maximum of directivity of received energy [5, 6]. So using horizontal array we can get both modes decomposition and directovity of the sound propagation.

Response of horizontal array so

$$u(\beta,\gamma) = \int_{-L/2}^{L/2} \Psi(\vec{R}+\vec{\rho}, z) e^{-ik\rho\cos\gamma} d\rho\,, \tag{6}$$

where γ is compensation angle of the array. In our case response of horizontal array is

$$u(\beta,\gamma) = L\sum_n \frac{\psi_n(z_1)\psi_n(z)}{\sqrt{q_n R}} \exp[i(q_n R)] \sin c\left[\left(q_n^0 + \frac{\zeta_s(y_R - vT)\nu_n(\omega)}{2q_n^0}\right)\cos\beta - k\cos\gamma\right]. \tag{7}$$

After calculation of the angle of compensation for different times and averaging over time we can get angle of horizontal refraction

$$\chi(\omega,\beta) = \arccos\left(\frac{k\cos\gamma}{q_n^0}\right) - \beta\,. \tag{8}$$

In the figures 3 and 4 we show temporal dependence of angles of horizontal refraction, calculated on the base of formula (8) . Dependence on frequency and modes number is determined by refraction index (formula 7).

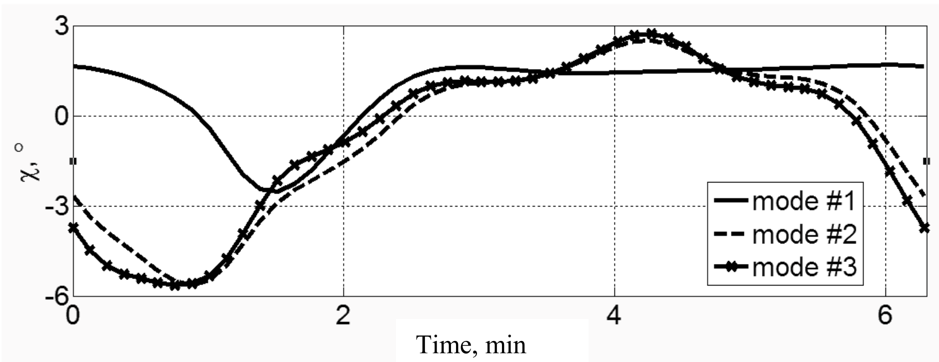

Fig. 3. Horizontal refraction angles for different modes versus time for environment SWARM'95 with horizontal line array of the length 400м at the angle α=2°, with track, frequency is 100 Hz, distance to the source is 15 km

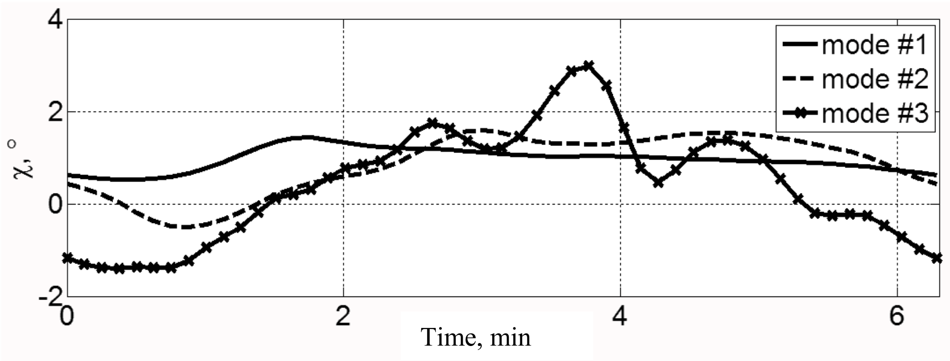

Fig. 4. Horizontal refraction angles for different modes versus time for environment SWARM'95 with horizontal line array of the length 400м at the angle $\alpha=2°$, with track, frequency is 200 Hz, distance to the source is 15 km

Remark in conclusion that for separate times angles of horizontal refraction for different modes are the same , however in general case situation is more complex and for determination of horizontal refraction it is necessary to carry out modes filtering.

Work was supported by RFBR, grant 06-05-64853

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