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ACOUSTIC EFFECTS DUE TO MODE COUPLING BY INTERNAL WAVES IN SHALLOW WATER

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Theoretical and experimental data analysis is carried out for the propagation of broadband sound pulses in shallow water in the presence of internal solitons (IS) which are moving approximately along an acoustic track. For this geometry, mode coupling is taken to be the main mechanism of influence of the IS on the sound field. The focus of this of research is on the sound intensity and arrival time fluctuations produced by moving solutions. Results of the theoretical analysis are compared with experimental data from the SWARM'95 experiment.

It is commonly known that internal waves in the coastal ocean are one of the main causes of sound field fluctuations for transmissions over different space and time scales. In this paper we continue research [1]. As an example, this effect can be observed if a IS (or a train) propagates approximately along an acoustic track. (see Fig.1).

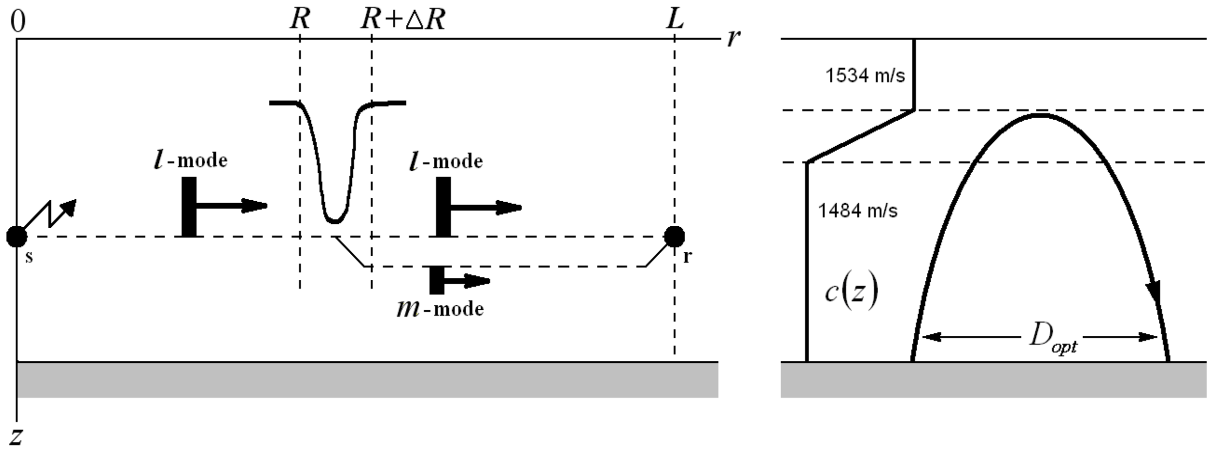

Fig.1.Left – creation of modes as a result of coupling, right – ray pattern.

In this situation, sound pulses radiated from the source, corresponding to a sum of separate normal modes, create additional modal pulses in the area of the waveguide perturbation, each propagating with their own group velocities, and these in turn change the sound field at the receiver.. We understand this variability as temporal fluctuations. Typical frequencies of fluctuations are ~1-10 cph. In this paper, we will carry out analysis of

- the arrival times of pulses, decomposed over normal modes, versus frequency;
- the temporal (or equivalently, range) correlation function of received signals versus frequency

The above mentioned analyses are appropriate to one of experiments we carried out within the framework of the SWARM'95 [3] shallow water acoustics experiment (see Fig.2). In this experiment, a sequence of broadband pulses (in the 30-200 Hz band) was radiated every minute from an airgun and received at a vertical line array (NRL VLA) 18 km distant from the source. During this experiment, several trains of IS were recorded propagating toward the shore. The wave fronts of these internal wave packets were directed at an approximately 35^{o} angle with the acoustical track. Details of this experiment (sound speed profile, bottom geoacostic data, etc.) were presented in the paper [3]. We note here some of the specific features of the waveguide and its oceanographic perturbation due to the IS, which will be taken as the environmental model for our analyses:

- a narrow thermocline layer (between 10 and 35m , of ~ 90m total depth);
- approximately constant soliton shapes and velocities (v_s ~ 0.5-1m/s);

- the small size ΔR of the IS, i.e. less horizontal extent than the acoustic track R (ΔR ~ 300-1000m << R ~15-20 km).

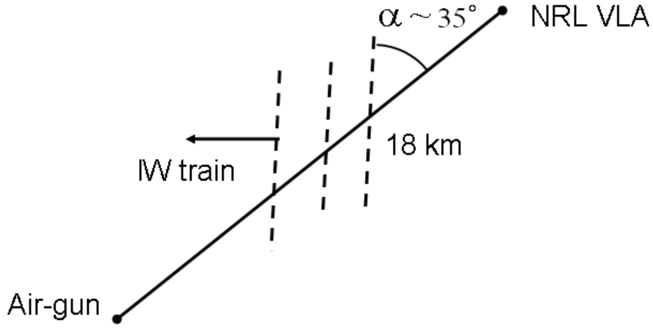

Fig.2

Let us consider a point source with spectrum $S(\omega)$, placed at the depth z_s in a shallow water waveguide. We assume that the unperturbed waveguide has constant depth H, a sound speed profile $c(z)$ and bottom parameters c_1, ρ_1. In our work here, we will neglect bottom attenuation, which in principle can be easily taken into account. At the point of observation (r,z) we have the Fourier decomposition of the sound field pressure:

$$P(r,z,t) = 2\int_0^\infty P_\omega(r,z)e^{-i\omega t}d\omega \qquad (1)$$

Our modeling techniques will be based on the decomposition of the sound field into waveguide modes $\psi_l(z)$, whose eigenvalues are denoted as q_l. In the absence of perturbations, we write

$$P_m = iS(\omega)\sum_l \frac{\psi_l(z_s)\psi_m(z)}{\sqrt{8\pi i q_m r}} S_{ml}\exp(i\Delta q_{lm}R) \qquad (2)$$

We next suppose that there is a perturbation of the sound field due to a localized, intense internal wave (generically called a soliton), moving with constant velocity v (if there is some angle α between the direction of the acoustic track and the wave front of the IS, then the real velocity of the IS is $v_s = v\sin\alpha$, see Fig.2) . At a given moment of time, denoted as T, this perturbation occupies an area at the range $R < r < R + \Delta R$, where $R = vT$,and ΔR is the “length” of the soliton. Thus as the result of the presence of internal waves, we have a small addition to the sound speed profile in the area $\delta c(r,z,T)$ which depends on the “slow” time T [4]. As is well known, this moving sound speed perturbation produces fluctuations of the sound field. Again, we suppose that mode coupling is the main mechanism responsible for the perturbation of the sound field. For this case, expression (2) is true in the area $r < R$. After acoustic interaction with the soliton, taking place in the area $R < r < R + \Delta R$, we have another modal decomposition for the sound field. We will describe this decomposition using S-matrix formalism, so that in the area $r > R + \Delta R$

$$P_\omega(r,z;R) = S(\omega)\sum_{m,l} \frac{\psi_l(z_s)\psi_m(z)}{\sqrt{8\pi i q_m r}} S_{ml}(R+\Delta R,\omega)\exp\left[i\left(q_m(r-R) + q_l R\right)\right] \qquad (3)$$

where the S-matrix satisfies the equation with “initial” condition:

$$\frac{d\mathbf{S}}{dr} = \mathbf{WS}, \quad \mathbf{S}(R) = \mathbf{I} \qquad (4)$$

In the above, $\mathbf{I}$ is the unit matrix, and the coefficient of interaction between modes in simple perturbation theory is

$$W_{ml}(r) = i\frac{k^2\exp[i(q_l - q_m)r]}{\sqrt{q_m q_l}}\int_0^H \frac{\delta c(r,z)}{c}\psi_m(z)\psi_l(z)dz \qquad (5)$$

Because the “slow” time is $T = R/v$, sound pressure at the receiver depends on time T.

Using Eq. (3), we can now find the amplitude of mode m created by mode coupling from all propagating modes l

$$P_m = S(\omega)\sum_l \frac{\psi_l(z_s)\psi_m(z)}{\sqrt{8\pi i q_m r}} S_{ml} \exp(i\Delta q_{lm} R) \tag{6}$$

where $S_{ml} = S_{ml}(R+\Delta R,\omega)$, and $\Delta q_{lm} = q_l - q_m$

The group velocity of an individual mode is

$$v_l^{gr} = \left(\frac{dq_l}{d\omega}\right)^{-1} \tag{15}$$

If the length of an acoustic track is L, then arrival time of a non-coupling mode is

$$t_l = \frac{L}{v_l^{gr}} \quad . \tag{16}$$

In this case, we have for its arrival time, denoted as t_{lm}, the following expression:

$$t_{lm} = \frac{R}{v_l^{gr}} + \frac{L-R}{v_m^{gr}} \tag{17}$$

So the corresponding diagram of arrival times contains set of curves: due to modes, passing without coupling and modes, created by soliton. Intensity of the corresponding signals depend on frequency according to (6) in accordance with intensity of coupling, in other words the more modes coupling the more is intensity of created mode. In our interpretation the most significant interaction takes place between adjacent modes having the turn point within thermocline. In ray language it corresponds to modes shown in the fig.1 in the right side. These rays have approximately the same ray cycle and, correspondingly the same group velocity (in our case it is 1463 m/s). So the most strong interaction takes place at the frequency where mentioned modes have this group velocity.

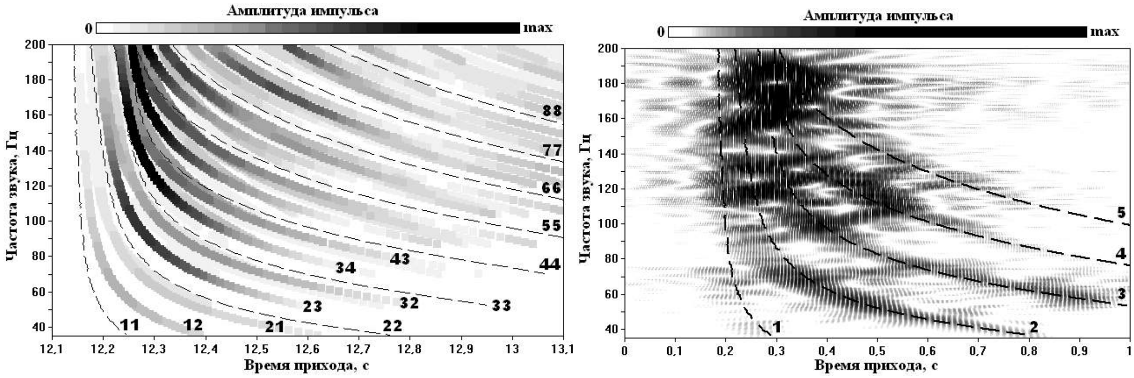

Fig.3 Arrival times of pulses, corresponding to separate modes versus frequency. Left – theory, right – one of experimental pulses. Number of modes are shown in the figure, two digits denote modes, created by soliton

In the fig.3 (left) theoretical dependence of intensity of created modes is constructed in gray scale of color according to (6). We can see that dark area do not depend on the sound frequency and . correspond to one and the same group velocity or arrival time 12.25 – 12.35 s In the right panel the corresponding experimental picture is shown, where dark spots are concentrated in comparatively narrow area of arrival times (0.25-0.35 s). In experimental figure we take another starting time than in theoretical figure as a result of processing of experimental data.

One of the important characteristics of the variability of the sound field is its correlation function, which is the subject of many studies (see for example the recent paper [5]). We can introduce the temporal correlation function for intensity at fixed frequency as:

$$\Gamma_\omega(\tau) = \int_0^{\Delta T} I_\omega(T) I_\omega(T-\tau)\, dT \tag{18}$$

If the temporal variability is a result of motion with a constant velocity, then we can connect range and time via $T = \dfrac{R}{v}$, $\tau = \dfrac{\Delta R_p}{v}$ and therefore we can work with the correlation functions $\Gamma_\omega(\Delta R_p)$.
Generally speaking, the horizontal (range dependent) correlation function is often created by theoretical modeling, because we must construct this function from data at many different positions, data that is sometimes not readily available. However, the temporal correlation function can be constructed more simply from a single experimentally measured temporal sequence of received pulses

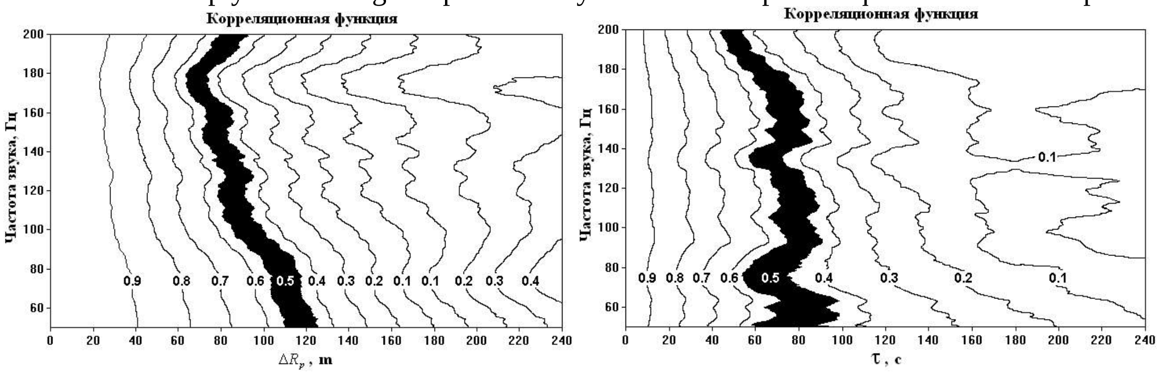

Fig.4 Correlation function. Left panel – theory, right panel - experiment

We can see that correlation time (correspondingly correlation length) depends on frequency smoothly. Correlation length, calculated in accordance with the level 0.5 corresponds to motion of the soliton through periodic interference structure with predominating scale $D_{opt} = 700 - 800$ m. Comparison with the experiment (using level 0.5) gives velocity of soliton $v = \Delta R_p / \tau = 1.1 - 1.2$ m/s.

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R E F E R E N C E S

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