

L.V.Gubko, I.N.Kanevsky, I.V.Timoshenko
THE ACCOUNT OF INFLUENCE OF ROTATIONAL ULTRASONIC STREAMS ON
PROCESSES HEAT TRANSMISSION AND MASS CARRY

FESTU, INSTITUTE OF INFORMATION SCIENCE,
 RADIO-ELECTRONICS AND ELECTRIC
 ENGINEERING, Department of hydroacoustics
 Pussia, 690600 Vladivostok, Pushkinckaya 10
 Telephone: (4232) 45-09-82; Fax: (4232) 45-45-77
 E-mail:ga_irieta@festu.ru

In the paper outcomes result of mathematical model operation of non-stationary process of heat transmission in a system solid – fluid, in conditions of influence of the small-scale rotational ultrasonic streams originating in a sound high-power field near to a boundary of liquid-solid phases. During model operation the population of the basic indexes of a considered physical system has been shown to the known differential heat conduction equation, which have been submitted concerning the dimensionless parameters. Such representation allows to generalize the obtained outcomes to processes mass carry, by replacement of the relevant expressions of the dimensionless parameters, in view of their common similarity. Expression for heat transmission coefficient in conditions of a ultrasonic convection, in which there are free of imperfections to earlier published outcomes. Required expression is submitted as a combination of the dimensionless parameters with numerical coefficients, and has been obtained by two methods. In the first case from a dimensional analysis the nonlinear regression with unknowns numerical coefficients composed. In the second case - as a result of an analytical solution of a boundary value problem for a heat conduction equation, in an approximation of a boundary layer. The analysis of the obtained solutions shows their analogousness.

Let's consider a problem about calculation of heat transmission parameters in the simplified statement by the example of a thermodynamic system with the one-dimensional thermal stream, diagrammatically shown on figure 1.

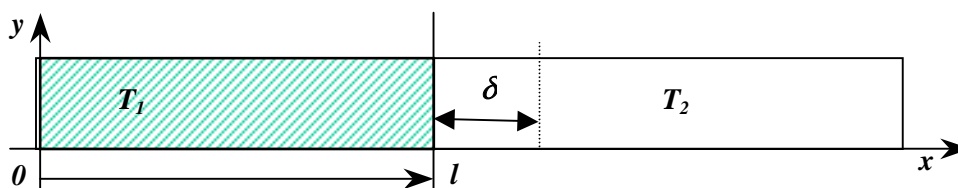


Fig.1 Pictorial model of a thermodynamic system.

Let's consider, that a medium with an index 1 – solid, and a medium 2 – a fluid. If in an initial instant $t=0$ in a fluid there is a uniform distribution of temperature T_2 , and in solid – T_1 , and $T_1 \neq T_2$, in a system will happen heat transmission from a medium to the greater temperature in a medium with smaller. Process will be completed, when in a system the state of a temperature balance will be established.

Problem about calculation of heat transmission parameters we shall formulate as follows. Unlimited on y the plate thickness l , having, in an initial instant $t=0$, in all points reference temperature T_0 , on the one side is washed by a fluid with temperature T_2 , other side of a plate is thermal protected. The temperature of a fluid during heat exchange is maintained by a stationary value. Heat of a plate happens on the part of a fluid. Thermalphysic parameters of a skew field and a medium (α , ρ , C) are known and do not vary during all process, thus the thermal conduction in the plate is more, than in a fluid: $\alpha_1 > \alpha_2$. In a fluid sound waves with known parameters (γ , f , c_0) from indefinitely removed on an axis x a radiant are spread. It is necessary to find a spatial distribution of temperature in the plate in the arbitrary instant $t > 0$.

The heat transfer in a solid medium will happen due to a thermal conduction. In a fluid influence on a heat transfer, except for a thermal conduction, mass transfer appearances render. In conditions of a sound field of major intensity, in a medium the effect of origin of the rotational streams, influencing to heat transmission will be observed, this appearance can be considered as a ultrasonic convection. It is typical for a ultrasonic convection is that the stationary values of force calling streams arise on boundaries with rigid mediums and consequently values of velocities of streams near to hindrances much more, than on removal from a demarcation.

For exposition of the appearances happening within the framework of model, we shall take advantage of a heat conduction equation for the one-dimensional case:

$$\partial T / \partial t = \chi \partial^2 T / \partial x^2, \quad (1)$$

где: $\chi = \alpha_1 / \rho C_V$ - a solid medium thermal diffusivity.

Let's consider, that the origin of coordinates of a considered thermodynamic system is in the beginning thermal protected side of a plate, then boundary requirements for the equation (1) will be noted as follows:

- 1) initial requirement at $t=0$ и $0 \leq x \leq l$ $T_1 = T_0$;
- 2) boundary requirement at $x=0$ и $t > 0$ $(\partial t / \partial x)_0 = 0$;
- 3) boundary requirement at $x=l$ и $t > 0$ $-\alpha_1 (\partial t / \partial x)_l = \lambda (T_C - T_2)$,

where T_C - temperature of a surface of the plate, washed by a fluid.

The solution of a problem in a general view can be presented as function of explanatory variables x and t and parameters of process $\chi, \lambda, \alpha_1, l, T_0, T_2$:

$$T = f(t, x, \chi, \lambda, \alpha_1, l, T_0, T_2). \quad (2)$$

To method of a similarity, we shall reduce requirements of a problem in the dimensionless aspect. For this purpose we shall take advantage of parameter of the dimensionless temperature:

$$\theta = \mathcal{G} / \mathcal{G}_0 = (T_2 - T) / (T_2 - T_0), \text{ где } \mathcal{G} = T - T_2 \text{ и } \mathcal{G}_0 = T_0 - T_2, \quad (3)$$

and the dimensionless coordinate: $X = x / l$. (4)

It means, that as a scale for measuring temperature magnitude \mathcal{G}_0 is used, and as a scale of length – magnitude l . After substitution (3) and (4) in (2) and necessary transformations [1] we shall receive:

$$\partial \theta / \partial Fo = \partial^2 \theta / \partial X^2, \quad (5)$$

where $\theta = (T_2 - T) / (T_2 - T_0)$ - the dimensionless temperature, $X = x / l$ - the dimensionless coordinate, $Fo = t \chi / l^2$ - Fourier number. If to consider magnitude l^2 / χ [second] as some time scale the Fourier number can be treated as the dimensionless time. In view of it, boundary requirements will be noted as:

$$1) \text{ at } Fo=0 \quad \theta = 1, \quad (6)$$

$$2) \text{ at } X=0 \quad (\partial \theta / \partial X)_0 = 0, \quad (7)$$

$$3) \text{ at } X=l \quad (\partial \theta / \partial X)_l = Bi \theta_l, \quad (8)$$

where $\theta_l = (T_2 - T_l) / (T_2 - T_0)$ - the dimensionless temperature on boundary of a solid body,

$$Bi = l \lambda / \alpha_1 \text{ - Bio number.}$$

Required function now has the following aspect:

$$\theta = f(Fo, Bi, X). \quad (9)$$

The equations noted in the dimensionless shape, gain the generalized sense as to a particular numerical value of Fo and Bi numbers there corresponds a set of problems with various values $(\chi, \lambda, \alpha_1, l)$, i.e. it is possible to consider a similarity of processes of a transient heat conduction. The obtained solutions can as be to extend to processes mass carry if to take advantage of expressions of Fo and Bi numbers for a diffusion.

In a fluid medium, near to a demarcation, at presence of convection, some area (boundary layer) is formed. The appearances happening in this area, determine a velocity of heat exchange through boundary, therefore in a considered case expediently to proceed from reviewing allocation of a velocity and temperature in all volume of a fluid medium, to their reviewing only in immediate proximity from a demarcation with a solid medium. The streams originating at it in a boundary layer, it is possible to count stationary. Let's count parameters of the ultrasonic field, not varying in time and feebly depending from temperature. These assumptions allow to use more simple equation of transposition of heat in a boundary layer.

The [2] calculated relations obtained for static process of diffusion in a stagnant sound wave at a solid wall are known. For calculations expressions for Rayleigh ultrasonic streams have been used. The obtained outcomes have shown, that ultrasonic streams render essential influence on a velocity of diffusion in fluid, that well agreed with experiment. Was as it is marked, that the good coordination with experiment has been obtained at rather small intensities of a ultrasonic field. At magnification of intensity, the experimental values of diffusion appeared much more, in some cases on the order, than calculated. It allows to make the supposition, that at major intensities the greater influence on a diffusion is rendered with the small-scale streams originating immediately in boundary layer (of Schlichting) and turbulizing of it.

For the account of these streams we shall take advantage of expression of a tangential component velocity of a ultrasonic stream in a viscous boundary layer near to the boundary obtained Schlichting [2]:

$$v_y = -(\mu - \mu^2)(\sin 2ky)v_0^2/4c_0; \quad (10)$$

where $\mu = x'/\delta$, $\delta = \sqrt{2\gamma/\omega}$ - thickness of a ultrasonic boundary layer,

$\omega = 2\pi f$ - circular frequency of ultrasonic oscillations, $\gamma = \eta/\rho$ - kinematic viscosity,

v_0 - amplitude of an oscillatory velocity, c_0 - velocity of a sound in a fluid medium.

For a determination of magnitude of heat transmission coefficient λ it is necessary to spot the functional relation between parameters (indexes of properties) defining its value. Follow principles of a similarity theory, it is more convenient to search for a required functional ratio through thermal transmission coefficient of boundary layer which can be presented as a combination of dimensionless complexes of its parameters (similitude parameters) and numerical coefficients. Using a procedure of the analysis of dimensionality, it is possible to receive expression for required magnitude in a general view. Starting from formulas (1) and (10) it is possible to spot, that the thermal transmission coefficient in conditions of a ultrasonic convection will depend on the following defined parameters:

$$\chi' = F(v, f, \chi, \gamma, c_0). \quad (11)$$

Following a procedure, it is possible to note outcome as product of the initial parameters submitted by their dimensionalities, in some degrees. Having lead necessary transformations we shall receive the criteria equation circumscribing heat exchange on a demarcation of phases at the presence of sound waves, concerning the dimensionless parameters:

$$\chi' = A \cdot \text{Pr}^{-c} \cdot (v_0/\sqrt{f\gamma})^a \cdot (c_0/\sqrt{f\gamma})^k \sqrt{f\gamma}, \quad (12)$$

$$\text{or } (\chi'/\sqrt{f\eta/\rho}) = A(p/f\eta)^B \text{Pr}^{-c}, \quad (13)$$

where $\text{Pr} = \gamma/\chi$ - Prandtl number, p - sound pressure,

$p/f\eta$ - the dimensionless number, describing a degree of influence of a ultrasonic field on thermal (and heat) transmission and mass carry,

$\chi'(f\eta/\rho)^{-1/2}$ - the dimensionless number describing thermal transfer of boundary layer in conditions of a ultrasonic convection.

a , c and k it is possible to spot numerical values of coefficients by mathematical model operation analytically or observationally, exploring a physical analog. The heat transmission coefficients can be determined from a ratio:

$$\lambda = \chi \rho C_v / \delta \quad (14)$$

For analytical definition of coefficients of the equation (12) it is possible to take advantage of the equation of a boundary layer (Prandtl equation) for temperature:

$$v_x \partial T / \partial x' + v_y \partial T / \partial y = \chi \partial^2 T / \partial x'^2 \quad (15)$$

Boundary conditions for it will be:

$$1) \text{ at } x = 0 \quad T = T_{cm} = T_2 - T_1; \quad 2) \text{ at } x \rightarrow \infty \quad \lim_{x \rightarrow 0} T(x) = 0 \quad (16)$$

For simplification of an input equation, due to elimination of a normal component velocity of a stream, we shall take advantage of a variable "flow function" (substitution of Mises)[3]. Let's enter a flow function such, that $\psi = const$ - a streamline, thus:

$$v_y = \partial\psi/\partial x \text{ and } v_x = -\partial\psi/\partial y. \quad (17)$$

Let's express derivatives from (15), through ψ :

$$\partial T/\partial x = \partial T/\partial\psi (\partial\psi/\partial x) = v_y \partial T/\partial\psi, \quad (18)$$

$$\partial T/\partial y = \partial T/\partial\psi (\partial\psi/\partial y) = -v_x \partial T/\partial\psi, \quad (19)$$

$$\partial^2 T/\partial x^2 = v_y \partial/\partial\psi (v_y \partial T/\partial\psi). \quad (20)$$

After substitution (18, 19, 20) in (14) and cuttings v_y , we shall receive the equation:

$$\partial T/\partial y = \chi \partial/\partial\psi (v_y \partial T/\partial\psi). \quad (21)$$

According to requirements of a considered problem we shall substitute in the equation (21) expression (10) for a tangential component velocity of a ultrasonic stream in a viscous boundary layer. At $x \rightarrow 0$ magnitude - $\mu \ll 1$, therefore the square-law term can be neglected, then expression (10) will become:

$$v_y = -(x'/\delta) \left(\nu_0^2/4c_0 \right) \sin 2ky, \text{ where } x' = x - l. \quad (22)$$

From expression for a flow function (17) since $x' \rightarrow 0$, supposing, that $x' = \partial x'$ and $\psi = \partial\psi$ it is possible to express $x' = \psi/\nu_y$, after substitution in (22) we shall receive:

$$v_y = -(\nu_0/2) \left(\sqrt{\sin 2ky} \sqrt{\psi} / \sqrt{c_0 \delta} \right) \quad (23)$$

Substitution (23) in (21) gives the equation:

$$\left(1/\sqrt{\sin 2ky} \right) \partial T/\partial y = -\left(1/\sqrt{c_0 \delta} \right) \left(\nu_0 \chi/2 \right) \partial/\partial\psi \left(\psi^{1/2} \partial T/\partial\psi \right). \quad (24)$$

This partial equation on two variables y and ψ . For cutting an amount of variables and, thus, transformations of the equation (24) to an aspect of the ordinary differential equation, suppose, that the considered physical system has scaling property. Then instead of variables y and ψ it is possible to enter some dimensionless variable, such, that:

$$T(y, \psi) = T(\xi), \quad (25)$$

where $\xi = A(Y(y))^\alpha (U(\psi))^\beta$ - the dimensionless complex of the functional parameters with stationary values in coefficients.

Generally, to spot a gang of magnitudes, defining ξ it is possible, having decided a nonlinear problem on eigen functions for the equation (24). For a considered case the aspect of the dimensionless variables can be spotted by immediate selection of combinations of parameters at partial derivatives. From available parameters the combination of four dimensionless complexes at which the problem has a correct solution is possible:

$$\xi = \sqrt{\psi} \nu_0 \delta \sqrt{\sin 2ky} / \chi \sqrt{c_0 \delta}, \quad (26)$$

$$\xi = \left(\sqrt{\psi} / \nu_0 \delta \right) \chi \sqrt{\sin 2ky} / \sqrt{c_0 \delta}, \quad (27)$$

$$\xi = \sqrt{\psi} \nu_0 \delta / \chi \sqrt{c_0 \delta \sin 2ky}, \quad (28)$$

$$\xi = \sqrt{\psi} \chi / \nu_0 \delta \sqrt{c_0 \delta \sin 2ky}. \quad (29)$$

Let's consider a solution of a problem concerning replacement (26). After substitution (26) in (24) we shall receive the equation:

$$\left(8c_0 k \cos 2ky / \nu_0^2 \sin^2 2ky \right) \psi \partial T/\partial\psi = -\partial^2 T/\partial\xi^2. \quad (30)$$

Let's express ψ from (26), as:

$$\psi = \xi^2 c_0 \chi^2 / v_0^2 \delta \sin 2ky. \quad (31)$$

Having substituted (31) in (30), after direct transformations we shall receive the equation:

$$d^2T/d\xi^2 + a\xi^2 dT/d\xi = 0, \quad (32)$$

$$\text{where } a = 8c_0^2 \chi^2 k \cos 2ky / v_0^4 \delta \sin^3 2ky. \quad (33)$$

The obtained equation (33) is ordinary differential second-kind equation. For lowering a degree of equation we shall produce replacement:

$$dT/d\xi = P, \quad (34)$$

and we shall pass to the linear differential equation of the following aspect:

$$dP/d\xi + a\xi^2 P = 0. \quad (35)$$

The common decision of the equation (35) is known and has the following aspect:

$$P = C \times e^{-\xi^3 a/3}. \quad (36)$$

Let's substitute a solution (36) in (34) and we shall integrate the obtained expression on ξ :

$$T = C_1 \int_0^\xi e^{-\xi^3 a/3} d\xi + C_2. \quad (37)$$

For definition C_2 we shall take advantage of a boundary requirement (16) for $x = 0$. Thus $\xi = 0$ and $T = T_{cm}$, from here it is possible to receive:

$$C_2 = T_{cm}. \quad (38)$$

Expression for C_1 can be received from a boundary requirement (16) for $x \rightarrow \infty$. Thus it is possible to note:

$$T = 0 = C_1 \int_0^\infty e^{-\xi^3 a/3} d\xi + T_{cm},$$

from here it is possible to express:

$$C_1 = -T_{cm} / \int_0^\infty e^{-\xi^3 a/3} d\xi. \quad (39)$$

Let's consider an integral in a denominator of expression (39):

$$\int_0^\infty e^{-\xi^3 a/3} d\xi \quad (40)$$

Let's enter a variable t such, that:

$$t = \sqrt[3]{a/3} \xi, \text{ then } dt = \sqrt[3]{a/3} d\xi. \text{ After substitution in (40) it is possible to receive:}$$

$$1/\sqrt[3]{a/3} \int_0^\infty e^{-t^3} dt. \quad (41)$$

The obtained integral in expression (41) is Γ -function and its value is determined from the table or by means of computer program (for example Mathcad). Having produced necessary evaluations, we shall receive:

$$1/\sqrt[3]{a/3} \int_0^\infty e^{-t^3} dt = 0,893/\sqrt[3]{a/3} = 1,29/\sqrt[3]{a}$$

Having substituted the obtained expression in (39) we shall receive:

$$C_1 = -T_{cm} \sqrt[3]{a}/1,29. \quad (42)$$

Substituting expressions (38) and (42) in the common decision (37), we shall receive:

$$T = T_{cm} \left(1 - \sqrt[3]{a}/1,29 \int_0^\xi e^{-a/3\xi^3} d\xi \right). \quad (43)$$

Let's express function $T(x)$ on boundary:

$$T|_{x=0} = T_{cm} \left(1 - \sqrt[3]{a}/1,29 \xi \right). \quad (44)$$

Let's discover expression for a variable ξ through x from formulas (22), (23) and (31). Equating (22) and (23) we shall receive: $\sqrt{\psi} = x \nu_0 \sqrt{\sin 2ky} / 2\sqrt{c_0 \delta}$,

from (31): $\sqrt{\psi} = \xi \sqrt{c_0 \delta} \chi / \nu_0 \delta \sqrt{\sin 2ky}$.

Having equated the obtained expressions and having expressed ξ it is possible to receive:

$$\xi = x \left(\nu_0 \sqrt{\sin 2ky} / 2\sqrt{c_0 \delta} \right) \cdot \left(\nu_0 \delta \sqrt{\sin 2ky} / \sqrt{c_0 \delta} \chi \right) = x \cdot \nu_0^2 \sin 2ky / 2c_0 \chi \quad (45)$$

Let's substitute (45) in (44):

$$T|_{x=0} = T_{cm} - T_{cm} \left(\sqrt[3]{a}/2,58 \right) \left(\nu_0^2 \sin 2ky / c_0 \chi \right) \cdot x. \quad (46)$$

Differentiating (46) on x , we shall receive:

$$\partial T / \partial x|_{x=0} = -T_{cm} \left(\sqrt[3]{a}/2,58 \right) \left(\nu_0^2 \sin 2ky / c_0 \chi \right) \quad (47)$$

Let's substitute in (47) expression for a from (33) and having lead direct transformations we shall express a current density of heat on boundary:

$$j = \chi' \partial T / \partial x|_{x=0} = -1,096 \cdot T_{cm} \cdot \sqrt[3]{\left(\nu_0^2 \chi^2 k / c_0 \delta \right) \cos 2ky}. \quad (48)$$

Let's discover a medial value \bar{j} on phase of rotational structure:

$$\bar{j} = -1,096 \cdot T_{cm} \cdot \Omega \cdot 1/\pi \int_0^\pi \sqrt[3]{|\cos 2ky|} dky = -0,903 \cdot T_{cm} \cdot \Omega,$$

where $\Omega = \sqrt[3]{\nu_0^2 \chi^2 k / c_0 \delta}$ [m/sec].

Having expressed Ω through the dimensionless complexes of parameters, we shall receive:

$$\bar{j} = -2,014 \cdot T_{cm} \cdot \left(\nu_0 / \sqrt{f\gamma} \right)^{2/3} \left(c_0 / \sqrt{f\gamma} \right)^{-2/3} (\gamma / \chi)^{-2/3} \cdot \sqrt{f\gamma} \quad (49)$$

From here it is possible to express average thermal transmission coefficient in the following aspect:

$$\chi' = \bar{j} / T_{cm} = 2,014 \cdot \text{Pr}^{-2/3} \cdot \left(\nu_0 / \sqrt{f\gamma} \right)^{2/3} \left(c_0 / \sqrt{f\gamma} \right)^{-2/3} \sqrt{f\gamma}, \quad (50)$$

where $\text{Pr} = \gamma / \chi$ – Prandtl number. In such aspect it does not differ under the shape from expression (12) for head transmission coefficient, obtained of a dimensional analysis. For passage to reviewing mass carry it is necessary to take advantage of the relevant expression for Prandtl number: $\text{Pr} = \gamma / D$, where D is molecular diffusion coefficient, in this case (50) will express mass carry coefficient on border in conditions acoustic convection.

REFERENCES

1. Under edition of professor. Guigo E.I. Theoretical bases of refrigerating engineering. Moscow: Agropromizdat, 1986, (in Russian).
2. Under edition Rozenberg L.D. Principal physics of a ultrasonic process engineering. Moscow: Nauka, 1970, (in Russian).
3. Samarsky A.A., Vabishevich P.N. Computing heat transfer. Moscow: Editorial URSS, 2003, (in Russian); Wiley, 1995.