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ABOUT STATISTICAL PROPERTIES OF THE VIBRATIONS INDUCED BY NEAR-WALL TURBULENT PRESSURE

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Statistical characteristics of fluctuations of resilient members under influence of a field near-wall turbulent pressure are considered. The main attention is given to Poisson model of a field which is based on conception of a turbulent pressure field generation due to spontaneous splashes in near-wall region of flow. Influence of a flow regime on parameters of characteristic functional of Poisson model of a field is discussed. Relation of these parameters and statistics of fluctuations of resilient members of a streamlined surface is analyzed. The executed modelling estimations show, in particular, that the member can be considered as a dot element when its characteristic size makes less than thirds of scale of splash.

Detailed statistical properties of fluctuations of construction units at a turbulent flow are essential in problems of maintenance of durability and a service life, especially at presence of the nonlinear interactions which are taking place at mutual contact of vibrating elements. The isolated and rather important problem connected with this question is diagnostics of stochastic properties of a turbulent stress field which is based on the statistical analysis of the signal generated by the resilient electrodynamic transducer of the finite size.

Further it is supposed, that relation between random speed v of a resilient member oscillatory moving at some control point and a turbulent pressure $p(\boldsymbol{x}, t)$ field on a streamlined surface $\boldsymbol{x}$ is linear and representable in the integral form:

$$v = \int p(\boldsymbol{x},t)\,\upsilon_v(\boldsymbol{x},t)\,d\boldsymbol{x}\,dt \cdot \tag{1}$$

Here $\upsilon_v(\boldsymbol{x}, t)$ is Green's function characterized the response v of the member on pulse action at the point $\boldsymbol{x}$ and the instant time t. Stochastic properties of speed v are completely defined by characteristic function $\phi_v(\lambda)$, that is Fourier image of frequency distribution function. In view of (1), characteristic function $\phi_v(\lambda)$ is represented by the relationship

$$\phi_v(\lambda) = \left\langle \exp(i\lambda v)\right\rangle = \left\langle \exp[i\int p(\boldsymbol{x},t)\,\lambda\upsilon_v(\boldsymbol{x},t)\,d\boldsymbol{x}\,dt]\right\rangle \tag{2}$$

(angular brackets $\langle\ldots\rangle$ mean statistical averaging).

The expression standing in the right part (2) represents nothing else than characteristic functional [1] of the turbulent pressure field, defined on the family of the functions proportional to Green's function $\upsilon_G(\boldsymbol{x}, t)$.

In case of stationary processes, for which spectral representation on random amplitudes $dP(x,\omega)$ is right

$$p(x,t) = \int e^{i\omega t} dP(x,\omega), \tag{3}$$

equality (2) can be presented in the form of

$$\phi_v(\lambda) = \left\langle \exp[i\int \lambda V_v(\boldsymbol{x},\omega)\,dP(x,\omega)\,d\boldsymbol{x}]\right\rangle, \tag{2a}$$

where function

$$V_v(\boldsymbol{x},\omega) = \int e^{i\omega t}\upsilon_v(\boldsymbol{x},t)\,dt \tag{4}$$

(complex conjugate to the Fourier image of $\upsilon_v(\boldsymbol{x}, t)$) corresponds to reaction of the element to harmonic loading in a point $\boldsymbol{x}$. As applied to a lateral oscillation of a thin infinite plate without taking into account reaction of environment

$$V_v(\boldsymbol{x},\omega) = \frac{1}{8\sqrt{mD}}[H_0^{(2)}(k_b r) - H_0^{(2)}(-ik_b r)] \tag{5}$$

[2], where m, D are accordingly mass per unit area and bending stiffness of plate, $k_b = (\omega^2 m / D)^{1/4}$ is bending wavenumber, r is distance from a current point $\boldsymbol{x}$ up to a control point, $H_0^{(2)}(y)$ is Hankel function of second kind and order zero.

In problems for which the field of influencing turbulent pressure can be considered not only stationary, but also homogeneous, the similar to (2a) frequency-wave representation of integrand in the right part (2) is convenient. At that relationship is true

$$\phi_v(\lambda) = \left\langle \exp[i\int \lambda W_v(\boldsymbol{\kappa},\omega)\,d\Pi(\boldsymbol{\kappa},\omega)] \right\rangle, \tag{2б}$$

in which expansion is carried out on random wave components $d\Pi(\boldsymbol{\kappa},\omega)$; function $W_v(\boldsymbol{\kappa},\omega)$ represents reaction of the element to excitation by a wave pressure defined by a wave vector $\boldsymbol{\kappa}$ and frequency ω. For a special case of a thin infinite plate

$$W_v(\boldsymbol{\kappa},\omega) = \frac{1}{i\omega m}\frac{k_b^4}{k_b^4 - \kappa^4}\,. \tag{6}$$

If the energy-transporting frequency-wave region of a turbulent pressure field satisfies to condition

$$|\boldsymbol{\kappa}| >> k_b\,, \tag{7}$$

that, obviously,

$$W_v(\boldsymbol{\kappa},\omega) = \frac{i\omega}{D}\frac{1}{\kappa^4}\,. \tag{8}$$

Passing to characteristic function $\phi_\eta(\lambda)$ of lateral displacement η of element, we shall note, that for it the analogue of function (8) is represented in a considered case in the form of

$$W_\eta(\boldsymbol{\kappa}) = \frac{1}{D\kappa^4} \tag{9}$$

and does not depend on frequency, hence, as a result probabilistic properties of displacements are defined by spatial characteristic functional of turbulent loading field:

$$\phi_\eta(\lambda) = \left\langle \exp[i\int \lambda W_\eta(\boldsymbol{\kappa})\,d\pi(\boldsymbol{\kappa})] \right\rangle, \tag{10}$$

where

$$d\pi(\boldsymbol{\kappa}) = \int_\omega \frac{\partial \Pi(\boldsymbol{\kappa},\omega)}{\partial\omega}d\omega\,. \tag{11}$$

For actual estimations of probabilistic vibrating characteristics (2, 10) the assignment of characteristic functional of turbulent pressure field generating fluctuation is necessary. In connection with the developed modelling representations [3] the Poisson field is of special interest, which the spatial characteristic functional

$$\Phi_P[\upsilon(\boldsymbol{x})] = \exp\{\nu\int[\chi(\int g(\boldsymbol{x}-\boldsymbol{y})\upsilon(\boldsymbol{x})d\boldsymbol{x}) - 1]d\boldsymbol{y}\} \tag{12}$$

is caused by a set of statistically independent equiprobability distributed chaotic splashes in wall zone of flow. The particular form of the functional (12) is evaluated by its internal parameters: average quantity ν of splashes per unit area, characteristic function $\chi(\mu)$ of pressure fluctuation in a splash core, and also by normalized influence function $g(\boldsymbol{r})$, that defines a characteristic scale l of the splash. With reference to a turbulent boundary layer the last one can be estimated as $l = (100\ldots1000)\nu/u^*$, where $u^* \equiv (\tau / \rho)^{1/2}$, ν is kinematic viscosity, τ is the friction force per unit area of wall surface acting from flow.

Wave representation of (12) is resulted by substitution of functional argument in the form of Fourier integral:

$$\Psi_p[W(\boldsymbol{\kappa})] = \Phi_p\left[\frac{1}{4\pi^2}\int e^{i\kappa x} W(\boldsymbol{\kappa}) d\boldsymbol{\kappa}\right]. \tag{13}$$

At that as argument of function χ in (12) the value

$$\frac{1}{4\pi^2}\int e^{i\kappa y}\, \gamma^*(\boldsymbol{\kappa})\, W(\boldsymbol{\kappa})\, d\boldsymbol{\kappa} \quad , \tag{14}$$

will serve, where $\gamma^*(\boldsymbol{\kappa})$ is complex conjugate to the Fourier image of influence function $g(\boldsymbol{r})$.

Further the statistics of fluctuations of the "piston" resilient member which has been built in a streamlined structure is considered. Current deformations of an member are supposed as linearly depended only from spatial distribution of pulsating loading on its receiving surface, where $\upsilon(\boldsymbol{x})$ = const. The given scheme models work of electrodynamic (in particular, piezoelectric) transducer under turbulent pressure field.

For the purpose of detailed estimations we shall consider special model of the Poisson turbulent pressure field formed by splashes, inducing "own" pressure which change on time is shown in fig.1a. The simplified piecewise-linear representation of dependence $P(t)$ is based on qualitative data and materials of observations [4] of wall splashes in a boundary layer and assumes instant increase of pressure at an initial stage of splash, then its sufficiently fast decrease with small «suction» and further gradual equalization with average pressure. Here we assume determinancy of function $P(t)$ owing to what the probability of a finding of magnitude P in some values range for statistical assembly of splashes is simply proportional to residence time of individual splash in this range. It is easy to see, that thus the frequency distribution function $f(P)$ of pressure P fluctuations in a splash core proves to be the step function shown in fig.1б.

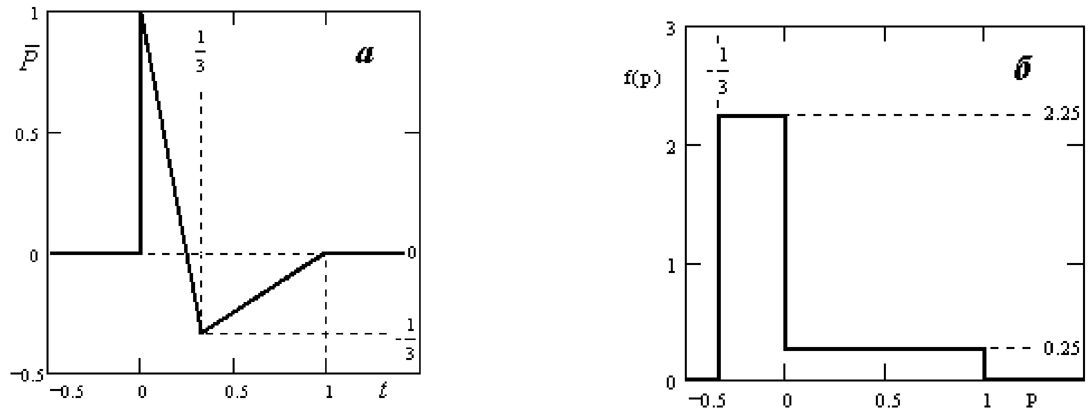

Fig.1. Parameters of Poisson field. ***a*** – temporal variation of "own" pressure P of individual splash; ***б*** – frequency distribution function of relative "own" pressure.

Accordingly the characteristic function $\chi(\mu)$ representing the Fourier image of frequency $f(P)$, is expressed by formula

$$\chi(\mu) = -[\frac{i}{4\mu}(8 - 9\cdot\exp(\frac{\mu}{3i}) + \exp(i\mu)] \tag{15}$$

We accept, that influence function $g(\boldsymbol{r})$ of splash is symmetric, i.e. $g(\boldsymbol{r})= g(r)$, and

$$g(r) = 1 - U(r - \rho), \tag{16}$$

where $U(x)$ is step function [$U(x) = 0$ at x <0 and $U(x) = 1$ at $x \geq 0$], ρ is radius of affected zone.

The round "piston" receiver of the unit area and unit sensitivity is considered, for which

$$\upsilon_0(\boldsymbol{r}) = 1 - U(1 - 1/\sqrt{\pi})\,. \tag{17}$$

Then from (12), in view of expressions for $g(\boldsymbol{r})$ and $\upsilon_0(\boldsymbol{r})$,

$$\varphi_{P\eta}(\lambda,\rho,\nu) = \Phi_P[\lambda\upsilon_0(\boldsymbol{r})] = \exp\{2\pi\cdot\nu\cdot\int_0^{\sqrt{1/\pi}+\rho}[X(\lambda,\rho,\eta)-1]\cdot\eta\cdot d\eta\} \tag{18}$$

where

$$X(\lambda,\rho,\eta) = \chi[\lambda \cdot \Sigma(\rho,\eta)], \qquad (19)$$

$\Sigma(\rho, \eta)$ is the area of intersection of unit circle with the circle of radius ρ at distance between its centres, equal η.

According to the general results [3] for the dot receiver (ρ>>1) characteristic function of fluctuations is equal

$$\varphi_s = \varphi_{sp}(\lambda,\rho,\nu) = \exp\{\nu \cdot \pi \cdot \rho^2 \cdot [\chi(\lambda) - 1]\} \cdot \qquad (20)$$

Otherwise, for the case of big receiver, when ρ<<1, equality should be carried out

$$\varphi_s = \varphi_{se}(\lambda,\rho,\nu) = \exp(-\frac{\lambda^2}{2}\nu \cdot \rho^4) \cdot \qquad (21)$$

Results of calculations of characteristic function of vibrations from the general formula (18) show, fig.2, that the statistics of fluctuations of the element as applied to concerned model Poisson field becomes practically the same as of the dot when its radius appears approximately three times less affected zone radius of splash. Accordingly, when the radius of the receiver exceeds affected zone radius of splash more than five times, vibrations of the element are sufficiently approximated by Gaussian distribution (21). In an interval between the specified magnitudes the size of a receiver surface exert essential influence on probabilistic parameters of vibrations of the piston receiver.

It is interesting, that the obtained quantitative estimations of notion limits about "dot" and "very big" receiver appear rather small depending from density splashes distribution on a surface. Therefore relative position of the curves presented on fig.2, is qualitatively remains at any values of $n = \pi\rho^2\nu$.

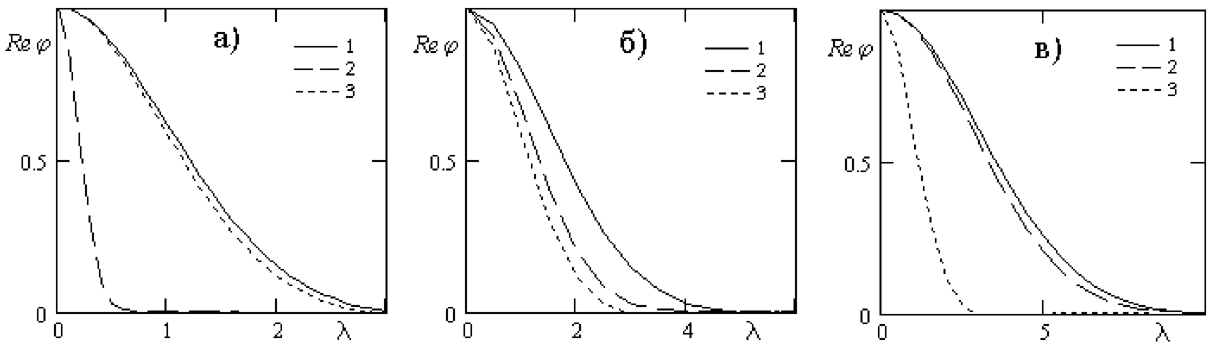

Fig.2. Comparison of characteristic function of vibrations of the piston receiver with limiting decisions. The quantity *n* of splashes on a platform of an affected zone is equal 10. Diagrams **а**) – ρ = 3; **б**) – 0.5; **в**) – 0.2. Curves: 1 – the formula (18), 2 – (21) (normal distribution), 3 – (20) (the dot receiver).

REFERENCES

1. E. B. Kudashev and L. R. Yablonik.. in *Proceedings of XI Session of the Russian Acoustical Society*, Vol. 1 (Moscow, 2001), p. 262
2. L.Cremer, M.Heckl, B.A.Petterson. Stucture-Born-Sound. 3rd edition. Springer-Verlag Berlin Heidelberg 2005.
3. E. B. Kudashev and L. R. Yablonik. Simple Models of the Characteristic Functional in Hydrodynamic Acoustics // Acoustical Physics, Vol. 48, No. 3, 2002, pp. 321–324.
4. Cantwell B.J. Organized motion in turbulent flow // Ann.Rev.Fluid Mech. 1981.V.13. P.457-515.