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NONLINEAR PROPAGATION OF SPHERICALLY DIVERGENT WAVES IN RELAXING MEDIUM: EXPERIMENT AND THEORETICAL MODELING

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Nonlinear wave propagation in media with relaxation is an important problem for many applications, such as sonic boom propagation, generated by supersonic aircrafts. Laboratory scale experiments and numerical modeling of sonic booms enable to investigate the behavior of the N-pulse duration and amplitude, as well as to predict variations in rise time. In this paper the modified Burgers equation is employed for modeling nonlinear propagation of spherically diverging N-waves in atmosphere. The results of numerical modeling are compared with the experimental data and show a good agreement. Numerical modeling is employed to study nonlinear and relaxation phenomena in N-wave duration and amplitude. It is shown that for the experimental conditions, the duration of the shock pulse is governed by nonlinear effect; whereas its amplitude depends both on nonlinear and relaxation effects.

Introduction

The problem of measuring the parameters of nonlinear shock pulses in relaxing media is an important problem for many applications. For example, sonic booms are generated during the supersonic aircraft flights and propagate through turbulent and inhomogeneous atmosphere. Temporal characteristics and spatial structure of the acoustic field produced by sonic boom close to the earth is influenced by aircraft trajectory, diffraction and scattering on the atmosphere inhomogeneities, nonlinear and relaxation processes [1]. To estimate possible effect of the sonic boom on the earth biosphere, it is necessary to predict acoustic field characteristics such as peak and average pressure, pulse duration and shock rise time. In this work the propagation of short-duration *N*-wave in steady atmosphere is studied in laboratory scaled experiments. Theoretical modeling is performed based on modified Burgers equation for nonlinear spherically divergent waves in homogeneous relaxing medium.

Experimental arrangement

For sonic boom laboratory investigations the experimental setup has been built [2], which allows generating short duration spherically divergent *N*-waves (about 30 μs) with high peak values (1000 Pa at 15 cm from the source). The experimental setup consists of electrical spark source based on tungsten electrodes with 6 mm gap between the electrodes, condenser microphone mounted into rigid baffle to avoid diffraction effects on its edges, amplifier and oscilloscope (Fig.1). To control the stability of the shape and peak characteristics of the propagating shock wave, an additional microphone placed at the fixed distance from the source is used. For time synchronization of the oscilloscope with the spark source an antenna detecting powerful electromagnetic radiation produced by spark is employed. The presented experimental setup makes possible to generate high amplitude single pulses, which transform to *N*-waves at the first centimeters of propagation due to nonlinear effects. Acoustical measurements of

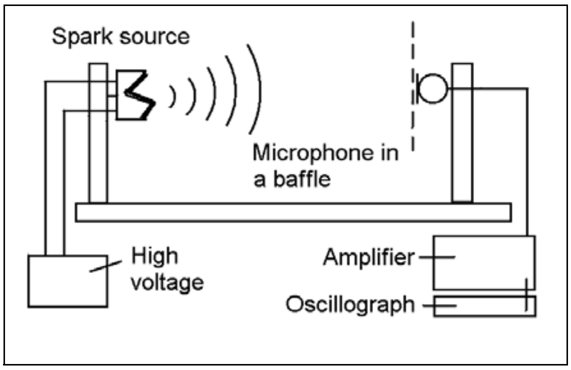

Fig.1 Experimental setup

shock waves are performed in homogeneous ambient air at the distances varying from 15 cm to 2.5 m from the spark source.

Theoretical model

For theoretical investigation of nonlinear spherically divergent wave propagation in homogeneous media with relaxation the modified Burgers equation is employed in this paper [3,4]:

$$\frac{\partial p}{\partial r} + \frac{p}{r} = \frac{\varepsilon}{\rho_0 c_0^3} p \frac{\partial p}{\partial \tau} + \frac{b}{2\rho_0 c_0^3} \frac{\partial^2 p}{\partial \tau^2} + \sum_{\nu=1}^{2} d_\nu \frac{\partial}{\partial t} \int_{-\infty}^{\tau} \exp(-\frac{\tau - \tau'}{\tau_\nu}) \frac{\partial p}{\partial \tau'} d\tau' . \quad (1)$$

Here p is the acoustic pressure, r - is the radius absolute value, $\tau = t - (r - r_0)/c_0$ - is the retarded time, ε - is the coefficient of nonlinearity in air, ρ_0 - the air density, c_0 is the equilibrium, small-signal sound speed, b - is the coefficient of viscosity. Each relaxation process is characterized by a relaxation time τ_ν and small-signal sound speed increment c_ν, $d_\nu = c_\nu / c_0^2$.

Modified Burgers Eq.(1) accounts for the effects of nonlinearity (first term on the right hand side), thermoviscous absorption (second term), and relaxation effects, associated with the excitation of oscillatory energy levels of oxygen and nitrogen molecules (third term). Spherical geometry is taken into account by the second term on the left hand side of Eq.(1).

Numerical algorithm

To perform numerical simulations it is convenient to rewrite the equation (1) in dimensionless form:

$$\frac{\partial P}{\partial \sigma} = P \frac{\partial P}{\partial \theta} + \frac{1}{\Gamma} \exp(\frac{x_p}{r_0}\sigma) \frac{\partial^2 P}{\partial \theta^2} + \sum_{\nu=1}^{2} D_\nu \exp(\frac{x_p}{r_0}\sigma) \frac{\partial}{\partial \theta} \int_{-\infty}^{\theta} \exp(-\frac{\theta - \theta'}{\theta_\nu}) \frac{\partial P}{\partial \theta'} d\theta' . \quad (2)$$

Here $P = pr/p_a r_0$ is the dimensionless acoustical pressure, normalized by the peak value in the initial profile p_a, r_0 is the distance from the spark source where initial profile is given, $\sigma = x_p \ln(r/r_0)/r_0$ is the spatial dimensionless propagation coordinate, where $x_p = \rho_0 c_0^3 / \varepsilon \omega_0 p_a$ is the shock formation distance; $\theta = \tau \omega_0$ is the dimensionless time, where ω_0 is the characteristic signal frequency, $\theta_\nu = \tau_\nu \omega_0$, $\Gamma = 2\varepsilon p_a / b\omega_0$ is the Goldberg number which determine relative influence of nonlinearity and absorption, and $D_\nu = \rho_\nu c_0 c_\nu / \varepsilon p_a$, $\nu = 1,2$ are two dimensionless parameters responsible for relaxation processes.

To obtain a solution for the dimensionless pressure P, the Eq.(2) is solved numerically in the time domain using operator splitting procedure. At the first step of the algorithm nonlinear effects are taken into account using central conservative scheme of the second-order of accuracy [4]. This scheme has small internal viscosity and it is enough to have only 2-3 points per shock to describe its evolution with high accuracy and stability. At the second step, dissipation effects are included using finite differences. Relaxation effects are calculated at the last step using the Crank-Nikolson algorithm of the second-order of accuracy on time θ and the first-order on propagation coordinate σ. To check the accuracy of nonlinear operator, numerical results were compared with known analytical solution of the simple wave equation [4], and to check the accuracy of the relaxation operator the comparison with the stationary Polyakova solution for monorelaxing media was performed [6]. For the following computational steps: $\Delta\sigma = 0.002$ и $\Delta\theta = 0.0262$, the maximum error in numerical solution was less then 1%.

Results

For numerical simulations of nonlinear *N*-wave propagation in relaxing medium the specific parameters of the medium, corresponding to those of the laboratory experiment are used: $\varepsilon = 1.21$, $b = 18.15 \cdot 10^{-6}$Pa·s, $\rho_0 = 1.29$ kg/m^3 , $c_0 = 343.67$ m/s. The parameters of relaxation processes were calculated using empirical expressions for the relative humidity 34%, temperature 293 K and ambient pressure level of 1 atmosphere: $c_1 = 0.11$ m/s, $\tau_1 = 6.0$ μs (O_2) , $c_2 = 0.023$ m/s, $\tau_2 = 531$ μs (N_2) [7].

For numerical simulations the initial waveform is an analytic *N*-wave with the amplitude and duration chosen according to the main characteristics of the experimental one. The initial waveform was chosen according to the *a priori* information, that fully developed signal should take a symmetric *N*-wave with short rise time (Fig.2). This assertion is based on those facts that the signal measured by

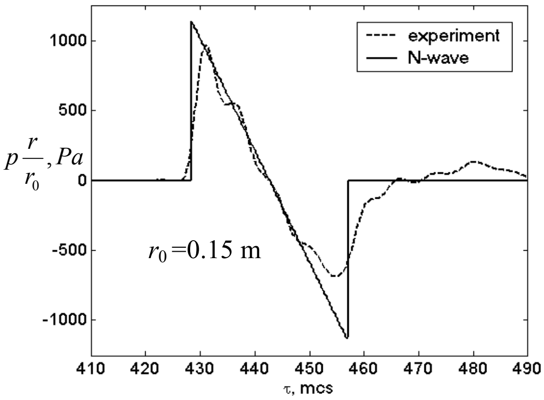

Fig.2 Initial experimental (at distance r_0 = 0.15 m) and analytical waveforms

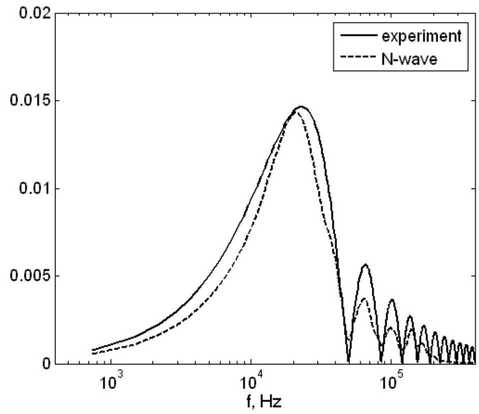

Fig.3 Spectra of the initial experimental and analytical waveforms

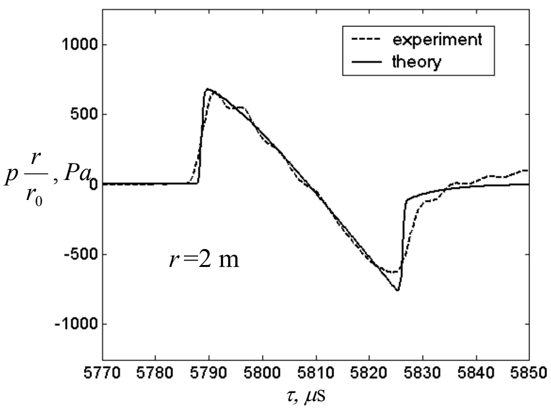

Fig.4. Experimental and computed waveforms at the distance r = 2 m from the spark source.

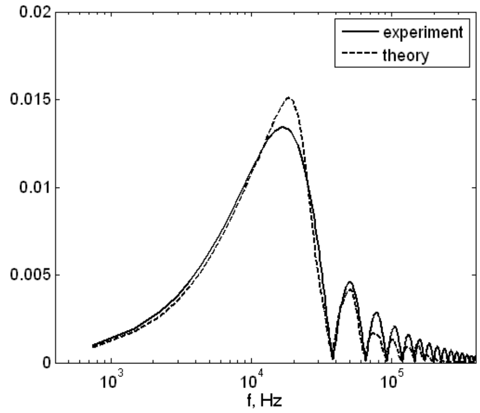

Fig.5. Spectra of the corresponding waveforms shown on fig.4

the microphone is distorted by residual diffraction effects at the microphone edge, and by the frequency response of the microphone (which is not flat at high frequencies).

N-wave duration can be defined based on the analysis of the experimental high frequency spectrum of the signal (Fig.3), as the zero values of the spectrum should not be affected by the frequency characteristic of the microphone. The initial waveform is defined by matching the zero values of the analytical signal spectrum to the zero values of the measured spectrum at the distance r_0=15 cm from the source. Actually the spectrum of the measured signal does not completely achieve zero level due to the presence of the diffraction "tail" in the signal but it is possible to define them. Using the described procedure it is possible to match 5-6 first zero values. The amplitude of the initial *N*-wave is defined by comparison between the experimental and theoretical dependences of the nonlinear shock front movement in retarded time and the shock amplitude on the propagation distance.

Shown in Figure 4 are the experimental and numerical waveforms obtained at the distance r = 2m from the source. To compensate the decrease of the shock amplitude due to spherical divergence of the wave, the pressure waveforms are multiplied by the coefficient r/r_0. The corresponding spectra are presented in Fig.5. The comparison of theoretical and experimental waveforms shows good agreement for the duration and amplitude of the pulse. Note, that zero points of the corresponding spectra also match each other. Some discrepancies can be seen for the shock rise time, but this problem could be resolved by applying the microphone frequency filter to the calculated waveform.

To compare the relative effect of relaxation and nonlinearity effects on *N*-wave propagation, several numerical simulations were performed: for the nonlinear propagation in a medium with (solid line) and without (dashed line) relaxation, and also for linear propagation in a medium with relaxation (dotted line). The results of numerical simulations are shown in Figs. 6 and 7 as the dependence of the

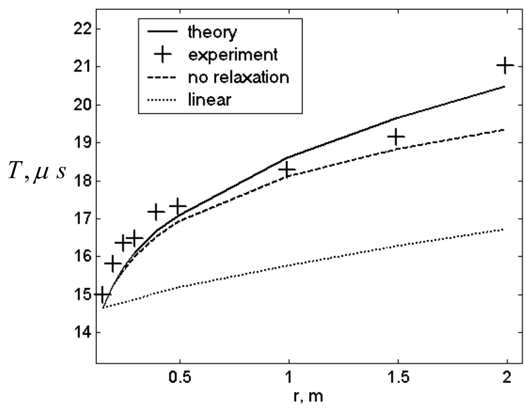

Fig.6. Dependence of the N wave positive half period duration on the propagation distance

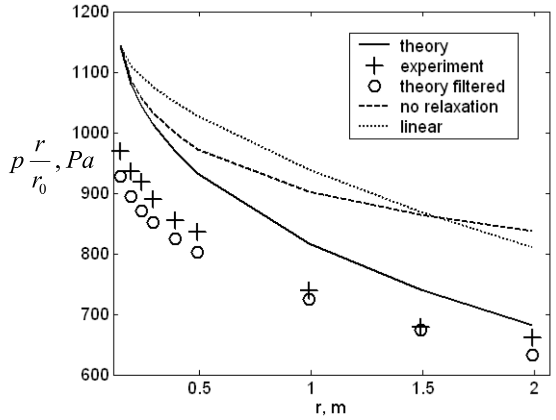

Fig.7. Dependence of the N wave peak positive pressure on the propagation distance

duration and peak positive pressure of the wave on propagation distance. Experimental data are shown on the figures as black crosses. Half duration of the pulse has been defined as the time interval between the point on the shock front of 10% peak pressure level and the point of zero pressure on the back slope. It is clearly seen that pulse duration is increasing mainly due to the influence of nonlinear effects, whereas peak positive pressure depends on both nonlinear absorption and relaxation phenomena. Starting from the distances of about 1.5 m when the peak pressure of the wave becomes lower and therefore nonlinear absorption is weakened, the influence of relaxation effects on wave amplitude becomes dominant. Note that the discrepancies between the theory and experiment for the positive peak pressure can be explained by taking into account the frequency filtering introduce by the microphone (see circles on fig.7). A good agreement between the results of numerical modeling and laboratory scale experiment data are thus obtained. In future works the modified Burgers equation for relaxing media will be generalized to the case of inhomogeneous media, and comparison will be held with the experimental results in turbulent media.

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