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PARAMETRICAL INTERACTION OF SOUND WAVES IN THE ACOUSTICAL ACTIVE MEDIA

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Peculiarities of forming of nonlinear acoustic impulse type structures in the non-equilibrium acoustically active media were analyzed. The conditions of non-stationary generation of giant parametrical impulses which amplitude is exceeded the amplitude of pumping wave were founded.

It is well known that in the acoustically active media the sound wave amplification is observed. [1,2]. The conditions of sound wave instability beginning, the mechanisms of its stabilization and the analysis of attendant phenomena were leaded in [1-3]. The set of dissipative structures which can formed in the acoustically active media were received in [4]. In [5] were received the condition of forming of the giant parametrical impulse. However, in [5] the Raileigh mechanism of sound wave instability don't take into account, although this mechanism can be essential in the wide range of media and wave parameters, Fig.1. Moreover, in [5] we were interested only by stationary solutions of running wave type.

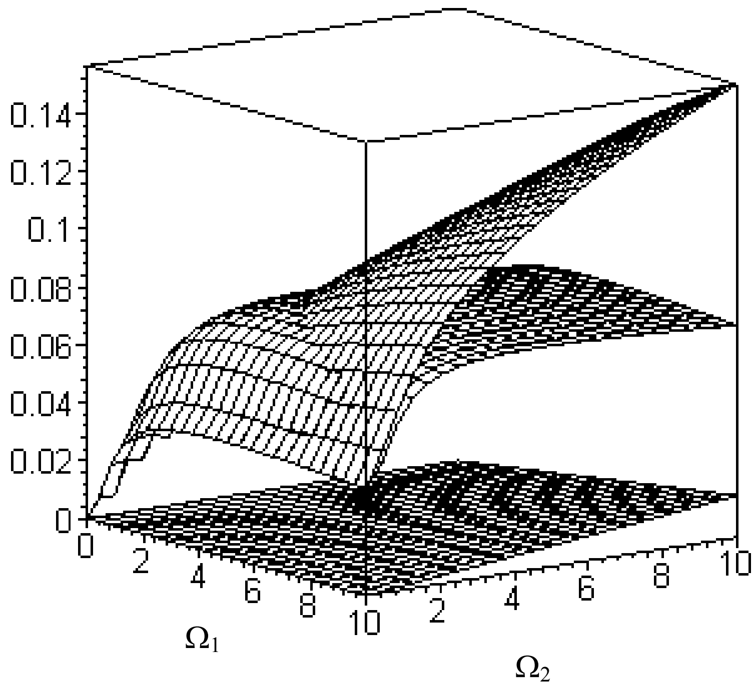

Fig. 1. The dependence of parametrical Γ_0, Raileigh $(\zeta_1+\zeta_2)/2$ and full increments from frequencies of interacting sound waves.

In present work the non-stationary mechanism of forming of giant parametrical impulses in media with Raileigh mechanism of sound wave instability were considered.

The basic system is included the Navier-Stoces equations, the equation of continuity, state, heat transfer and vibrational energy. After standard transformations [5] this system is reduced to equations:

$$C_{V\infty}\tau_0\frac{\partial}{\partial t}\left(\frac{\partial^2 u}{\partial t^2}-u_{S\infty}^2\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial z^2}\right)+\frac{\partial^2}{\partial x\partial t}\left(\frac{u^2+w^2}{2}+\frac{\gamma_\infty u_{S\infty}^2}{2\rho_0^2}\rho^{(1)2}\right)-\mu_\infty\frac{\partial}{\partial t}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial z^2}\right)\right)+$$
$$+C_{V0}\left(\frac{\partial^2 u}{\partial t^2}-u_{S0}^2\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial z^2}\right)+\frac{\partial^2}{\partial x\partial t}\left(\frac{u^2+w^2}{2}+\frac{\gamma_0 u_{S0}^2}{2\rho_0^2}\rho^{(1)2}\right)-\mu_0\frac{\partial}{\partial t}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial z^2}\right)\right)=0,$$
$$C_{V\infty}\tau_0\frac{\partial}{\partial t}\left(\frac{\partial^2 w}{\partial t^2}-u_{S\infty}^2\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial z^2}\right)+\frac{\partial^2}{\partial z\partial t}\left(\frac{u^2+w^2}{2}+\frac{\gamma_\infty u_{S\infty}^2}{2\rho_0^2}\rho^{(1)2}\right)-\mu_\infty\frac{\partial}{\partial t}\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial z^2}\right)\right)+ \qquad (1)$$
$$+C_{V0}\left(\frac{\partial^2 w}{\partial t^2}-u_{S0}^2\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial z^2}\right)+\frac{\partial^2}{\partial z\partial t}\left(\frac{u^2+w^2}{2}+\frac{\gamma_0 u_{S0}^2}{2\rho_0^2}\rho^{(1)2}\right)-\mu_0\frac{\partial}{\partial t}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial z^2}\right)\right)=0,$$
$$\frac{\partial\rho^{(1)}}{\partial t}+\rho_0\left(\frac{\partial u^{(1)}}{\partial x}+\frac{\partial w^{(1)}}{\partial z}\right)=0,\ \frac{\partial u^{(1)}}{\partial z}-\frac{\partial w^{(1)}}{\partial x}=0.$$

This system is described the nonlinear evolution of plane acoustical disturbances with wide spectrum in the non-equilibrium gas in accuracy to second order on amplitude. In one-dimensional case system is reduced to equation, received in [5]. Here $\tau_0 = \tau(T_0,\rho_0)$ is time of VT-relaxation at stationary means of the parameters T_0, $\rho_{0.}$, $u_{S\infty} = (C_{P\infty}T_0/C_{V\infty}M)^{1/2}$, $u_{S0} = (C_{P0}T_0/C_{V0}M)^{1/2}$, $\gamma_\infty = C_{P\infty}/C_{V\infty}$, $\gamma_0 = C_{P0}/C_{V0}$, $\mu_\infty = 4\eta/3 + \chi(1/C_{V\infty}-1/C_{P\infty})$, $\mu_0 = 4\eta/3 + \chi(1/C_{V0}-1/C_{P0})$ is high- and low frequency coefficients of dissipation, M is molecular mass.

Let us find the solution of system, received above in the sum of three waves $u=\sum_{j=0}^{2}u_jA_j(\theta x,\theta z,\theta t)\exp(ik_jr-i\omega_jt)+\text{к.с.}+\theta U_j(x,z,t)$, $w=\sum_{j=0}^{2}w_jA_j(\theta x,\theta z,\theta t)\exp(ik_jr-i\omega_jt)+\text{к.с.}+\theta W_j(x,z,t)$, $\rho=\sum_{j=0}^{2}\rho_jA_j(\theta x,\theta z,\theta t)\exp(ik_jr-i\omega_jt)+\text{к.с.}+\theta\Re_j(x,z,t)$.

Here $A_j = A_{k_j}$. As a result after standard transformations we have a system

$$\frac{\partial A_0}{\partial t}+v_0\nabla A_0+\delta_0A_0=U^{(0)}A_1A_2\exp(i\Delta\omega t),$$
$$\frac{\partial A_1}{\partial t}+v_1\nabla A_1+\delta_1A_1=U^{(1)}A_0A_2^*\exp(i\Delta\omega t), \qquad (2)$$
$$\frac{\partial A_2}{\partial t}+v_2\nabla A_2+\delta_2A_2=U^{(2)}A_0A_1^*\exp(i\Delta\omega t),$$

where δ_j is acoustical increment, v_j is group velocity, $\eta_{0;\infty}=\mu_{0;\infty}/2\tau_0u_{S\infty}^2$ $U^{(l)}_{k_l;k_m;k_n}$ is operator of interaction, $U^{(j)}=U^{(j)}_{k_0k_1k_2}$, $v_j=v_{k_j}$, $\Delta\omega=\omega_1+\omega_2-\omega_0$ is frequency detuning.

In the approach of slow changing of the wave amplitudes and constancy of pumping wave amplitude $A_2(0)$ = const, $A_0(0) >> A_2(0)$, $A_1(0)$ in the case of degenerated three-wave interaction ($\omega_2 = 2\omega_1$), system (2) is reduced to equations

$$\frac{\partial A_1}{\partial x}+\frac{1}{v_{X1}}\frac{\partial A_1}{\partial t}+\alpha_1A_1=\Xi^{(1)}A_2A\exp(i\Delta kx),\ \frac{\partial A_2}{\partial x}+\frac{1}{v_{X2}}\frac{\partial A_2}{\partial t}+\alpha_2A_2=-\Xi^{(1)}A_1^2\exp(i\Delta kx), \qquad (3)$$

where $\alpha_j=\alpha_{k_j}=\delta_j/v_{Xj}$, $i\Xi^{(j)}=U^{(j)}/v_{Xj}$.

This system must be added to boundary conditions $A_1(x=0)=a_1(t)$, $A_2(x=0)=a_2(t)$. We further have analized the case of long pumping impulse $A_2 \approx A_{20}$ and weak initial subharmonic impulse $A_1(t) << A_{20}$. We were also proposed that phase detuning is small $\Delta k \to 0$.

It was shown [1] that under condition of neglecting of spatial Raileigh increment ($\alpha_{1;2} = 0$) we have such results. On first stage the amplitude of subharmonic wave A_1 is grown to amplitude of pumping wave a_{20} . On the next stage the interaction of waves become nonlinear. The scenario of this stage is strongly dependent from the form of profile. If $a_1 = a_{10} =$ const or $\nu_{12} = 0$ on the length $L_0 = \Gamma_0^{-1}\ln(2a_{20}/a_{10}) = (\Psi(\omega_1)k_1M_{20})^{-1}\ln(2M_{20}/M_{10})$ the amplitude of subharmonic wave have a peak $A_{1max} \approx a_{20}$, in later zone $x > L_0$ is decreased, since some energy go to generation of pumping wave. Here $\Gamma_0 = |A_0(0)|\sqrt{\Xi^{(1)}\Xi^{(2)}}$ is parametrical increment, $\nu_{12} = v_{X1}^{-1} - v_{X2}^{-1}$ is group velocities detuning.

In the opposite case of exponential front of signal wave

$$a_1(\xi_1) = a_{10}\exp(-\xi_1/\tau_1)$$

together with the group velocities detuning (only if $u_{S0} > u_{S\infty}$) the energy interchange is different for front and tail of subharmonic wave. Practically all pumping power is immersed in the front of impulse whereas its tail is sharped. These conditions could be realized only in the non-equilibrium media where $u_{S0} > u_{S\infty}$. The further signal wave evolution is depended from relation from the slope of front τ_1 and critical duration of disturbance $\tau_C = \nu_{21}\Gamma_0^{-1}$.

If $\tau_C < \tau_1$ that on length $0 < x < L_0$, the perturbation is grown with linear increment Γ_0. In the point $x = L_0$ the amplitude of subharmonic signal is grown to value $A_1 = a_{20}$. On nonlinear stage of evolution $x > L_0$ the tail of impulse is forming.

If $\tau_C < \tau_1$ on nonlinear stage of evolution in the zone $0 < x < L_0$ under condition $\Gamma_0 x >> 1$ the quasi-stationary impulse

$$A_1 \approx a_{20}(1 + \tau_C/\tau_1)^{1/2}\,\mathrm{sech}(\Gamma_0(x - L_1) - \xi_1/\tau_1)$$

is formed. This forming is come to the end on length

$$L_1 \approx (2\Gamma_0)^{-1}\ln\left[2M_{20}^2(\tau_1 + \tau_C)/M_{10}^2\tau_1\right] > L_0 .$$

Thus the amplitude of impulse under condition $\tau_C > \tau_1$ is exceeded the pumping wave amplitude, in other words the giant parametrical impulse is formed. It is shown that peak values of time τ_C are achieved in region of maximal increment of Raileigh instability $\omega_1\tau_0 \sim 1$. Therefore, the inequality $\omega_1\tau_0 >> 1$ can take place in the wide spectral range. Note, that in region $\omega_1\tau_0 \sim 1$ have a maximum a Raileigh increment too, Fig.1 since this neglecting is correct only on short time interval.

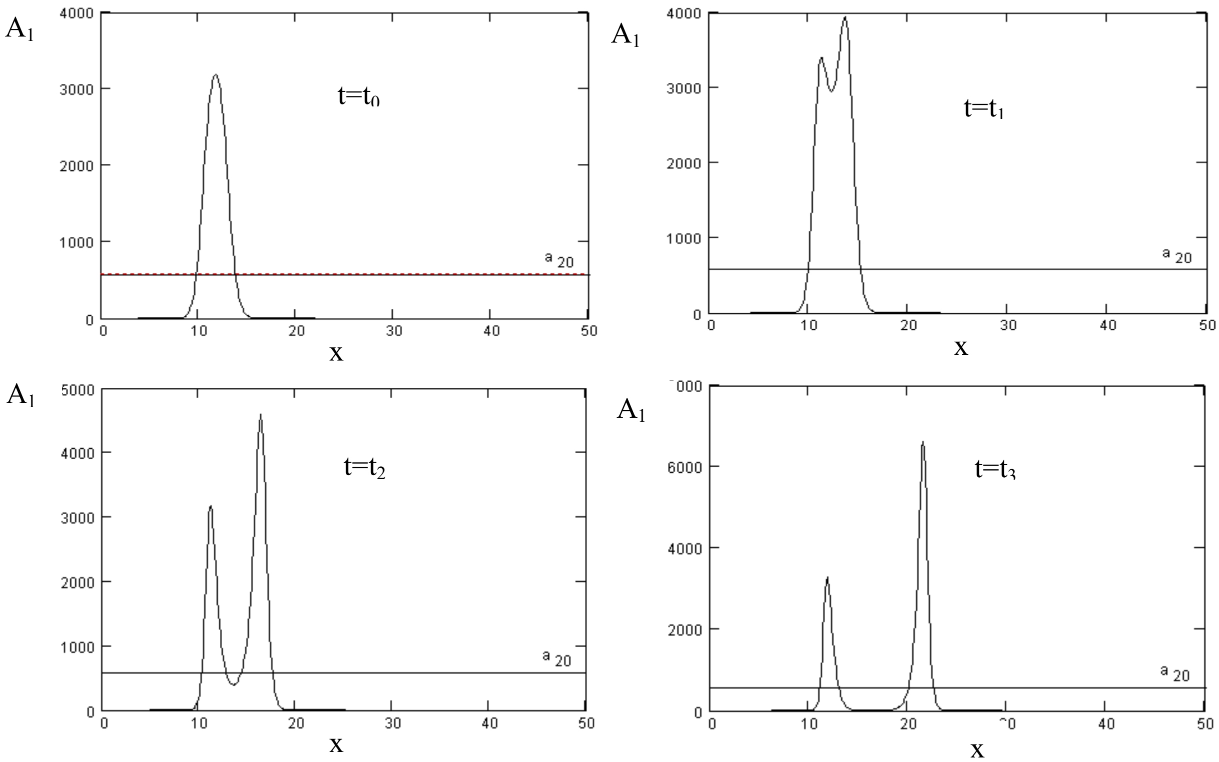

Fig.2 Forming of consequence of giant impulses in the acoustically active media in times $t_0 < t_1 < t_2 < t_3$.

The integration of system (2) in this case in limit of degenerated three-wave interaction were conducted numerically with parameters of typical laser medium CO_2:N_2:He =1:2:3, normal conditions, $ф_0=9\cdot10^{-6}$с, $щ_1ф_0$= 1, a_{10}=11.84, a_{20}=592, ν_{12}= $-2.118\cdot10^{-7}$c/sm, $\Gamma_0=3\cdot10^{-2}$см$^{-1}$, $ф_1=10^{-6}$с, $ф_C/ф_1$=7. It were received the following results.

On first stage (time $t = t_0$ on Fig.2), under condition $\tau_C > \tau_1$ the giant parametrical impulse is formed. Later it become unstable (time $t = t_1$ on Fig.2) and decomposed to consecution of giant impulses (see Fig.2 in times $t = t_{2;3}$). Velocity of first impulse is greater than velocity of second impulse.

The similar regime were observed in [4], where non-stationary solutions of system (1) in one-dimensional case were analyzed.

Therefore we can have a conclusion about role of Raileigh amplification during the forming of dissipative structures of impulse type in the acoustically active media. The Raileigh instability is answered for initiation of back front instability, where concentrated the low-frequency part of impulse spectrum and role of negative low-frequency second viscosity is especially essential [1,3] and for swapping of energy from media to impulse.

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REFERENCES

1. Kogan E.Ya., Molevich N.E. The excitation of waves in non-equilibrium gas with VRT-relaxation // Tech. Phys. 1985. V. 55. № 4. P. 754-757. (In Russian)

2. Osipov A.I., Uvarov A.V. Kinetic and gasdynamic processes in non-equilibrium molecular // Uspekhi Fis. Nauk. 1992. T. 162. № 11. P. 1-23. (In Russian)

3. Molevich N.E., Oraevsky A.N. Waves in medium with negative second viscosity // Proc. Lebedev’s Physical. Inst. 1992. V. 222. P. 45-97. (In Russian)

4. Makaryan V.G., Molevich N.E. New stationary structures in the acoustically active media // Tech. Phys. Letters. 2003. V. 68. № 3. P. 194-199. (In Russian)

5. Zavershinskii I P, Kogan E Ya Resonant interaction of acoustic waves in a non-equilibrium gases // High Temperatures. 2005. V. 43 P.1-7. (In Russian)