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RESONANT PROPERTIES OF ACOUSTIC LENS ANTENNAE

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This article suggests a model of wave equation, in which wave number is expressed by total energy of wave packet. It offers to take into consideration nonlinear phenomena in the medium while making series expansion of a function which describes energy. The article also suggests to take into account resonant phenomena in acoustic lenses of small wave size for calculation of energy parameters of focusing systems.

Wave transmission through lens antenna leads to the transformation of wavefront set. This transformation allows to solve many applied tasks on acoustics, optics, radiotechnics, particularly to focus waves, generate prescribed antenna directivity diagrams, to change the shape of focal spot in focal region and many other tasks.

If the linear sizes of the lens are much larger than wave length, then classical transformation of the wave phase front. If the linear sizes of the lens are comparable to the wave length, then together with phase front transformation we can expect different resonant phenomena in the lens. This phenomena can be explained in the context of classical wave models and in the context of mathematical analogies with quantum-mechanical models also.

The classical mathematical model of wave equation gives good approximation for wave transmission in homogenous media. If the homogenous media is bounded by one or several closed surfaces (layer-like structure), then wave equation is also a correct problem of mathematical physics. And this problem can be solved in a closed species using analytic function.

In heterogeneous medium wave processes can be described by a wave equation, in which phase velocity depends on coordinates. This model doesn't provide rigorous solution compared with steady speed, but is of great importance for applied problems.

This work contains generalized approach to the solution of wave equation. As an initial mathematical model we chose the equation as it was written in the work [1].

$$\Delta\Phi - \frac{p^2}{\mathrm{E}^2}\frac{\partial^2\Phi}{\partial t^2} = 0 \qquad (1)$$

where Φ is a vibrational speed potential of elastic wave, E – energy of wave packet, p – mechanical moment of wave packet movement, λ - Laplace operator. Generally Φ depends on spatial coordinates x_1, x_2, x_3 and on time t.

For one-dimensional case the equation can be written as follows:

$$\frac{\partial^2\Phi}{\partial x^2} - \frac{p^2}{E^2}\frac{\partial^2\Phi}{\partial t^2} = 0 \qquad (1a)$$

$$\upsilon = \sqrt{\frac{E^2}{p^2}} = \frac{E}{P}$$ wave packet propagation velocity.

If energy E and impulse "p" are continuous constants during wave transmission process, then the solution of the latter equation can be written as follows:

$$\Phi(x,t) = \frac{1}{2}\left[f(x - \frac{E}{p}t) + f(x + \frac{E}{p}t) \right] \qquad (1б)$$

In the context of the solution of equation (1a) with Dalamber's method, the solution (1б) describes more wide physical range of problems than acoustic waves. In particular, it can be wave transmission on water surface, string oscillation and so on.

If the volume unit E(s, u_{ik}) internal energy function of deformation tensor U_{ik} and entropy S is acquainted, formally it can be equated in expanded form as follows:

$$E = E_0 + \frac{\partial E}{\partial U_{ik}}\delta U_{ik} + \frac{1}{2!}\frac{\partial^2 E}{\partial U_{ik}^2}\delta U_{ik}^2 + \frac{1}{3!}\frac{\partial^3 E}{\partial U_{ik}^3}\delta U_{ik}^3 + ...$$

where E_0 is a specific internal energy at zero deformations, $\frac{\partial^n E}{\partial U_{ik}^n}$ - zero value argument derivatives at constant entropy S.

In this case phase velocity υ can formally be set out as follows:

$$\upsilon = \frac{E}{p} = \frac{E_0}{p} + \frac{1}{p}\frac{\partial E}{\partial U_{ik}}\delta U_{ik} + \frac{1}{p}\frac{1}{2!}\frac{\partial^2 E}{\partial U_{ik}^2}\delta U_{ik}^2 + ... = \upsilon_0 + \upsilon_1 + \upsilon_2 + ...$$

For harmonic process let us introduce a new constant H such that E=Hω, $\frac{2\pi}{\lambda} = k = \frac{p}{H}$.

In this case wave equation may be written as:

$$\Delta\Phi + \left(\frac{p}{H}\right)^2 \Phi = 0$$

or (2)

$$\Delta\Phi + \left(\frac{p}{E}\omega\right)^2 \Phi = 0$$

For enclosed volumes given the following bound conditions:

$$\left.\begin{aligned} \frac{\partial \Phi_1}{\partial n}|_S &= \frac{\partial \Phi_2}{\partial n}|_S \\ \rho_1 \Phi_1 |_S &= \rho_2 \Phi_2 |_S \end{aligned}\right\} \qquad (3)$$

ρ_1, ρ_2 – media density.

Let's write the solution of equations (2), (2a) as follows:

$$\Phi_1 = \frac{e^{i\frac{p_1}{H}R}}{R} + K_{отр}\frac{e^{-i\frac{p_1}{H}R}}{R}$$

$$\Phi_2 = K_{пр}\frac{e^{i\frac{p_2}{H}R}}{R}$$

$K_{отр}$, $K_{пр}$ – wave reflection coefficient and the coefficient of wave leak (transmission) through the media boundary S, obeying (3).

By the "lens" model we shall mean any mathematical model, which describes wave field in enclosed volume with different phase velocity and density on both sides of surface S boundary. Depending on the "lens" wave sizes different solution methods for equations (2) and (2a) at the boundary conditions (3) can be used. If the wave sizes are big, it is possible to use Reley or Debai approximations.

However, at small wave sizes it is necessary to use rigorous solution methods for equations (2) and (2a) of the (4) solution type. In this case resonating characteristics of the enclosed volume must become apparent. Therewith for every natural volume frequency there must be matched sets of discrete energies E_n.

Total number of oscillation modes N lying between frequency ω_0 и $\omega_0+\Delta\omega$ is proportionate to squared frequency ω_0^2.

$$N = \frac{AW}{\upsilon^3}\omega_0^2\Delta\omega$$

where A is a constant, W – "lens" volume, υ – phase velocity of the elastic waves inside of the volume.

This formula with practically absolute exactness coincides with the resonance of spherical volume bounded by a rigid surface.

It must be emphasized that "lenses" resonance have some analogies with energy quantization in quantum mechanics.

Therefore in case of lens sound gain and focusing at low frequencies it is possible that basic coefficients, which characterize operating efficiency of the lens, would be discrete.

If the operating frequency is raised, the discrecity of the spectrum of energy lines decreases. That is why in high frequency acoustic lenses and optical focusing systems there is no characteristics discrecity.

It is necessary to take into account the resonating characteristic of the "lenses" in seismoacoustics, research of low-frequency wave processes in closed bays when linear dimensions commensurate with wave lengths.

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