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MAIN FEATURES OF A NON-UNIFORM WAVE SCATTERING BY AN INCLINED SURFACE

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Scattering a Rayleigh wave within the framework of a linear theory of an elasticity by a bounding plane surface is considered. By means of the spectral method a solution of a problem of interaction between a non-uniform Rayleigh wave and an inclined surface is constructed. Is obtained in the explicit form a solution of the problem of a Rayleigh wave diffraction. The numerical analysis of phase- amplitude characteristics of a Rayleigh wave transmitted to the surface was made. Are found the displacement amplitudes of longitudinal and transverse waves diverging from the surface.

The structure is revealed of acoustic field originating on an inclined boundary aroused by a surface sharp fracture. Wave disturbances having spatially non-uniformity on an inclined plan generated by the incident non-uniform wave generate the scatted waves field.

This non-uniformity leads to the appearance of surface waves as well as longitudinal and transverse waves. In this report the standard methods construct a precise solution of a problem for acoustic field on an inclined plane, disposed at an arbitrary angle with the surface where the wave propagates.

For a plate problem formulation the field of displacements and velocities at different instants is searched. In a system of coordinates (x, z) which can be obtained by rotating the starting system of coordinates (ε, η) by an angle θ (Fig. 1).

Solution must satisfy equation of motion:

$$\Delta\Phi + k_\ell^2\Phi = 0, \qquad \Delta\psi + k_t^2\psi = 0, \qquad (1)$$

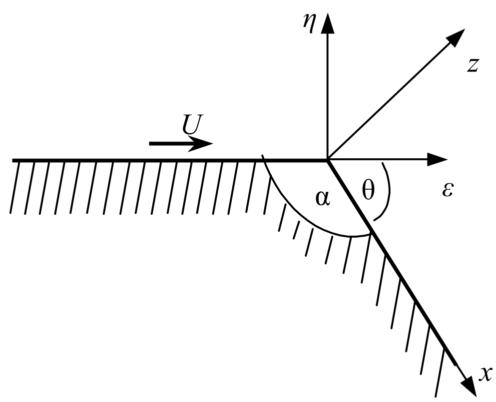

Fig.1. System of coordinates.

and boundary conditions: tensor tension components, generated by the incident and secondary waves should be equal to zero.

$$\sigma^o_{zz} + \sigma_{zz} = 0, \qquad \sigma^o_{xz} + \sigma_{xz} = 0. \qquad (2)$$

The oscillations of particles are formed by an incident plate Rayleigh wave, it's spatial structure is considered known:

$$\Phi_r = \exp\left[i\,(k_r\varepsilon - \omega t) - q_r\eta)\right],$$

$$\psi_r = p\exp\left[i\,(k_r\varepsilon - \omega t) - s_r\eta)\right], \qquad (3)$$

here $q_r = \sqrt{k_r^2 - k_\ell^2}$, $s_r = \sqrt{k_r^2 - k_t^2}$, $p = -\sqrt{\dfrac{q_r}{s_r}}\,i$,

(The dependence on time exp(–iωt) is omitted further).

The problem solution we'll consider for wedge angles α ($\alpha > 90°$). In this case, the structure of the wave being incident on the second wedge side – remains invariable.

The solutions are to be seached in the form of Fourier series of plane waves [1]:

$$\Phi = \frac{1}{2\pi}\int_{-\infty}^{\infty}\Phi^*(k)e^{-ikx}dk, \qquad \psi = \frac{1}{2\pi}\int_{-\infty}^{\infty}\psi^*(k)e^{-ikx}dk. \tag{4}$$

The obtained solution is expressed in the form of quadratures linking complex amplitudes of potentials to an angle θ:

$$\Phi = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left\{\frac{\left[\left(\frac{P(k_{1x})}{k-k_{1x}} - p\frac{Q(k_{1x})}{k-k_{2x}}\right)S(k_r) + \left(\frac{R(k_{1x})}{k-k_{1x}} - p\frac{S(k_{2x})}{k-k_{2x}}\right)Q(k_r)\right]}{4k^2qs - (k^2+s^2)^2}\cos(\theta) + \right.$$

$$\left. + \frac{\left[\left(\frac{P(k_{1z})}{k-k_{1z}} - p\frac{Q(k_{2z})}{k-k_{2z}}\right)S(k_r) + \left(\frac{R(k_{1z})}{k-k_{1z}} - p\frac{S(k_{2z})}{k-k_{2z}}\right)Q(k_r)\right]}{4k^2qs - (k^2+s^2)^2}\sin(\theta)\right\}\exp(qz)dk, \tag{5}$$

$$\psi = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left\{\frac{\left[\left(\frac{P(k_{1x})}{k-k_{1x}} - p\frac{Q(k_{1x})}{k-k_{2x}}\right)R(k_r) + \left(\frac{R(k_{1x})}{k-k_{1x}} - p\frac{S(k_{2x})}{k-k_{2x}}\right)P(k_r)\right]}{4k^2qs - (k^2+s^2)^2}\cos(\theta) + \right.$$

$$\left. + \frac{\left[\left(\frac{P(k_{1z})}{k-k_{1z}} - p\frac{Q(k_{2z})}{k-k_{2z}}\right)R(k_r) + \left(\frac{R(k_{1z})}{k-k_{1z}} - p\frac{S(k_{2z})}{k-k_{2z}}\right)P(k_r)\right]}{4k^2qs - (k^2+s^2)^2}\sin(\theta)\right\}\exp(sz)dk. \tag{6}$$

The integrals of this form frequently arise in problems of a diffraction and are evaluated in the explicit form. The applying of a saddle point method to the integral representation of a solution result in the formulas for directivity patterns of diverging longitudinal and transverse waves, and the residues in poles of an integrand determine Rayleigh waves spreading along boundary.

Let's mark, that the amplitude of a transmitted Rayleigh wave will be decreased with the appearance of longitudinal and transverse waves carrying energy of the incident wave into the depth of a medium.

These waves are shaped in that case, when the projection of a wave vector of an incident wave is less than a wave vector of a transverse wave. Such situation arises at an angle position of a plane $\theta > \theta_0$, where

$$\theta_0 = \arccos k_t / k_r.$$

Thus, at major wedge angles close to 180° the degeneration of a problem on an angle stipulated by physical reasons takes place.

From fig. 2 it is visible, the longitudinal and transverse waves appear at $\alpha < \alpha_0$.

At these angles there is a decomposition of oscillations on a volumetric undular component, which one brings in the noticeable contribution to a power engineering of process, and boundary driving of ground waves along a plane (Fig. 3).

As demonstrates the analysis, the amplitude and phase of a transmitted wave on a plane for the degenerated case remains invariable (Fig. 3, 4), and for other - wears composite character, growing out combined actions both competitions of normal and shift component undular perturbations called by an incident wave.

The amplitude of displacements will increase in accordance with a diminution of a wedge angle,

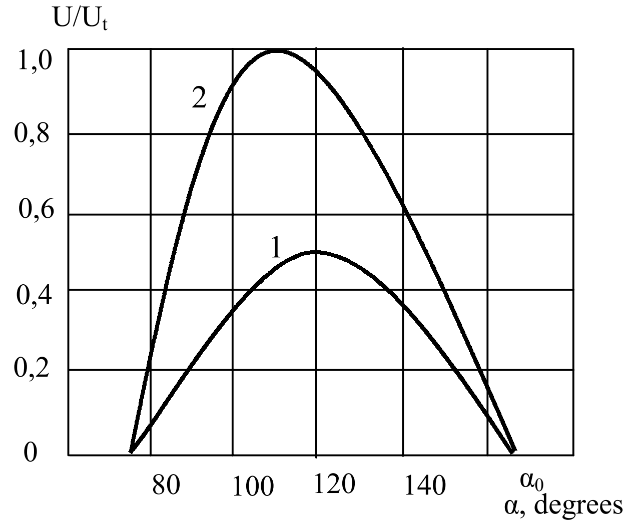

Fig. 2. Amplitudes longitudinal (1) transverse (2) waves depending on a wedge angle.

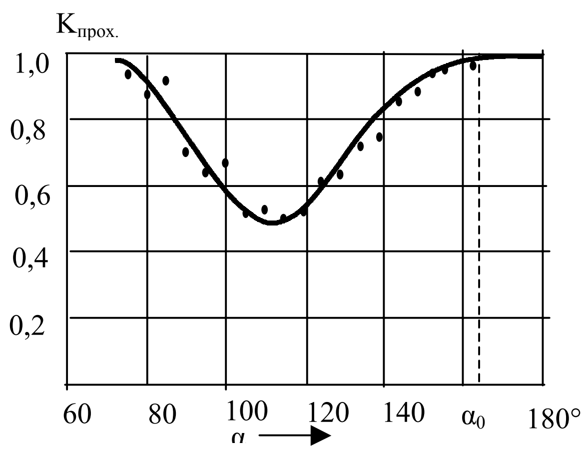

Fig. 3. Dependence of transmission coefficients on a wedge angle, (test data from [2], for a duralumin wedge).

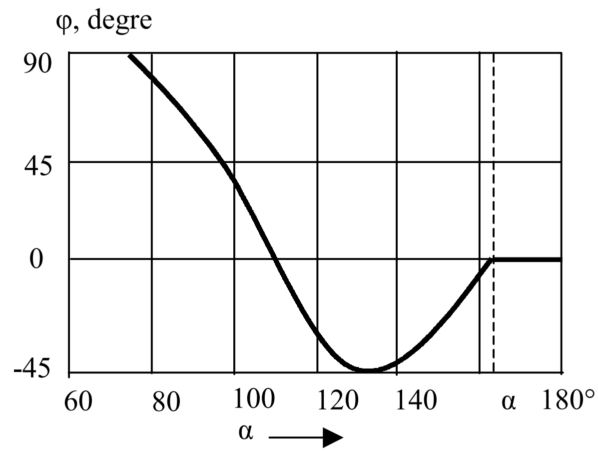

Fig. 4. Oscillation phase in the transmitted wave depending on a wedge angle.

reaching a maximum at angles close to 120° (fig. 3), and then up to zero point is moderated. The generation of these waves, growing out combined actions both competitions of a longitudinal and shift component of the incident wave, depends on frame of a profile of undular perturbations on the second side of a wedge.

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