methods are suitable for treating deep-seated tumors, precise, safe, repeatable, bloodless and economic.

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EFFECT OF THE SHOCK WAVE ENVELOPE ON CAVITATION BUBBLE DYNAMICS

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The Rayleigh equation describing nonlinear oscillations of a gas bubble in a liquid has been analyzed using the theory of groups. The group of scale transformations was calculated and then used as the basis for constructing the solutions to the Rayleigh equation. The analytical description of the essentially nonlinear dynamics of a bubble allows one to use the aforementioned solutions as a model for analyzing such phenomena as shock wave propagation in liquids with phase inclusions. The study of effectiveness of expansion and collapse of a single bubble driven by periodically prolonged scaling acoustical field has been performed. The key to understanding the transition from weakly oscillating to strongly collapsing bubbles lies in the existence of a threshold for spontaneous bubble expansion known as the Blake threshold. Blake threshold is never achieved at this form of driving and this is the main reason for ineffectiveness of bubble extension.. The small, in comparison with the pressure drop at the leading edge, stretching

pressure at the trailing edge of the scaling pulse increases insignificantly bubble radius, thus decreasing the probability of tissue damage due to cavitation that can occur at this phase of the shock wave.

Theoretical analysis of the acoustic cavitation is based on the Rayleigh equation describing the behavior of a gas bubble in an external acoustic field:

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 + \frac{P_0}{\rho_0}\left[1 - \left(\frac{R_0}{R}\right)^{3\gamma}\right] = -\frac{P(t) - P_0}{\rho_0}. \quad (1)$$

Here, P_0, P and R_0, R are the equilibrium and current values of the pressure in the liquid medium and the bubble radius, respectively; ρ_0 is the liquid density; γ is the polytropic exponent. Traditionally, the cavitation phenomena have been studied for the external driving in the form of a harmonic signal $P(t) = P_0 - P_m \cos(\omega t)$, where P_m and ω are the amplitude and frequency of the acoustic field. Previously, a complete symmetry group for the Rayleigh equation has been found, which allowed the form of an external field giving rise to internal dynamic symmetry in gas bubble pulsation to be determined [1, 2]. In particular, the scale invariance takes place when the external pressure has the following form: $P(t) = P_m \left[t_0 /(t + t_0)\right]^{6\gamma/(2+3\gamma)}$, where t_0 is the fall time. Then, a quite natural question arises concerning the efficiency of the bubble extension and collapse under exposure of a periodically prolonged scale-invariant external field,

$$P(t) = \sum_{k=0}^{\infty} P_m \left[\frac{t_0}{(t - kT) + t_0}\right]^{6\gamma/(2+3\gamma)} \left[\Theta(t - kT) - \Theta(t - (k+1)T)\right] \quad (2)$$

(T is the continuation period) in comparison to the harmonic case [3].

The periodically prolonged external field (2) may correspond to a real pressure field in an experimental cell only provided that the total flux over the period is zero:

$$\int_t^{t+T} dt' \left[P(t') - P_0\right] = 0. \quad (3)$$

This condition established a certain relationship between the pressure drop at the leading front P_m, the period T, and the characteristic fall time t_0:

$$\frac{P_m}{P_0} = \frac{3\gamma - 2}{3\gamma + 2}\left(\frac{T}{t_0}\right)\left[1 - \left(1 + T/t_0\right)^{-(3\gamma-2)/(3\gamma+2)}\right]^{-1}. \quad (4)$$

It should be noted that expression (2) reduces in the limiting cases to the well-known objects of nonlinear acoustics. In particular, for $(T/t_0) << 1$ we deal with a sawtooth wave with the pressure varying over a period $(0 \le t \le T)$ as

$$P(t) \approx P_0 \left[1 - \left(6\gamma/3\gamma + 2\right)(t - T/2)t_0^{-1}\right]. \quad (5)$$

For $(T/t_0) \geq 1$, expression (2) describes a sequence of shock waves similar to those arising in the case of an ultrasonic lithotripter. The character of the nonlinear pulsation of a bubble depends on the shape of the envelope of a high-power ultrasonic wave [4]. This dependence can be studied in the general case only by numerically solving the Rayleigh equation. The results of such a comparative analysis for the cases of harmonic, symmetric, and asymmetric sawtooth waves were reported in [5].

In the present study, the system is analyzed predominantly by analytical methods. If the period is shorter than the fall time (i.e., $(T/t_0) << 1$), the wave amplitude in (5) is also small and we can find an exact solution of a linearized Rayleigh equation.

$$R = R_0 + \Delta R\,,\ \xi = \dot{R} + i\Omega_0 \Delta R\,,\ \frac{d\xi}{dt} - i\Omega_0 \xi = F\,,$$

$$F = \frac{6\gamma P_0}{\rho_0 R_0 (2+3\gamma)} \sum_{k=0}^{\infty} \frac{[t-(k+1/2)T]}{t_0} \left\{ \Theta(t-kT) - \Theta[t-(k+1)T] \right\}$$

$$\xi(t) = e^{i\Omega_0 t} \left[\int_0^t dt' F(t') e^{-i\Omega_0 t'} + \xi_0 \right] = \frac{6\gamma P_0}{\rho_0 R_0 (2+3\gamma)} e^{i\Omega_0 (lT+\Delta t)} \left\{ \frac{e^{-il\Omega_0 T} - 1}{e^{-i\Omega_0 T} - 1} \int_0^T dt' \left(\frac{t'-T/2}{t_0} \right) e^{-i\Omega_0 t'} + \right.$$

$$\left. + e^{-il\Omega_0 T} \int_0^{\Delta t} dt' \left(\frac{t'-T/2}{t_0} \right) e^{-i\Omega_0 t'} + \xi_0 \right\}$$

$$\Delta R = \Omega_0^{-1} \operatorname{Im} \xi = \frac{2R_0}{(2+3\gamma)} \left\{ \left(\frac{\Delta t - T/2}{t_0} \right) + \left[\frac{T}{2t_0} \cos(\Omega_0 \Delta t) - \frac{\sin(\Omega_0 \Delta t)}{2(t_0 \Omega_0)} \right] + \right.$$

$$\left. + \frac{[(\Omega_0 T)\cos(\Omega_0 T/2) - 2\sin(\Omega_0 T/2)]}{(t_0 \Omega_0)} \left[\cos(\Omega_0 \Delta t) \left(\frac{\sin(l\Omega_0 T)}{2\sin(\Omega_0 T/2)} \right) - \sin(\Omega_0 \Delta t) \left(\frac{1-\cos(l\Omega_0 T)}{2\sin(\Omega_0 T/2)} \right) \right] \right\}. \quad (6)$$

Here, $\Omega_0 = (3\gamma P_0 \rho_0^{-1} R_0^2)^{1/2}$ is the fundamental frequency of linear pulsation of the bubble. In the case of a resonance between the n-th harmonic of the external periodic field and the fundamental frequency, $\Omega_0 T = 2\pi n$, the coefficients in the third term (describing the contribution from *l* complete periods) linearly increase with the time (i.e., with the number of periods *l*). This growth is naturally limited if we take into account the damping. For $(\Omega_0 T) << 1$, the bubble does not substantially change its size during the period (in the first order in $(T/t_0) << 1$). For $(\Omega_0 T) >> 1$, the maximum size $R_{\max} \sim R_0 + 2R_0 (2+3\gamma)^{-1} (T/t_0)$ (differing but little from the equilibrium radius) is attained by the moment of completion of the phase of extension under the exposure of the external field. When the field period increases, the wave amplitude also grows according to relation (5).

In the case of $(T/t_0) > 1$, we have to analyze a substantially nonlinear pulsation regime. If the external action upon the bubble varies slowly in comparison to the period of eigenoscillations $2\pi/\Omega_0 < t_0$, we can obtain an analytical solution using the condition of existence of the adiabatic invariant. It was shown [1] that a change in the bubble radius in the asymptotic limit $t >> t_0$ can be considered as a superposition of two motions:

$$R(t) = R_0\left(\frac{P_0}{P_m}\right)^{1/4}\left(\frac{t+t_0}{t_0}\right)^{1/3} + 0.01R_0\left(\frac{P_m}{P_0}\right)\left(t_0\Omega_0\right)^{1/3}\sin\left\{\left(\frac{P_m}{P_0}\right)^{3/4}\left(t_0\Omega_0\right)\ln\left[\Omega_0\left(t+t_0\right)\right]+\alpha\right\} \quad (7)$$

Here, the first motion (the first term) represents the invariant (automodel) solution of Eq. (1) describing the power growth with an exponent different from that in the Rayleigh law. The second motion has the form of oscillations with a steady amplitude and a logarithmically increasing period. Expression (7) corresponds to γ = 4.3 for a bubble occurring in rest at the moment of the shock wave front propagation. In all the subsequent periods of external action, the structure of solution (7) is retained, only the amplitude and phase of the second term exhibit variation. At a sufficiently large time *t* (for $T >> t_0$), such that transient processes are accomplished during the period, the first term in (7) predominates and the maximum bubble size is attained before arrival of the next shock wave front. For an arbitrary γ, the maximum radius is

$$R_{\max} = R(T) = R_0\left(P_0/P_m\right)^{1/3\gamma}\left(T/t_0\right)^{2/2+3\gamma} \quad (8)$$

Using relation (4) between the field amplitude P_m and the period *T*, we obtain $R_{\max} \sim R_0\left(P_m/P_0\right)^{(3\gamma-2)/[3\gamma(3\gamma+2)]}$. An extremely small exponent (for γ = 4.3, $R_{\max} \sim R_0\left(P_m/P_0\right)^{1/12}$ and the natural restrictions imposed on the ratio $\left(P_m/P_0\right)$ by the condition of applicability of the Rayleigh equation result in that the maximum radius cannot significantly exceed the equilibrium value ($R_{\max} \sim R_0$).

The case, when the time t_0 is comparable with or shorter than the period of intrinsic oscillations ($t_0 < 2\pi/\Omega_0$) and the external action is rather intense ($\left(P_m/P_0\right) \geq 1$), does not admit analytical description. This case will be illustrated by the results of numerical calculations. Let us take the following values of the main parameters: γ = 4.3 and $T = 10\pi t_0$. Substituting these values into relation (4), we obtain an expression for the pressure amplitude:

$$\left(P_m/P_0\right) = (10\pi/3)\left[1-(1+10\pi)^{-1/3}\right]^{-1} \approx 15.26 \quad (9)$$

The results of numerical calculations of the time variation of the bubble radius in a stationary regime with $(t_0\Omega_0) = 1$ are presented in Fig. 1a, where the dashed line shows the corresponding scale-invariant solution. Figure 1b shows the time variation of the external pressure (2) for the amplitude determined by formula (9). Although the analytical solution (7) in this case is formally inapplicable ($t_0 < 2\pi/\Omega_0$), it still qualitatively correctly reflects the behavior of the curve in Fig. 1a.

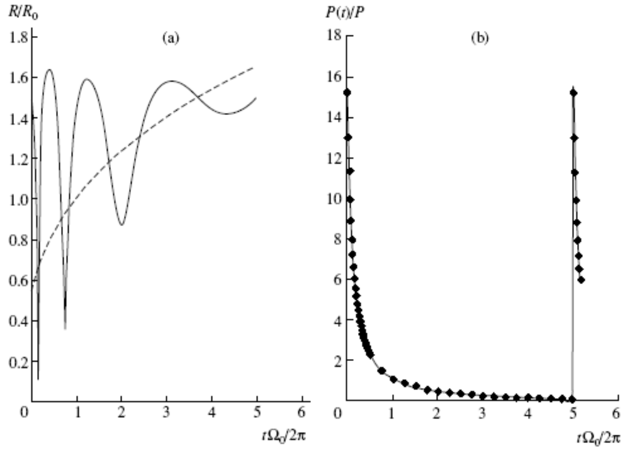

Fig. 1. Time variation of (a) the bubble radius in a stationary regime and (b) the external pressure inducing these oscillations.

The most important feature of this numerical calculation is that (similar to all the cases considered above) the periodically prolonged scale-invariant driving is relatively inefficient in extending the bubble, even at a pressure on a megapascal level.

The key to understanding the transition from weakly oscillating to strongly collapsing bubbles lies in the existence of a threshold for spontaneous bubble expansion known as the Blake threshold [6]. Is this threshold realized at scaling driving? Note that the fall time t_0 should be long compared to the time scale of bubble eigenoscillations $2\pi/\Omega_0 < t_0$ to consider external pressure as quasi-static and follow Blake's arguments. Consider bubble at the static condition than the Rayleigh equation (1) reduces to

$$\left(P_0 + \frac{2\sigma}{R_0}\right)\left(\frac{R_0}{R}\right)^3 - \frac{2\sigma}{R} = P(t), \tag{10}$$

where for the gas pressure the isothermal ideal gas law was used, which is certainly an excellent approximation for the static situation and surface tension was included.

For $P(t) > 0$ equation (10) has exactly one solution. If $P(t) < 0$ but small in absolute magnitude, then two equilibrium states exist, the one at large R being unstable, i.e. a bubble with large radius would grow indefinitely. Finally at critical $P_B(t) < 0$ (Blake threshold pressure) the two equilibrium points merge and disappear. In this situation the gas pressure is always larger than $P(t) + 2\sigma / R$ and (10) can't be fulfilled for any radius [6]. Since according to (2) $P(t)$ is always positive for the scaling driving, Blake threshold is never achieved and this is the main reason for ineffectiveness of bubble extension.

This circumstance, on the one hand, makes the scaling driving considered here not very promising from the standpoint of investigation of the stable sonoluminescence of a single bubble — the acoustic cavitation effect extensively studied in recent decade [7]. On the other hand, it is the relatively low efficiency of extension that makes the use of this pulse shape attractive from the standpoint of lithotripsy by – technique used shock waves to break up stones in the kidney. The signal employed in medical practice comprises a train of about 1000 shock waves with a pressure jump of up to 60 MPa at the leading front and a fall time of several microseconds. The shock wave envelope of the majority of manufactured lithotripters: Dornier, Siemens lithostar, Wolf piezolith, Edapalt lead to excess of Blake threshold at the stretching phase of the wave where a dynamical expansion follows which can only be stopped when $P(t)$ has again become large. A relatively small extending pressure (as compared to the pressure jump at the leading edge) in the scale-invariant driving does not significantly increases the bubble size, thus significantly reducing the probability of tissue damage as a result of cavitation caused by the extension phase of the shock pulse.

Acknowledgments. This study was supported by the Russian Foundation for Basic Research, project no. 04-02-16412.

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