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TRANSMISSION THROUGH AND REFLECTION FROM A FLUID ELASTIC LAYER:QUASI-LINEAR APPROXIMATION

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Second Harmonic Generation (SHG) in a three fluid layer system, is investigated in the quasi-linear approximation. Considering an initially sinusoidal plane wave normally incident upon the central layer, the pressure radiated from the later is evaluated at double frequency. Analytical expressions for the reflected and transmitted pressure fields are derived from series of reflected and refracted waves. Results are discussed as a function of the geometrical and material characteristics of the system.

1. Introduction

Transmission and reflection coefficients are known to be useful tools in linear acoustics. In particular, they can be used to predict the acoustic field on both sides of a lens. As the field intensity increases, harmonic contribution has to be considered. This paper is an attempt to derive analytical expressions at the second order for the pressure fields transmitted through and reflected from a fluid elastic layer. The study is restricted to non dissipative system, normally insonified by an initially sinusoidal plane wave.

For a weak bulk nonlinearity, the perturbation method can be applied to derive second order approximation of the wave equation (see Ch.10 of [1]). The equation contains a source term proportional to the square of the linear field solution. Hence, the resolution procedure is identical for linear and quasi-linear approximations. Considering the analogy, a methodology used in the linear case to build reflection and transmission coefficients from series of reflected and refracted waves in the layered system is here transposed [2, 3]: expressions for the reflected and transmitted pressure fields at twice the fundamental frequency are derived from the basic solution to the second order wave equation (section 3). Numerical results highlight the influence of the dimensional characteristics and the properties of the system (section 4).

2. Second Harmonic Generation (SHG)

SHG in canonical shape waveguides is under current investigation (see [4, 5] and references therein). These studies show that a necessary condition to obtain cumulative SHG is fulfilled as the phase velocities of the fundamental and the second harmonic components, $c^{(1)}$ and $c^{(2)}$ respectively, are equal (or very close) to one another. Then, the second harmonic field grows linearly with the propagation distance. This condition is automatically verified in the case of an initially sinusoidal plane wave at $z = 0$ propagating in an unbounded ideal fluid.

The solution to the quasi-linear wave equation for the pressure field is (see Ch.2 of [1]) :

$$P^{(2)} = \hat{P}^2 \frac{\beta k z}{2 j \rho c^2} e^{j2(kz-\omega t)} \quad (1)$$

where $\omega = 2\pi f$ is the angular frequency, $k = \omega/c$ the wave number of the primary wave, $2k = 2k^{(1)} = k^{(2)}$, c the longitudinal velocity, ρ the fluid density and $\hat{P}$ the amplitude of the initial linear component. The coefficient of nonlinearity β is linked to the nonlinearity parameter *B/A* by: $\beta = 1 + B/2A$. In the following, the time dependence is dropped out.

At the interface between two media, SHG is interrupted. Right after reflection/transmission, two contributions can be distinguished in the second harmonic component : the first one is due to the free propagation at double frequency of the previously generated wave, and the second one corresponds to SHG from the reflected/transmitted fundamental wave [6].

3. Pressure fields at double frequency

The system under consideration is sketched in Fig. 1. A plane fluid layer (fluid 1) is encased between two identical semi infinite media (fluid 0). Both fluids are assumed to be homogeneous and non viscous. The central layer is insonified normally with respect to the interface (θ = 0), by a primary sinusoidal plane acoustic wave of frequency *f* and peak value $\hat{P}$, placed at *z* = 0. Expressions at 2*f* for the transmitted $P_t^{(2)}$ and reflected $P_r^{(2)}$ pressure fields received at $z = z_t$ and $z = z_r$, are derived in section 3a. and 3b., respectively.

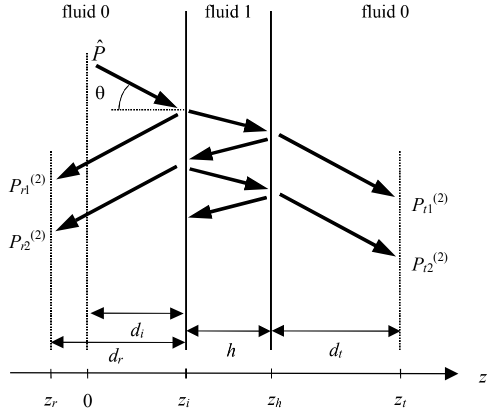

Fig. 1. Sketch of the three fluid layer system. Both the two first contributions to the reflected and transmitted second harmonic pressure fields are indicated: $P_{r1}^{(2)}$, $P_{r2}^{(2)}$ and $P_{t1}^{(2)}$, $P_{t2}^{(2)}$, respectively. For the sake of clarity, wave travel paths are shown for any incidence angle θ. However, the study is restricted to the case of normal incidence (θ = 0).

3a. Transmitted pressure field: $P_t^{(2)}$

The transmitted pressure field received at $z = z_t$ at twice the fundamental frequency can be written as the sum of m partial components for which SHG is achieved in q steps:

$$P_t^{(2)} = \sum_{m=1}^{\infty} \sum_{q=1}^{2m+1} P_{t\ m\ q}^{(2)} \tag{2}$$

with, for $q = 1$, $1 < q < 2m+1$ and $q = 2m+1$, respectively,

$$P_{t\ m\ q}^{(2)} = \begin{cases} \hat{P}^2 \dfrac{\beta_0 k_0 d_i t_{01} t_{10} r_{10}^{-1}}{2j\rho_0 c_0^2} \left(r_{10} e^{j2k_1 h}\right)^{2m-1} e^{j2k_0(d_i+d_t)} \\ \hat{P}^2 \dfrac{\beta_1 k_1 h t_{01}^2 t_{10} r_{10}^{q-3}}{2j\rho_1 c_1^2} \left(r_{10} e^{j2k_1 h}\right)^{2m-1} e^{j2k_0(d_i+d_t)} \\ \hat{P}^2 \dfrac{\beta_0 k_0 d_t t_{01}^2 t_{10}^2 r_{10}^{-2}}{2j\rho_0 c_0^2} \left(r_{10} e^{jk_1 h}\right)^{2(2m-1)} e^{j2k_0(d_i+d_t)} \end{cases} \tag{3}$$

and

$$\begin{aligned} r_{ab} &= (Z_b - Z_a)/(Z_a + Z_b) \\ t_{ab} &= 2Z_b/(Z_a + Z_b) \\ Z_a &= \rho_a c_a \\ Z_b &= \rho_b c_b \end{aligned} \tag{4}$$

where a and b indicate the medium index (0 or 1). For instance, there are three contributions to $P_{t1}^{(2)}$ $\left(P_{t1}^{(2)} = P_{t11}^{(2)} + P_{t12}^{(2)} + P_{t13}^{(2)}\right)$, according that SHG occurs before (q=1), during (q=2), or after (q=3) propagation of the primary wave in the central layer. More generally, relation (2) can be rewritten as a sum of three terms: the first and third ones correspond to SHG in fluid 0 before the first interface and after the second interface, respectively. The intermediate term is related to SHG in fluid 1. It yields:

$$P_t^{(2)} = \sum_{m=1}^{\infty} P_{t\ m\ 1}^{(2)} + \sum_{m=1}^{\infty} \sum_{q=2}^{2m} P_{t\ m\ q}^{(2)} + \sum_{m=1}^{\infty} P_{t\ m\ q=2m+1}^{(2)} \tag{5}$$

with

$$\sum_{m=1}^{\infty} P_{t\ m\ 1}^{(2)} = \hat{P}^2 \frac{\beta_0 k_0 d_i}{2j\rho_0 c_0^2} \frac{t_{01} t_{10} e^{j2\left[k_1 h + k_0(d_i+d_t)\right]}}{\left[1 - \left(r_{10} e^{j2k_1 h}\right)^2\right]} \tag{6a}$$

$$\sum_{m=1}^{\infty}\sum_{q=2}^{2m} P^{(2)}_{t\,m\,q} = \hat{P}^2 \frac{\beta_1 k_1 h}{2j\rho_1 c_1^2} \frac{t_{10} t_{01}^2 \left[1 + r_{10}^3 e^{j4k_1 h}\right] e^{j2\left[k_1 h + k_0 (d_i + d_t)\right]}}{\left[1 - \left(r_{10} e^{jk_1 h}\right)^4\right]\left[1 - \left(r_{10} e^{j2k_1 h}\right)^2\right]} \tag{6b}$$

$$\sum_{m=1}^{\infty} P^{(2)}_{t\,m\,q=2m+1} = \hat{P}^2 \frac{\beta_0 k_0 d_t}{2j\rho_0 c_0^2} \frac{t_{01}^2 t_{10}^2 e^{j2\left[k_1 h + k_0 (d_i + d_t)\right]}}{\left[1 - \left(r_{10} e^{jk_1 h}\right)^4\right]} \tag{6c}$$

3b. Reflected pressure field: $P_r^{(2)}$

On the same principle, the pressure field received at $z = z_r$ can be expanded as follow:

$$P_r^{(2)} = \sum_{m=1}^{\infty} P^{(2)}_{r\,m} \tag{7}$$

with, for $m = 1$

$$P^{(2)}_{r1} = \hat{P}^2 \frac{\beta_0 k_0 r_{01}\left[d_i + r_{01} d_r\right]}{2j\rho_0 c_0^2} e^{j2k_0(d_i + d_r)} \tag{8}$$

and for $m > 1$

$$P^{(2)}_{r\,m} = \sum_{q=1}^{2m} P^{(2)}_{r\,m\,q} \tag{9}$$

where, for $q = 1$, $1 < q < 2m$ and $q = 2m$, respectively :

$$P^{(2)}_{r\,m\,q} = \begin{cases} \hat{P}^2 \dfrac{\beta_0 k_0 d_i t_{10} t_{01} r_{10}^{-1}}{2j\rho_0 c_0^2} \left(r_{10} e^{j2k_1 h}\right)^{2(m-1)} e^{j2k_0(d_i + d_r)} \\ \hat{P}^2 \dfrac{\beta_1 k_1 h t_{01}^2 t_{10} r_{10}^{q-3}}{2j\rho_1 c_1^2} \left(r_{10} e^{j2k_1 h}\right)^{2m} e^{j2k_0(d_i + d_r)} \\ \hat{P}^2 \dfrac{\beta_0 k_0 d_r t_{01}^2 t_{10}^2 r_{10}^{-2}}{2j\rho_0 c_0^2} \left(r_{10} e^{jk_1 h}\right)^{4(m-1)} e^{j2k_0(d_i + d_r)} \end{cases} \tag{10}$$

Similarly to (6a-c), (7) can be rewritten as :

$$P_r^{(2)} = \sum_{m=1}^{\infty} P^{(2)}_{r\,m\,1} + \sum_{m=2}^{\infty}\sum_{q=2}^{2m-1} P^{(2)}_{r\,m\,q} + \sum_{m=1}^{\infty} P^{(2)}_{r\,m\,q=2m} \tag{11}$$

with

$$\sum_{m=1}^{\infty} P^{(2)}_{r\,m\,1} = \hat{P}^2 \frac{\beta_0 k_0 d_i}{2j\rho_0 c_0^2} \left(r_{01} + \frac{t_{01} r_{10} t_{10} e^{j4k_1 h}}{\left[1 - \left(r_{10} e^{j2k_1 h}\right)^2\right]} \right) e^{j2k_0(d_i + d_r)} \tag{12a}$$

$$\sum_{m=2}^{\infty}\sum_{q=2}^{2m-1} P^{(2)}_{r\,m\,q} = \hat{P}^2 \frac{\beta_1 k_1 h}{2j\rho_1 c_1^2} \frac{r_{10} t_{10} t_{01}^2 \left(1+r_{10}\right) e^{j2\left[2k_1 h + k_0\left(d_i + d_r\right)\right]}}{\left[1-\left(r_{10} e^{jk_1 h}\right)^4\right]\left[1-\left(r_{10} e^{j2k_1 h}\right)^2\right]} \qquad (12b)$$

$$\sum_{m=1}^{\infty} P^{(2)}_{r\,m\,q=2m} = \hat{P}^2 \frac{\beta_0 k_0 d_r}{2j\rho_0 c_0^2}\left(r_{01}^2 + \frac{t_{01}^2 r_{10}^2 t_{10}^2 e^{j4k_1 h}}{\left[1-\left(r_{10} e^{jk_1 h}\right)^4\right]}\right) e^{j2k_0\left(d_i + d_r\right)} \qquad (12c)$$

4. Numerical examples

Consider a plane plate made of glass, immersed in water. In the case of normal incidence and neglecting diffraction effects, shear wave contributions are sufficiently small to be ignored. Using (5), total and partial transmitted pressure fields are evaluated at $z = z_t$ as a function of frequency, at the vicinity of a resonance frequency of the central layer (Fig. 2). The parameters used in the computations for both media are listed in Tab. 1. Results are normalized by the SHGenerated in water on distance $d_i + h + d_t$:

$$P^{(2)}_{ref} = \hat{P}^2 \frac{\beta_0 k_0 (d_i + h + d_t)}{2j\rho_0 c_0^2} e^{j2k_0(d_i + h + d_t)} \qquad (13)$$

Tab. 1. Parameters used in the computations.

	ρ (kg/m^3)	c (m/s)	β
Water	1000	1500	3.5
Glass	2200	6000	6

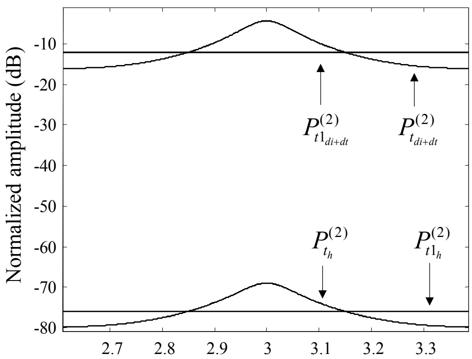

Fig. 2. Transmitted pressure field at $z = z_t$. $P^{(2)}_{t1_{di+dt}}$ (SHG in water: first contribution), $P^{(2)}_{t_{di+dt}}$ (SHG in water: total contribution), $P^{(2)}_{t1_h}$ (SHG in glass: first contribution) and $P^{(2)}_{t_h}$ (SHG in glass: total contribution).

Fig. 2 shows that SHG mainly occurs in water on the distance $d_i + d_t$ ($d_i = d_t = 100h = 0.5m$). Note that the conditions $h \ll d_i$ and $h \ll d_t$ avoid any stationary waves in fluid 0. It can be noticed too that SHG in the central layer reaches a maximum value at the resonance frequency and then exceeds for approximately 10dB the first contribution level. Authors have shown that the third order elastic parameter value may roughly be evaluated from the first contribution to the second harmonic transmitted pressure field [6, 7]. Total pressure field measurement at a resonance frequency of the plate appears to be a suitable configuration for such NDE purpose.

5. Conclusion

This paper investigates SHG in a three fluid layer planar system, assuming identical properties of exterior media, in the case of an initially sinusoidal wave at normal incidence. Attenuation and diffraction effects are ignored. Explicit description of multiple travel paths for second harmonic generated and freely propagating in the system, is proposed. Concise expressions for transmitted and reflected pressure fields at twice the fundamental frequency are obtained. It is shown that the transmitted and reflected pressure fields can be separated in three contributions due to the fact that SHG occurs before, during or after propagation in the central layer. Through numerical examples, this repartition highlights that SHG preponderantly happens in the softer medium and is greater at a resonance frequency of the central layer.

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