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NONLINEAR PARAMETER ESTIMATION IN WATER-SATURATED SANDY SEDIMENT WITH DIFFERENCE FREQUENCY ACOUSTIC WAVES

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Nonlinear acoustic responses of water-saturated sediments are very important to understand nonlinear phenomena of gassy ocean sediments. Especially, the second harmonic, the sum and the difference frequency acoustic waves in water-saturated sediments can provide practical criteria to estimate the nonlinear parameter of gassy sediments. In this paper, the difference frequency acoustic wave in water-saturated sandy sediment was observed in a water tank experiment with a pulse transmission technique. Its pressure level was 12 dB higher than the background noise level at a maximum undistorted driving pressure of source acoustic transducer. The experimental results were compared with a theoretical estimation of the parametric acoustic array. The nonlinear parameter of water-saturated sandy sediment was also estimated as 73 with their comparison. This value can be utilized as the background information to estimate gas void fraction in the water-saturated gassy sandy sediment.

1. Introduction

The porous media, such as soil, rocks, and sediments, exhibit high nonlinearity in comparison to the nonporous media [1,2]. The nonlinearity of sediments can play an important role for oil field prospecting and ecological monitoring in the ocean. Most of the sediments in the ocean contain a lot of bubbles, which show very sensitive nonlinear responses for acoustic waves. These responses for the bubbles can be useful to estimate the gas void fraction in gassy sediments [3]. Generally, the values of nonlinear parameter for the gassy sediments with the bubbles can be two to three orders of magnitude greater than those for the non-gassy sediments [1,3]. According to the granular model, the values of nonlinear parameter for fluid-saturated solid grains can reach up to 10^3 in some case [1,2]. Since the non-gassy sediments can be considered as the water-saturated solid grain medium, they also may have some remarkable values for the nonlinearity parameter like the gassy sediments. This means that for nonlinear acoustic investigation of the gassy sediments, nonlinear acoustic properties of the water-saturated sediments need to be first investigated as background acoustic data.

Nonlinear acoustic waves, such as the subharmonic, the ultraharmonic, the second harmonic, the sum and the difference frequency acoustic waves, can be generated by the nonlinearity of water-saturated sediments. The difference frequency wave can be utilized in many nonlinear underwater acoustic fields, because it is low frequency wave compared to primary frequency acoustic wave and shows good directivity beam pattern [4-6]. It also can provide practical criterion to estimate the gas void fraction from the nonlinear parameter of gassy sediments.

In this paper, the generation of the difference frequency acoustic wave in water-saturated sandy sediments was observed by using pulse transmission techniques. The experimental results are predicted through the theory of the parametric acoustic array. The nonlinear parameter value of the water-saturated sandy sediments was also estimated through a relation between the experiment and the theory.

2. Theory

Nonlinear wave equation to describe the propagation of acoustic waves in a medium is well known to the Khokhlov-Zabolotskaya-Kuznetsov(KZK) equation, which account for the nonlinearity of the medium, the attenuation and the diffraction of acoustic waves in the medium. It is expressed, for the propagation of acoustic waves in the z-direction as [7]:

$$\frac{\partial^2 p}{\partial z\ \partial \tau} = \frac{c_0}{2}\nabla_\perp^2 p + \frac{\delta}{2\ c_0^3}\frac{\partial^3 p}{\partial \tau^3} + \frac{\delta}{2\ \rho_0 c_0^3}\frac{\partial^2 p}{\partial \tau^2}, \tag{1}$$

where p is the sound pressure, $\tau = t - (z/c_0)$ is the retarded time, c_0 and ρ_0 are the sound speed and the density of the medium, δ is the diffusivity of sound, $\beta = 1 + (B/2A)$ is the nonlinear parameter of the medium, $\nabla_\perp^2 = \partial^2/\partial r^2 + r^{-1}\partial/\partial r$ is the Laplacian in (*x*,*y*) plane with $r = \sqrt{x^2 + y^2}$.

For axisymmetric sources, a quasilinear solution of Eq. (1) can be assumed as the form of

$$p = p_l + p_c , \tag{2}$$

where p_l is the linear solution of Eq. (1) at angular frequency ω, p_c is a small correction term of p_l, which is $p_c << p_l$. For the propagation of two primary acoustic waves with finite amplitudes in the medium, p_l and p_c can be expressed as the forms of

$$p_l(r,z,\tau) = \frac{1}{2j}[q_1(r,z)e^{j\omega_1\tau} + q_2(r,z)e^{j\omega_2\tau}] + \text{c.c.}, \tag{3}$$

$$\begin{aligned} p_c(r,z,\tau) = \frac{1}{2j}[&q_{1\_2nd}(r,z)e^{j2\omega_1\tau} + q_{2\_2nd}(r,z)e^{j2\omega_2\tau} \\ &q_+(r,z)e^{j\omega_+\tau} + q_-(r,z)e^{j\omega_-\tau} \;] + \text{c.c.}, \end{aligned} \tag{4}$$

where q_1, q_2, q_{1_2nd}, q_{2_2nd}, q_+, and q_- are complex pressure amplitudes of the primary, the second harmonic, the sum, and the difference frequency waves, respectively. When two primary acoustic waves with finite amplitudes propagate in the medium, Eq .(4) means that nonlinear acoustic waves are generated by the nonlinearity of the medium. When Eqs. (2), (3), and (4) are substituted into Eq. (1) and the source condition is given by

$$p(r,0,t) = \frac{1}{2j}[q_1(r,0)e^{j\omega_1 t} + q_2(r,0)e^{j\omega_2 t}] + \text{c.c.}, \tag{5}$$

q_- is expressed by

$$q_-(r,z) = -\frac{\pi\beta k_-}{\rho_0 c_0^2}\int_0^z\int_0^\infty q_1(r',z')q_2^*(r',z')G_-(r,z\,|\,r',z')r'dr'dz', \tag{6}$$

with

$$G_-(r,z\,|\,r',z') = \frac{jk_-}{2\pi(z-z')}J_0\left(\frac{k_- rr'}{z-z'}\right)\exp\left[-\alpha_-(z-z') - \frac{jk_-(r^2+r'^2)}{2(z-z')}\right], \tag{7}$$

where G_- is Green's function, $k_- = \omega_-/c_0$ and α_- are the wave number and the attenuation coefficient of the difference frequency wave, respectively. If we assume that two primary frequency acoustic waves are collimated plane waves radiated by a circular piston with a radius *a*, q_1 and q_2 in Eq (6) can be given by

$$q_1 = p_{01}H(a-r)e^{-\alpha_1 z}, \;\; q_2 = p_{02}H(a-r)e^{-\alpha_2 z}, \tag{8}$$

where $H(a-r)$ is the step function (*H*=0 for *r* > *a*, *H*=1 for *r* < *a*). When the nonlinear interaction zone of two primary acoustic waves is confined by their near field zone, Eq. (6), for the far field zone of the difference frequency acoustic wave, is simplified by

$$q_-(\theta,z) \approx -\frac{jp_{01}p_{02}\beta\omega_-^2 a^2}{4\rho_0 c_0^4\alpha_T}\frac{e^{-\alpha_- z}}{z}D_w(\theta)D_A(\theta)\exp\left(-\frac{1}{2}jk_- z\tan^2\theta\right), \tag{9}$$

where $\alpha_T = \alpha_1 + \alpha_2 - \alpha_-$ is the total attenuation in the interaction zone, $D_w(\theta)$ and $D_A(\theta)$ are Westervelt directivity and the aperture factor of the circular piston, they given by

$$D_w(\theta) = \frac{1}{1 + j(k_-/2\alpha_T)\tan^2\theta}, \;\; D_A(\theta) = \frac{2J_1(k_- a\tan\theta)}{k_- a\tan\theta}. \tag{10}$$

Then, the directivity of the difference frequency acoustic wave is expressed as

$$D(\theta) = D_w(\theta)D_A(\theta). \tag{11}$$

If the beam width in Westervelt directivity is small, the aperture factor $D_A(\theta)$ in Eq. (11) is ignored. When θ is zero, the nonlinear parameter β from Eq. (9) is easily determined by

$$\beta = \frac{4\rho_0 c_0^4\alpha_T z}{p_{01}p_{02}\omega_-^2 a^2 e^{-\alpha_- z}}\,|\,q_-\,|. \tag{12}$$

3. Experimental Setup

Figure 1 shows the experimental setup for the measurement of the difference frequency acoustic wave generated in the water-saturated sandy sediment. The water-saturated sandy sediment was packed in a thin film box with a volume of $100 \times 100 \times 50$ mm^3 in a large anechoic water tank. To make the large incident pressure amplitude and the collimated sound beam conditions, the sediment was located at distance of 10 mm from the transducer. The porosity and the mean grain diameter of the sediment were 43.3% and 450 μm, respectively. The transducer was simultaneously driven at two primary frequencies 76 kHz and 114 kHz. The signals transmitted through the water-saturated sandy sediment at two primary frequencies were tone burst signals with a pulse duration of 5 ms and a repetition time of 200 ms, respectively. We used this driving method to get a condition of continuous waves with high amplitude. The signal sources were two function generators(Agilent 33250A) and the signals were amplified by 50 dB through a power amplifier(Amplifier Research AR 75A 250) before transmitted through the transducer. The transmitted signal was received by a hydrophone(B&K 8103) and it was placed at a distance of 400 mm from the sediment and the distance was far field distance of the transducer. A movement of the hydrophone was controlled through the positioning system. The received signals were acquired using a 500-MHz digital storage oscilloscope(LeCroy LT342) and stored on a computer for off-line analysis.

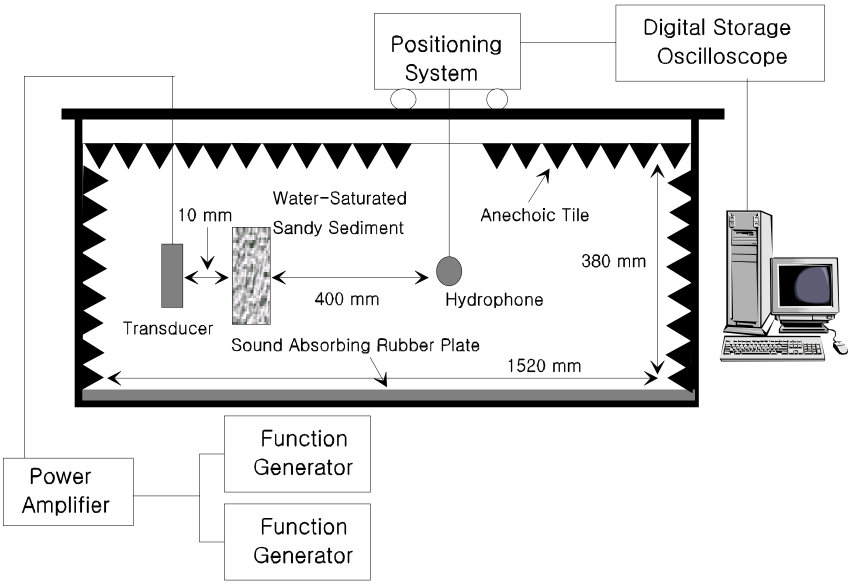

Fig. 1. Experimental setup for measurements of the difference frequency acoustic wave generated in water-saturated sandy sediment

3. Results and Discussion

For measurement of the difference frequency acoustic wave, Figs. 2(a) and (b) show the frequency spectra of the received signals with and without the water-saturated sandy sediment at two primary frequencies of 76 kHz and 114 kHz. For primary acoustic wave of 76 kHz, incident acoustic pressure on the water-saturated sandy sediment was 140 kPa. It was 150 kPa for primary acoustic wave of 114 kHz. The incident acoustic pressures were maximum pressures that the transducer could be simultaneously driven at two primary frequencies. As shown in Fig. 2(b), the difference frequency acoustic wave is clearly generated at the frequency of 38 kHz and its generation is due to the nonlinearity of the water-saturated sandy sediment. The amplitude of difference frequency acoustic wave with the water-saturated sandy sediment was 12 dB higher than that without the sediment. Figure 2(b) also shows the generation of sum frequency acoustic wave at the frequency of 190 kHz. Since the difference of amplitude with and without the water-saturated sandy sediment was less than 3 dB, the experimental investigation for sum frequency wave is not considered in this study.

As shown Fig. 2, since the nonlinearity of the water-saturated sandy sediment is higher than that of the water, the nonlinear interaction zone can be confined to the sediment. Then, ρ_0, c_0, and

α_T in nonlinear source terms of Eq. (9) are the density, the sound speed, and the total attenuation in water saturated-sandy sediment, respectively. Since the difference frequency acoustic wave generated in the sediment is propagated through the water, α_- , k_-, and z in propagation terms of Eq. (9) are the attenuation, the wave number of the difference frequency wave in waver and the distance from the sediment to the observation point. Table.1 shows the parameter values used in Eq. (9).

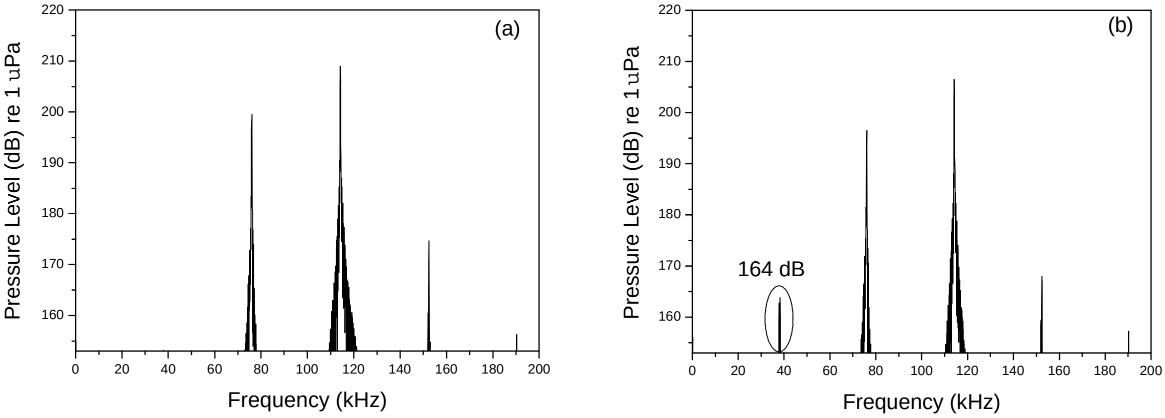

Fig. 2. Frequency spectra of the signals transmitted through (a) the pure water and (b) the water-saturated sandy sediment.

Table 1. Parameter values used in Eq. (9).

Parameter	Value
Density of the sediment, ρ_0	2616 kg/m^3
Sound speed in the sediment, c_0	1680 m/s
Sound attenuation at primary frequency 76 kHz in the sediment, α_1	4.85 Np/m
Sound attenuation at primary frequency 114 kHz in the sediment, α_2	5.38 Np/m
Sound attenuation at difference frequency 38 kHz in the sediment, α_d	4.24 Np/m
Total sound attenuation in the sediment, $\alpha_T = \alpha_1 + \alpha_2 - \alpha_d$	5.98 Np/m
Sound attenuation at difference frequency 38 kHz in the water, α_-	2.65 Np/m
Distance from the sediment to the observation point, z	0.4 m

Figure 3 shows the theoretical and experimental results for the far field beam directivity pattern of the difference frequency acoustic wave at the frequency of 38 kHz with the water-saturated sandy sediment. The theoretical results are calculated from Eq. (11). As shown in Fig. 3, the

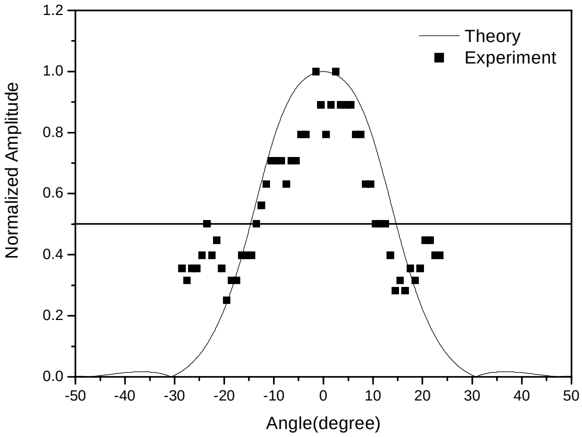

Fig. 3. Far field beam directivity patterns of the difference frequency waves at 38 kHz with the water-saturated sandy sediment.

theoretical results do not almost show the side lobes for the difference frequency acoustic wave. But, the experimental results show some small side lobes. It means that the phase cancellation between two primary acoustic waves is not enough progressed during the propagation to the observation point. Theoretical results in Fig. 3 also shows that the beam width of the difference frequency acoustic wave is 30°, which is narrower than the beam width of 41° for the circular piston at the same frequency with the difference frequency [6]. The difference frequency acoustic wave has the narrow beam width is one of the characteristics of the parametric acoustic array.

Table 2. Pressure levels of the difference frequency acoustic waves at the primary frequencies of 76 kHz and 113.5 – 115 kHz with the water-saturated sandy sediment.

Difference Frequency (kHz)	Pressure Amplitude (Pa)
37.5	181
38.0	146
38.5	219
39.0	186

Table 2 shows the pressure amplitudes of the difference frequency acoustic waves at the primary frequencies of 76 kHz and 113.5 – 115 kHz with the water-saturated sandy sediment. Incident acoustic pressure of the primary wave at the frequency of 76 kHz was 140 kPa and it was 112 kPa at the primary frequency range 113.5 – 115 kHz. Then, the nonlinear parameter of the water-saturated sandy sediment estimated through Eq. (12) was $\beta = 73$. This is the same value with the nonlinear parameter of 73 estimated by using the second harmonic frequency data, which is measured at fundamental frequency of 76 kHz and at distance of 100 mm from the sediment. The nonlinear parameter estimation method with the second harmonic frequency data is referred in Ref. 8. The estimated nonlinear parameter value is not remarkable value as compared with that of the gassy sediment [3] and closed to the data obtained in [9]. However, since the gassy sandy sediment has a low gas bubble concentration as compared with the gassy muddy sediment [3], its nonlinear parameter value can be low. For this case, the estimated nonlinear parameter can be used as the background data to estimate the parameter of the gassy sandy sediment with a low gas void fraction.

4. Conclusions

The difference frequency acoustic wave in water-saturated sandy sediment was significantly observed in a water tank experiment due to the nonlinearity of the sediment. Its directivity beam pattern agrees well with that estimated by a parametric acoustic array theory. The nonlinearity parameter is also estimated as $\beta = 73$. This value can be used as a practical criterion to estimate the nonlinear parameter of the gassy ocean sandy sediment of similar porosity and average grain size with the sample sediment used in this paper.

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