A.K. Batrin, A.M. Gavrilov

NONLINEAR INTERACTION OF WAVES WITH MULTIPLE FREQUENCIES AND ANY AMPLITUDE-PHASE RATIOS

The Taganrog State radio engineering University
Russia, 347900, Taganrog, Shevchenko 2
Ph.: (8634437-17-95; 321-483;
E-mail: gavr_am@mail.ru

The importance of amplitude, phase and frequency ratio is considered at nonlinear interaction of two acoustic waves with multiple frequencies in the quadratically nonlinear medium without a dispersion. The solution of Riman's equation for a biharmonic wave of final amplitude (WFA) with multiple frequencies in preshock areas is obtained. For preshock area a decrease in influence of phase ratio is shown at increase in the relation of frequencies on process of formation of shock in time profile (TP) and on behavior of primary and secondary waves. By the example of interaction of waves with frequencies, refer as 1:3, it is shown, that selection of amplitude-phase ratio it is possible to achieve various practically significant effects, for example, an interdiction of generation of the second harmonic, easing or amplification of nonlinear attenuation of the first harmonic, etc. The experimental researches which have been lead in fresh water, confirm theoretically received laws.

Basic difference of regular waves of final amplitude (WFA) from casual waves is presence of phase dependence of nonlinear processes occurring at their distribution [1, 2]. Until recently the question of joint influence of frequency, amplitude and phase ratios in an primary spectrum regular WFA on process of its distribution in the nonlinear medium remains poorly studied, despite of its urgency for applied tasks. Most full the role of amplitude-phase ratios is considered in special cases of frequency ratios - tasks of the degenerative parametrical interaction (DPI) [1, 3] and a nonlinear acoustic radiator with three-frequency primary wave [4, 5]. It is shown, that amplitude-phase ratios in a spectrum radiate WFA it is possible to change a direction of swapping of energy from primary waves, to brake and strengthen nonlinear attenuation, to forbid generation of separate secondary waves, to initiate a dispersion of speed of a sound, etc.

The laws inherent in the above-stated cases, it is trivial are not distributed to other frequency ratios. So, at interaction of low-frequency and high-frequency waves [6] which frequencies correspond as 1:12, phase dependence of nonlinear processes is not found out neither theoretically (four-frequency approximation), nor experimentally. Reception of the exact decision for the common case biharmonic WFA with multiple frequencies $\omega_1 = \omega$ and $\omega_2 = N\omega$, where $N = 1, 2, 3, 4,\ldots$, and any amplitude-phase ratios in an initial spectrum will allow to consider from uniform positions various cases of frequency ratio including considered earlier [3, 6].

Collinear distribution of two waves to the ideal quadratically nonlinear medium without a dispersion is described by the equation of simple waves [1]:

$$\frac{\partial v}{\partial x} - \frac{\varepsilon}{c_0^2} v \frac{\partial v}{\partial \tau} = 0, \qquad (1)$$

where ε - parameter of nonlinearity of medium; c_0 - speed of distribution of waves in not indignant medium; $\tau = (t - x/c_0)$ - time in accompanying system of coordinates. The boundary condition represents biharmonic wave with frequencies $\omega_1 = \omega$ and $\omega_2 = (N\omega)$

$$v(\tau, x = 0) = v_1 \sin \omega_1 \tau + v_2 \sin(\omega_2 \tau + \varphi_0), \qquad (2)$$

Where $\varphi_0 = (\varphi_2 - N\varphi_1)$ - phase invariant, describing a ratio of phases for two waves with different frequencies which size in linear approach does not depend from passable flat waves of distance and time; φ_1 and φ_2 - initial phases according to the first and the second waves. For a boundary conditions (2) private decision of the equation (1) looks like

$$u(z,\tau) = sin(\omega \cdot \tau + z \cdot u) + A \cdot sin(N \cdot \omega \cdot \tau + N \cdot z \cdot u + \varphi_0), \qquad (3)$$

where $A = v_2 / v_1$; $u = v/v_1$; $z = x(\varepsilon \cdot \omega \cdot v_1 / c_0^2) = x/x_P$; x_P - distance of formation of shock in a wave profile of the first wave (ω_1), a second wave extending in absence; A - the relation of initial amplitudes of the first (ω_1) and the second (ω_2) waves; $u(\tau,x)$ - the current value of oscillatory speed extending biharmonic

WFA, normalized on initial amplitude of the first wave. The further analysis $u(z,\tau)$ we shall lead, having set it in the parametrical form:

$$u = \sin\xi + A\cdot \sin(N\cdot\xi+\varphi_0); \quad \omega\cdot\tau = \xi - z\cdot u = \xi - z\cdot[\sin\xi + A\cdot\sin(N\cdot\xi+\varphi_0)]. \quad (4)$$

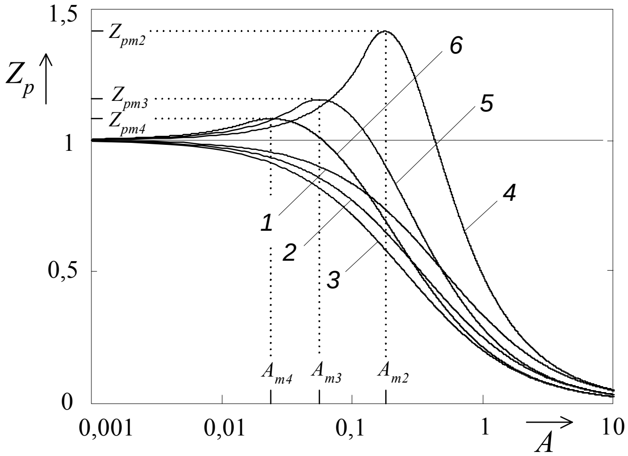


Fig. 1. Dependence of length of area of formation of shock TP biharmonic WFA from amplitude of the second wave at various ratio of frequencies: curves 1 and 4 ($N = 2$); 2 and 5 ($N = 3$); 3 and 6 ($N = 4$); 1, 2, 3 - at $\varphi_0 = 0$; 4, 5, 6 - at $\varphi_0 = 180^0$

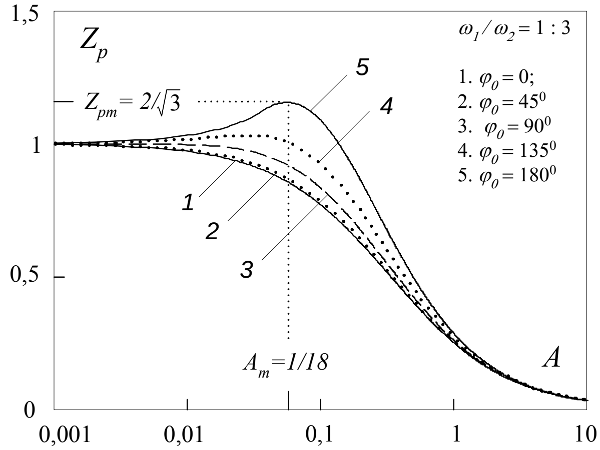
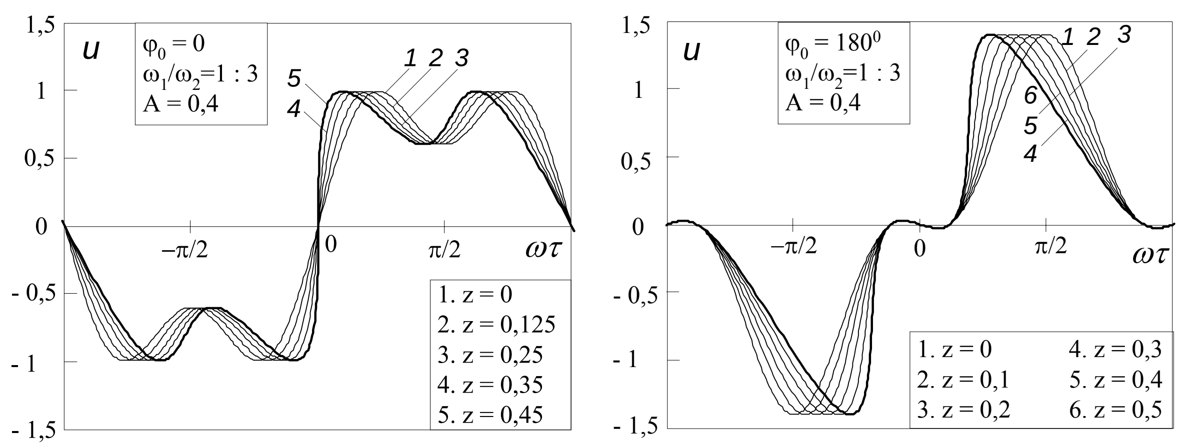


Fig. 2. Dependences of length of area of formation of shock TP two-frequency WFA with a ratio of frequencies $N = 3$ from amplitude of the second wave at various values initial ($z = 0$) phase invariant

One of criteria of intensity of nonlinear distortions in a wave $u(z,\tau)$ is the distance of formation of shock z_P, passing which in a structure of a wave is formed a site with an infinite gradient of oscillatory speed. The size z_P is determined from system of the equations:

$$(\omega\cdot\tau)'_{\xi} = 1 - z\cdot[\cos\xi + N\cdot A\cdot\cos(N\cdot\xi+\varphi_0)] = 0;$$
$$(\omega\cdot\tau)''_{\xi\xi} = z\cdot[\sin\xi + N^2 A\cdot\sin(N\cdot\xi+\varphi_0)] = 0. \quad (5)$$



a) б)

Fig. 3. Transformation of a time profile biharmonic WFA with distance ($A = 0{,}4$; $N = 3$) at initial values phase invariant $\varphi_0 = 0$ (a) and $\varphi_0 = 180^0$ (б)

On fig. 1 the designed dependences $z_P(A)$ for N = 2, 3 and 4 are shown at extreme values phase invariant $\varphi_0 = 0$ and $\varphi_0 = 180^0$ on which reduction of influence of phase ratio by process of formation of shock in TP biharmonic WFA with growth N is traced. It is obvious, that at $N \to \infty$ this influence becomes vanishing small, confirming a correctness received in [6] conclusions. At $\varphi_0 = 45^0$, 90^0 and 135^0 dependences $z_P(A)$ occupy intermediate position, as well as in case DPI [3], on fig. 2 case $N = 3$ is shown. Dynamics of distortions and formations of shock in TP for various N and equal values A and φ_0 as have shown calculations, is qualitatively similar. On fig. 3 it is shown, that at $\varphi_0 = 0$ and $\varphi_0 = 180^0$ shocks are formed in different places of a profile (case $N = 3$), that, as well as in case DPI [1, 3], results in their qualitatively distinguished further behavior - to formation of shock front or movement of breaks.

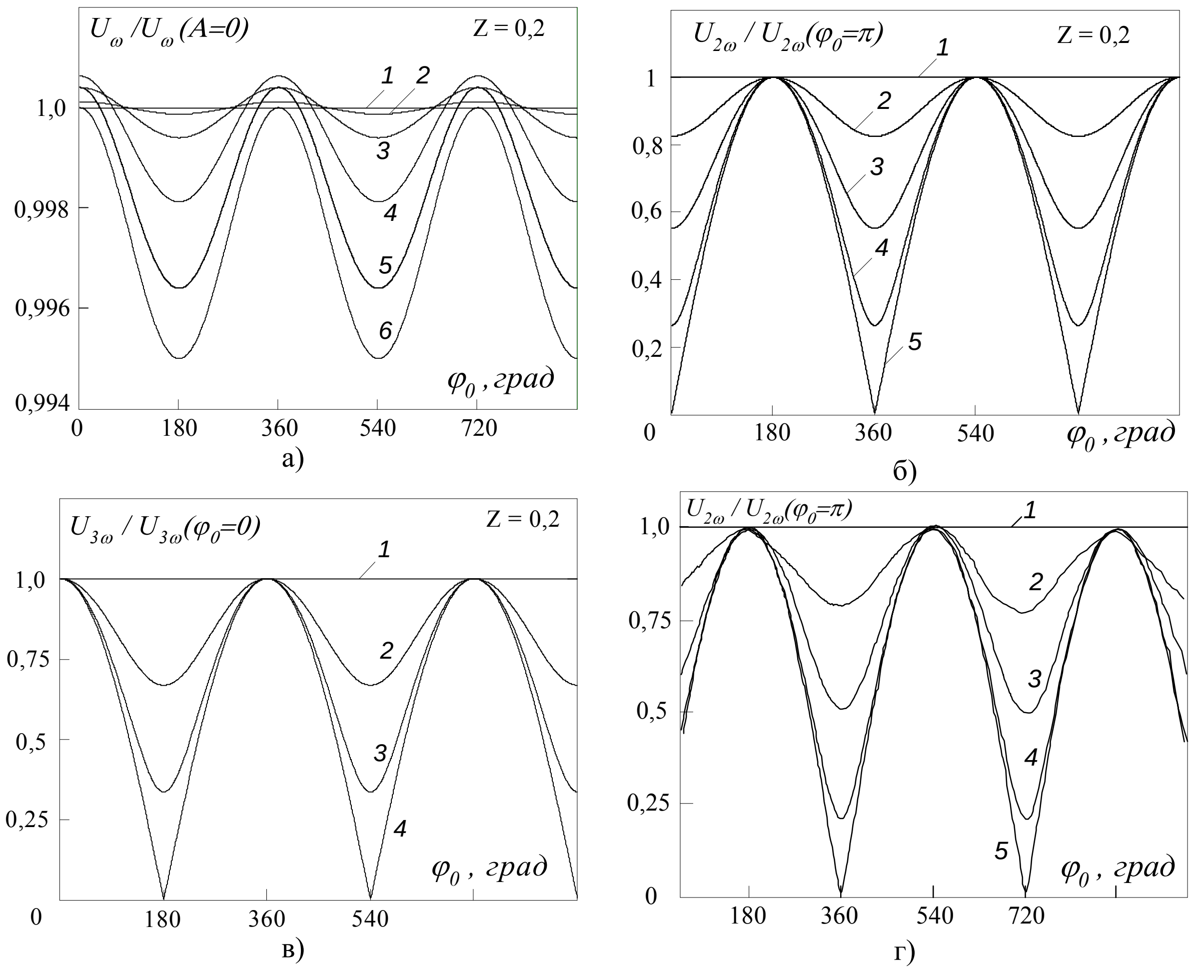


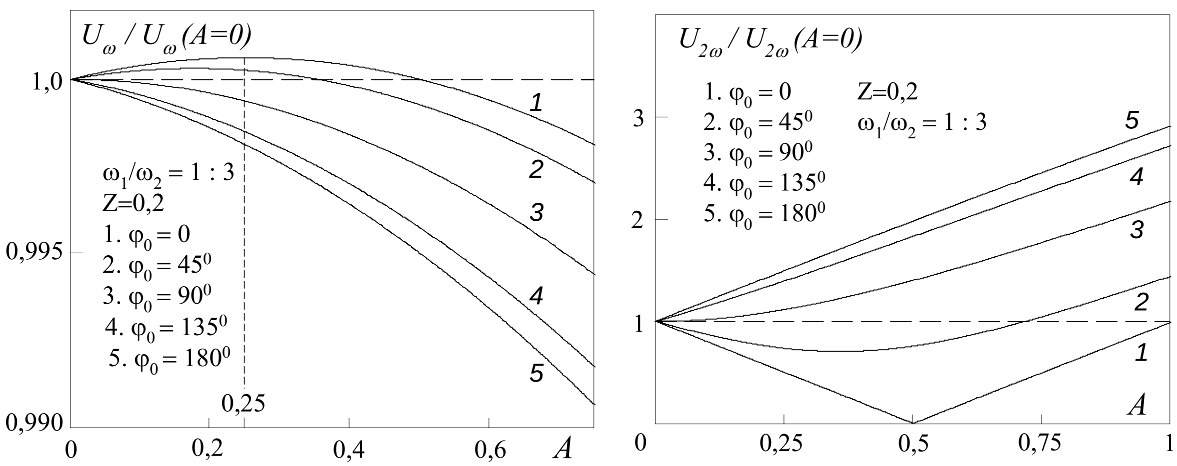
Fig. 4. Amplitude-phase characteristics of the first (а), the second (б) and the third (в) harmonics at various ratio of amplitudes in primary biharmonic WFA at $N = 3$:
а) 1. - $A = 0$; 2. - $A = 0{,}025$; 3. $A = 0{,}1$; 4. - $A = 0{,}25$; 5. - $A = 0{,}4$; 6. - $A = 0{,}5$;
б) 1. - $A = 0$; 2. - $A = 0{,}05$ and 2,03; 3. - $A = 0{,}15$ and 1,36; 4. - $A = 0{,}3$ and 0,82; 5. - $A = 0{,}5$;
в) 1. - $A = 0$; 2. - $A = 0{,}0032$ and 0,08; 3. $A = 0{,}008$ and 0,0325; 4. - $A = 0{,}016$;
г) experiment: 1 - 5 at values A on fig. 4-в

Within the framework of the spectral analysis of the common decision (3) expression for amplitude n-th harmonics in a spectrum originally biharmonic WFA is received

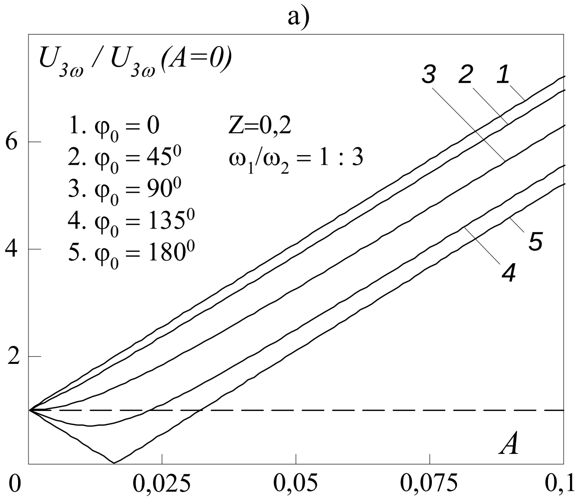
$$U_n(z,A,\varphi_0,N)=\left|\frac{1}{in\cdot z}\sum_{l=-\infty}^{\infty}J_{n-l\cdot N}(n\cdot z)J_l(A\cdot n\cdot z)\cdot exp(il\cdot\varphi_0)\right|. \qquad (5)$$

At $A = 0$ this expression (5) passes in known Bessel-Fubiny decision for single monochromatic WFA [1]. The designed and experimental amplitude-phase characteristics of first three harmonics biharmonic WFA ($N = 3$) reflect phase-depending character of nonlinear processes both for primary, and for secondary waves, fig. 4.

On fig. 5 dependences of the normalized amplitudes of first three harmonics in a spectrum biharmonic WFA ($N = 3$) from amplitude of the second wave are shown A at various values phase invariant. It is visible, that at $\varphi_0 = 0$ and $0 < A < 0{,}5$ amplitude of the first harmonic exceeds a level which it has in case of monochromatic WFA though and to a lesser degree, than at DPI [3]. It occurs because of delay of its nonlinear attenuation and partial swapping in it of energy of the second wave ($\omega_2 = 3\omega$). Intensity of generation of the second harmonic (2ω) also can be adjusted over a wide range amplitude-phase ratio down to its full interdiction ($A = 0{,}5$ at $\varphi_0 = 0$).



а) б)



в)

Fig. 5. Dependences of the normalized amplitudes of the first (a), the second (б) and the third (в) harmonics biharmonic WFA at $N = 3$ and $z = 0{,}2$ from amplitude ratio in an primary wave

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