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ACTIVE-PASSIVE THERMOACOUSTIC TOMOGRAPHY: RESULTS OF THE MODELING EXPERIMENTS

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The process of correlation measurements that serves as the basis for thermoacoustic devices [1] is briefly considered. We also discuss the new possibilities arising when additional sources of thermal fields or specially generated random fields (so-called "illumination fields") are introduced into the system. A possibility of reconstructing the local values of absorption and inhomogeneity of sound velocity from analyzing the correlation dependences based on difference delays and with the help of a controlled anisotropic acoustic illumination is demonstrated.

Recent studies [2] demonstrated that the tendency to increase to the maximum extent the spatial resolution and information content of images in problems of acoustic thermotomography offers considerable promise for a combined utilization of data in the active-passive mode. The approach uses a detection of the fields in a correlation tomographic system due to both the intrinsic thermal radiation of an object under investigation and the scattering of external acoustic fields by this object, including the case of simultaneous utilization of an additional thermal or quasi-random acoustic irradiation. This active-passive mode provides an opportunity to realize a unified approach to the problem and to statistically estimate the temperature [3] as well as acoustic characteristics of a medium, which will increase the diagnostic capabilities of the acoustic methods. It is possible within the framework of this approach to suggest a scheme of acoustic correlation thermotomography of a medium that is inhomogeneous in temperature, absorption coefficient, ultrasonic phase velocity, and, possibly, density. The theoretical basis and the experimental methods of active-passive thermoacoustic tomography are more comprehensive described in [2, 4].

A model is considered that provides an opportunity to obtain the results important for the following consideration. For the proving major theoretical conclusions we developed and manufactured an experimental setup, which replicate the scheme at fig. 1. Two identical plane receivers (transducers 1 and 2 in fig. 1) and also an inhomogeneity of absorption and sound phase velocity in the form of a liquid thin (much thinner than the average wavelength λ) layer are located in a volume. The volume is filled with a weakly absorbing liquid medium and bounded by walls absorbing acoustical radiation. The transducers separate, from the thermal radiation of the absorbing layer, plane waves propagating in directions that are perpendicular to the planes of the transducers.

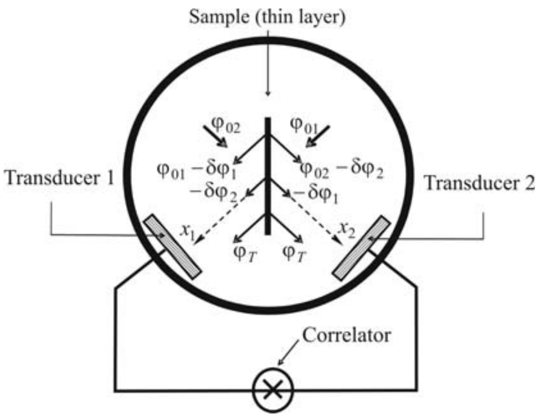

Fig. 1. Correlation measurements of thermoacoustic fields of a thin layer

Let us take the plane wave φ_0 radiated by a part of the wall. The wave equation for the potential of the particle velocity φ in a medium inhomogeneous in viscosity and sound phase velocity $c(\mathbf{r})$ with a constant density ρ_0 has the form, in the quasi-monochromatic approximation $\sim \exp(-i\omega t)$:

$$\Delta\varphi + \frac{\omega^2}{c_0^2}\varphi = \frac{ib\omega}{\rho_0 c_0^2}\Delta\varphi + \nu\varphi + F_0 \ , \text{ where } \ \nu(\mathbf{r}) = \left(\frac{\omega^2}{c_0^2} - \frac{\omega^2}{c^2(\mathbf{r})}\right) \ , \ b = (4/3)\eta + \xi \ ; \qquad (1)$$

η and ξ are the coefficients of shear and bulk viscosity; and f_0 are the external sources produced by fluctuation processes in the walls of the volume; c_0 is the phase velocity in the surrounding liquid.

Let the x_1 axis coincide with the axis of the near wave field of receiving transducer 1. The one-dimensional problem along this axis is considered; the Green function is $G(x,x') = -(i/2k_0)\exp(ik_0\,|\,x-x'\,|)$. In the first Born approximation, the solution of Eq. (1) has the form:

$$\varphi(x_1) \equiv \varphi_0(x_1) - \delta\varphi(x_1) \approx \varphi_0(x_1) - \frac{k_0^2}{2\rho_0 c_0}\int b(x')\,\varphi_0(x')\,\exp(ik_0|\,x_1 - x'|)\,dx' - \\ -\frac{i}{2k_0}\int \nu(x')\,\varphi_0(x')\,\exp(ik_0|\,x_1 - x'|)\,dx' \;\equiv\; \varphi_0(x_1) - \delta\varphi_b(x_1) - i\,\delta\varphi_\nu(x_1) \quad . \qquad (2)$$

If external radiation from the walls is incident on the layer, the secondary sources producing the scattered field $-\delta\varphi$ are generated in the layer. In Born approximation we have $-\delta\varphi \approx -\delta\varphi_b - i\,\delta\varphi_\nu$, and the constituent $-\delta\varphi_b$ has the phase opposite to the phase of the initial field φ_0. The imaginary unit before the second integral term in Eq. (2) is the evidence of a phase shift by $\pm\pi/2$ in the constituent $-i\,\delta\varphi_\nu$.

In the case of the mode of phasing by ***difference delays***, the thin layer is oriented parallel to the bisectrix of the angle between the planes of receiving transducers 1 and 2 in fig. 1. Mutually uncorrelated primary (background) waves φ_{01} and φ_{02} from the corresponding parts of the basin walls arrive at the layer lying within the region of intersection of the sensitivity zones of the receivers. After scattering of these waves from the layer, the total field $(\varphi_{01} - \delta\varphi_1)$ and the scattered field $(-\delta\varphi_1)$ propagate in the directions symmetric with respect to the layer and combine with $(-\delta\varphi_2)$ and $(\varphi_{02} - \delta\varphi_2)$. Moreover, the absorbing layer produces its own thermoacoustic radiation φ_T, which also goes to both receivers. The cross-coherence function of signals (i.e., the correlation function in the case of a complex representation of signals) is:

$$\Gamma_{12}(\tau) = \Big\langle (\varphi_{01} - \delta\varphi_1 - \delta\varphi_2 + \varphi_T)\,\Big|_{\,t} \times (\varphi_{02} - \delta\varphi_2 - \delta\varphi_1 + \varphi_T)^*\,\Big|_{\,t+\tau} \Big\rangle \;, \qquad (3)$$

where the angular brackets $\langle\ \rangle$ mean averaging over realizations. In the case of the delay $\tau = \tau_{12}^-$ equal to the difference of propagation times from the layer to receivers 2 and 1, taking into account the independence of signals, we obtain:

$$\Gamma_{12}(\tau = \tau_{12}^-) = \Big\langle \big|\,\delta\varphi_1\big|^2 \Big\rangle + \Big\langle \big|\,\delta\varphi_2\big|^2 \Big\rangle - \Big\langle \varphi_{01}\,\delta\varphi_1^* \Big\rangle - \Big\langle \varphi_{02}^*\,\delta\varphi_2 \Big\rangle + \Big\langle \big|\,\varphi_T\big|^2 \Big\rangle \;. \qquad (4)$$

Mutual orientation of the layer and transducers 1 and 2 provides an in-phase contribution to the value $\Gamma_{12}(\tau = \tau_{12}^-)$ from all parts of the layer.

In the *isothermal case*, the presence of refractive-absorbing regions does not change the power and correlation properties of the received signals. In the *nonequilibrium case* the temperature of the background radiation $T_{bg}(\mathbf{r})$ differs from the intrinsic temperature $T(\mathbf{r})$ of some absorbing region of the medium. Then a "nonequilibrium" distributed source of intrinsic and scattered thermal noise arises with the spatial density of power distribution, proportional to: $\sim \alpha(\mathbf{r})\,[T(\mathbf{r}) - T_{bg}(\mathbf{r})]$, where absorption coefficient $\alpha(\mathbf{r}) = b(\mathbf{r})\,\omega^2/(\rho_0 c_0^3)$.

The intrinsic thermal field φ_{0T} generated within an absorbing object undergoes multiple scattering from the inhomogeneities of viscosity and sound velocity, in the same way as the field φ_0 incident on the object from outside. Therefore, by analogy with the total field $\varphi \equiv \varphi_0 - \delta\varphi$, the total intrinsic thermal field φ_T can be written in the form $\varphi_T \equiv \varphi_{0T} - \delta\varphi_T$, where $(-\delta\varphi_T)$ is the scattered component. In this case, we have:

$$\Big\langle \big|\,\varphi_T\big|^2 \Big\rangle = \Big\langle \big|\,\varphi_{0T}\big|^2 \Big\rangle - 2\,\mathrm{Re}\Big\langle \varphi_{0T}\,\delta\varphi_T^* \Big\rangle + \Big\langle \big|\delta\,\varphi_T\big|^2 \Big\rangle \;. \qquad (5)$$

Representation of the scattered fields $-\delta\varphi_1$, $-\delta\varphi_2$ and $-\delta\varphi_T$ in the form of the Born-Neumann series (their principal terms are given by Eq. (2)) provides an opportunity to express Eq. (4) using the acoustic and temperature parameters b, ν, T, and T_{bg} of the thin layer. Thus, keeping the series terms linear in b and ν in Eq. (4), have the form:

$$\mathrm{Re}\,\Gamma_{12}(\tau=\tau_{12}^{-}) = 0.5A(b)\left[T-T_{bg}^{(1)}\right] + 0.5A(b)\left[T-T_{bg}^{(2)}\right] = A(b)\left[T-(T_{bg}^{(1)}+T_{bg}^{(2)})/2\right] \;; \quad (6)$$

$$\mathrm{Im}\,\Gamma_{12}(\tau=\tau_{12}^{-}) = -\mathrm{Im}\left\langle\varphi_{01}\,\delta\varphi_1^*\right\rangle - \mathrm{Im}\left\langle\varphi_{02}^*\,\delta\varphi_2\right\rangle = C(\nu)\left[T_{bg}^{(1)} - T_{bg}^{(2)}\right] \;, \quad (7)$$

where the coefficients $A(b)$ and $C(\nu)$ are proportional to b and ν, respectively. Here and later the upper index at the temperature of background irradiation (for example, $T_{bg}{}^{(1)}(\mathbf{r})$) signifies the correspondence of this temperature to the field with the same index (i.e. φ_{01}). In the case of isotropic background radiation with the temperature $T_{bg} \equiv T_{bg}^{(1)} = T_{bg}^{(2)}$, it follows from Eqs. (6) and (7) that:

$$\mathrm{Re}\,\Gamma_{12}(\tau=\tau_{12}^{-}) = A(b)\left[T-T_{bg}\right] \;; \qquad \mathrm{Im}\,\Gamma_{12}(\tau=\tau_{12}^{-}) = 0 \;. \quad (8)$$

Therefore, the value of the inhomogeneity of phase velocity can't be reconstructed in the conditions of isotropic background radiation in the case of signal phasing by only difference delays. It was proposed in [2, 4] the scheme of correlation measurements in the case of signal phasing by summary delays to reconstruction the $\nu(\mathbf{r})$. However, the implementation of the scheme of correlation measurements based on summary delays is sufficiently difficult, since it needs utilization of an antenna array made of sound-transparent receiving transducers. Therefore, separate reconstruction of both scattering components $b(\mathbf{r})$ and $\nu(\mathbf{r})$ in a scheme with difference delays in the case of utilization of anisotropic background radiation seems to be promising for real tomographic systems. This is connected with the appearance of a nonzero imaginary part of a phased coherence function with the principal term proportional to $\nu(\mathbf{r})$. Indeed, if the basin walls are heated nonuniformly in such a way that the background fields φ_{01} and φ_{02} have different temperatures $T_{bg}^{(1)} \neq T_{bg}^{(2)}$, then, according to Eq. (7), $\mathrm{Im}\,\Gamma_{12}(\tau_{12}^{-}) \neq 0$. Let us consider two simplest versions of realization of background radiation at a fixed intrinsic temperature T of a thin layer under investigation. In the first version, only the temperature of the field φ_{01} differs from T, $T_{bg}^{(1)} = T + \delta T^{(1)}$ and $T_{bg}^{(2)} = T$. In this case, for the coherence function $\Gamma_{12} \equiv \Gamma_{12}^{(\mathrm{I})}$ from Eqs. (6) and (7) it follows that:

$$\mathrm{Re}\,\Gamma_{12}^{(\mathrm{I})}(\tau=\tau_{12}^{-}) = 0.5A(b)\left[T-T_{bg}^{(1)}\right] = -0.5A(b)\,\delta T^{(1)} ; \quad \mathrm{Im}\,\Gamma_{12}^{(\mathrm{I})}(\tau=\tau_{12}^{-}) = C(\nu)\,\delta T^{(1)} . \quad (9)$$

In the second version, on the contrary, the temperature of the field φ_{02} differs from T, $T_{bg}^{(1)} = T$ and $T_{bg}^{(2)} = T + \delta T^{(2)}$. In this case, we have for $\Gamma_{12} \equiv \Gamma_{12}^{(\mathrm{II})}$:

$$\mathrm{Re}\,\Gamma_{12}^{(\mathrm{II})}(\tau=\tau_{12}^{-}) = 0.5A(b)\left[T-T_{bg}^{(2)}\right] = -0.5A(b)\delta T^{(2)} ; \quad \mathrm{Im}\Gamma_{12}^{(\mathrm{II})}(\tau=\tau_{12}^{-}) = -C(\nu)\,\delta T^{(2)} . \quad (10)$$

Therefore, in the case of anisotropic background radiation, the principal term of the function $\mathrm{Re}\,\Gamma_{12}(\tau_{12}^{-})$ is proportional to the absorption coefficient, and the principal term of the function $\mathrm{Im}\,\Gamma_{12}(\tau_{12}^{-})$ is proportional to inhomogeneity of sound velocity. These properties provide an opportunity to reconstruct both b and ν, while working only with difference delays. The effect can be amplified and the precision of reconstruction can be increased by using summary and difference combinations of Eqs. (9) and (10). The sum $\Gamma_{12}^{(\mathrm{I})} + \Gamma_{12}^{(\mathrm{II})}$ coincides with the result of Eqs. (6) and (7) in the case of a simultaneous irradiation $T_{bg}^{(1)} = T + \delta T^{(1)}$ and $T_{bg}^{(2)} = T + \delta T^{(2)}$:

$$\begin{cases} \mathrm{Re}\,\Gamma_{12}^{(\mathrm{I})}(\tau_{12}^{-}) + \mathrm{Re}\,\Gamma_{12}^{(\mathrm{II})}(\tau_{12}^{-}) = -A(b)\left[\delta T^{(1)} + \delta T^{(2)}\right]/2 \\ \mathrm{Im}\,\Gamma_{12}^{(\mathrm{I})}(\tau_{12}^{-}) + \mathrm{Im}\,\Gamma_{12}^{(\mathrm{II})}(\tau_{12}^{-}) = C(\nu)\left[\delta T^{(1)} - \delta T^{(2)}\right] \end{cases} .$$

The difference $\Gamma_{12}^{(\mathrm{I})} - \Gamma_{12}^{(\mathrm{II})}$ yields:

$$\begin{cases} \mathrm{Re}\,\Gamma_{12}^{(\mathrm{I})}(\tau_{12}^{-}) - \mathrm{Re}\,\Gamma_{12}^{(\mathrm{II})}(\tau_{12}^{-}) = -A(b)\left[\delta T^{(1)} - \delta T^{(2)}\right]/2 \\ \mathrm{Im}\,\Gamma_{12}^{(\mathrm{I})}(\tau_{12}^{-}) - \mathrm{Im}\,\Gamma_{12}^{(\mathrm{II})}(\tau_{12}^{-}) = C(\nu)\left[\delta T^{(1)} + \delta T^{(2)}\right] \end{cases} .$$

The particular case of $\delta T \equiv \delta T^{(1)} = \delta T^{(2)}$ provides an opportunity to demonstrate the advantages of the combinations $\Gamma_{12}^{(\mathrm{I})} \pm \Gamma_{12}^{(\mathrm{II})}$:

$$\mathrm{Re}\,\Gamma_{12}^{(\mathrm{I})}(\tau_{12}^{-}) + \mathrm{Re}\,\Gamma_{12}^{(\mathrm{II})}(\tau_{12}^{-}) = -A(b)\,\delta T\ ,\quad \mathrm{Im}\,\Gamma_{12}^{(\mathrm{I})}(\tau_{12}^{-}) + \mathrm{Im}\,\Gamma_{12}^{(\mathrm{II})}(\tau_{12}^{-}) = 0\ ; \tag{11}$$

$$\mathrm{Re}\,\Gamma_{12}^{(\mathrm{I})}(\tau_{12}^{-}) - \mathrm{Re}\,\Gamma_{12}^{(\mathrm{II})}(\tau_{12}^{-}) = 0\ ,\quad \mathrm{Im}\,\Gamma_{12}^{(\mathrm{I})}(\tau_{12}^{-}) - \mathrm{Im}\,\Gamma_{12}^{(\mathrm{II})}(\tau_{12}^{-}) = 2C(\nu)\,\delta T\ . \tag{12}$$

The real part of the summary combination of Eq. (11) separates the contribution from the absorbing component of the layer by doubling it (figs. 2a and 3a). In this case, the contribution of the inhomogeneity of sound velocity to the imaginary part is compensated. Conversely, the imaginary part of the difference combination of Eq. (12) separates and doubles the contribution from the inhomogeneity of sound velocity (figs. 2b and 3b). At the same time, the contribution of the absorbing component to the real part is compensated. Burst of the sum of the correlation functions is absent in fig. 3a, because the refraction layer has not an absorption. The analogous situation takes place in fig. 2b that corresponds to the measurements with the absorbing layer, which has not an inhomogeneity of sound velocity.

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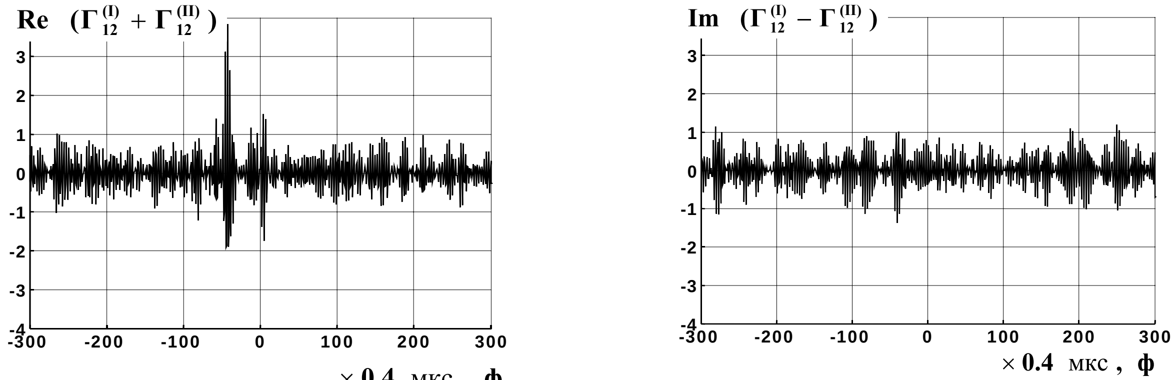
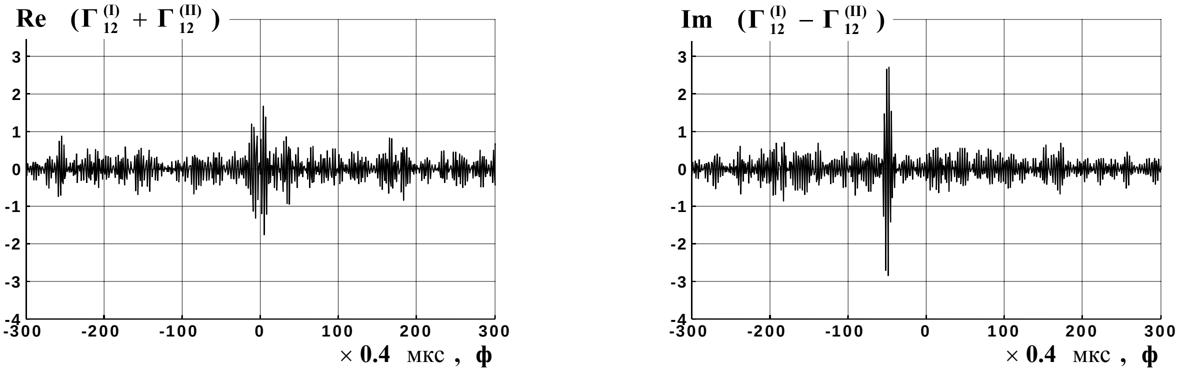


2a ***2b***

Fig. 2. (*2a*) Sum and (*2b*) difference of cross-coherence functions corresponding to an asymmetric irradiation of a thin absorbing layer sequentially from two directions.



3a ***3b***

Fig. 3. (*3a*) Sum and (*3b*) difference of cross-coherence functions corresponding to an asymmetric irradiation of a thin refractive layer sequentially from two directions.

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