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VARIATIONS OF MODAL STRUCTURE IN A RANGE-DEPENDENT WAVEGUIDE

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Ray method for evaluation of the mode amplitudes has been applied for analysis of wave fields in deep water acoustic waveguides with refractive index inhomogeneities of two types: (i) weak inhomogeneities that cause small perturbations of ray paths and (ii) strong inhomogeneities with large spatial scales. Simple analytical relations for description of mode amplitude variations in the presence of such inhomogeneities have been obtained. It has been found out under what conditions mode scattering at weak fluctuations of refractive index can lead to significant redistribution of energy between modes. The derivation of a new criterion for applicability of the adiabatic approximation is another result of this work. Our criterion agrees with the well-known requirement that range variations should be small at the cycle length and with a statement that the adiabatic approximation accuracy enhances at low frequencies. But our new criterion allows one to account for accumulation of errors with range and to estimate limiting ranges at which the adiabatic approximation still can be applied. In order to illustrate and test our analytical results numerical simulations of wave fields in deep water waveguides have been carried out.

Applicability of the normal mode approach for the analysis of wave field in range-dependent waveguides is restricted by the complexity of mode coupling equations describing the dynamics of mode amplitudes [1,2]. In spite of the fact that fast and accurate codes for evaluation of wave fields by the coupled mode method are available [3], the derivation of approximate analytical approaches that may allow one to establish connection between variations of the modal structure variations of medium parameters that cause these variations are still of interest. In Refs. [4-7] it has been shown that in the high frequency approximation the mode amplitude can be unexpectedly simply expressed through solutions of the ray equations.

Mathematically, this results is formulated as follows. Let us expand the wave field at every range point into a sum over local modes. The latter are also called the modes of the reference waveguide, i.e. modes of an imaginary range-independent waveguide whose cross section coincides with the cross section of the real waveguide at the given range r . An eigenfunction of the m -th mode $\phi_m(z)$ in the WKB approximation can be represented as a sum of two terms with amplitudes being smooth functions of coordinates and rapidly varying phases:

$$\phi_m(z) = \phi_m^+(z) + \phi_m^-(z), \quad \phi_m^\pm(z) = Q_m e^{\pm ikS_m(z)}. \quad (1)$$

Explicit expressions for amplitudes Q_m and phases ϕ_m are well known [1,2] and therefore we do not present them here. The mode representation of the wave field in the range-dependent waveguide has the form:

$$u(r, z) = \sum_m B_m(r) \phi_m(r, z). \quad (2)$$

In Refs. [4-7] it is shown that in the geometrical optics approximation the amplitude of the m -th mode $B_m(r)$ is determined by contributions from the so-called *mode rays*. This term denotes standard rays paths whose action variables at the given range r are equal to the action variable of the m -th mode $\pi(m + 1/2)$. Then

$$B_m(r) = \sum_v A_{m,v} e^{ik\Phi_{m,v}}, \quad (3)$$

where $A_{m,v}$ and $\Phi_{m,v}$ are an amplitude and phase of the v -th mode ray, respectively. Explicit expressions for these quantities see in Refs. [4-6].

In the range-independent waveguide at every range point there are two mode rays for each mode. These mode rays leaves the point source at launch angles equal to grazing angle of the Brillouin wave forming the mode. Figure 1 graphs trajectories of mode rays in a waveguide with the canonical sound

speed profile (Munk profile). The field is excited by a monochromatic point source at a carrier frequency 200 Hz.

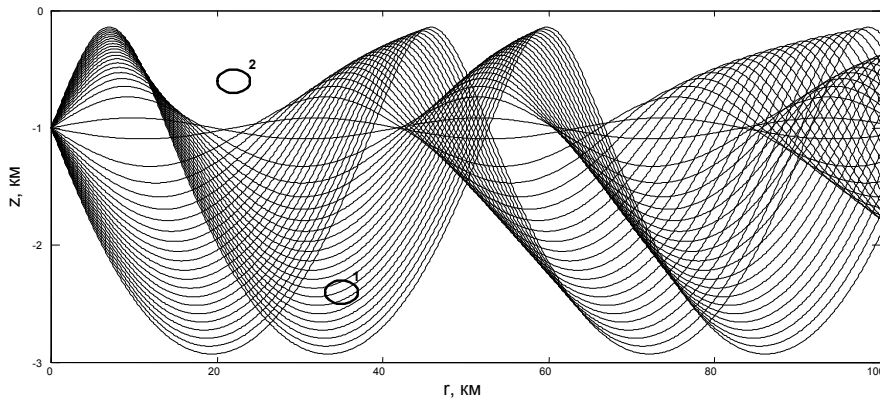


Fig. 1. Trajectories of mode rays in a waveguide with the canonical sound speed profile (Munk profile) are shown for each fifth out of 100 modes. Ellipses 1 and 2 indicate areas where inhomogeneities of refractive index are located.

In the present paper the normal mode method is applied to analyze mode coupling, i.e. redistribution of the sound energy between modes, in a range-dependent waveguide. We consider two problems. The first one deals with weak inhomogeneities that in the standard geometrical optics are accounted for by introducing increments for the ray eikonals [1]. In our approach similar increments are introduced for eikonal of mode rays. In the presence of a weak perturbation of the refractive index δn the amplitude of the mode ray does not change but its eikonal $\Phi_{m,\nu}$ acquire an increment

$$\delta\Phi_{m,\nu} = k \int_{\Gamma} \delta n ds, \tag{4}$$

where k is a wave number, ds is an arc length, and integration goes over an unperturbed mode ray. It is important to note that even weak inhomogeneities whose influence on the ray amplitude is negligible can cause an increment $\delta\Phi_{m,\nu} > \pi$. Then the sum (3) may vary significantly. In other words, inhomogeneities that does not affect ray amplitudes may cause strong variations of mode amplitudes.

As an example we have considered weak inhomogeneities of refractive index determined by the relation $\delta n = \delta n_0 \exp(-(z - z_0)^2 / \Delta z^2 - (r - r_0)^2 / \Delta r^2)$. At ellipses 1 and 2 shown in Fig. 1 the inhomogeneities are e -times weaker than compared to there maximum values δn_0 . According to Eq. (4) the inhomogeneity affects only those modes whose rays intersect it. Results presented in Fig. 2 confirm this statement: only amplitudes of modes with numbers $m=55-85$ significantly vary. One of mode rays for each of these modes (see Fig. 1) either cross the ellipsis 1, or pass in the close vicinity of the ellipsis. Note that the mode variations shown in the upper panel of Fig. 2 have been obtained by to different methods: using Eqs. (1)-(4) or on the basis of numerical solution of the parabolic equation. Both method gave similar results.

In Fig. 1 it is clearly seen that all mode rays for first 100 modes passé away from the ellipsis 2. Correspondingly, amplitudes of these modes remain unchanged in the presence of the second inhomogeneity. These example demonstrate an interesting effect: the mode can “ignore” and inhomogeneity even if the latter is located between the mode turning points. A qualitative explanation to this phenomenon has been given in Ref. [7]. Here we only notice that this can occur only if a large number of modes propagate simultaneously. Due to scattering at an inhomogeneity the mode can loose a part of its energy but under some conditions – these conditions are expressed by the requirement that the mode rays do not intersect the inhomogeneity - the loss is compensated by contributions from other modes. If there is only one incident mode scattering at the inhomogeneity the mode amplitude can only decrease.

Now turn our attention to the case of large-scale inhomogeneities of refractive index and apply our ray formulas for mode amplitudes in order to derive a new criterion for validity of the adiabatic

approximation. A generally used criterion states that the adiabatic approximation is valid under the condition [1,2]

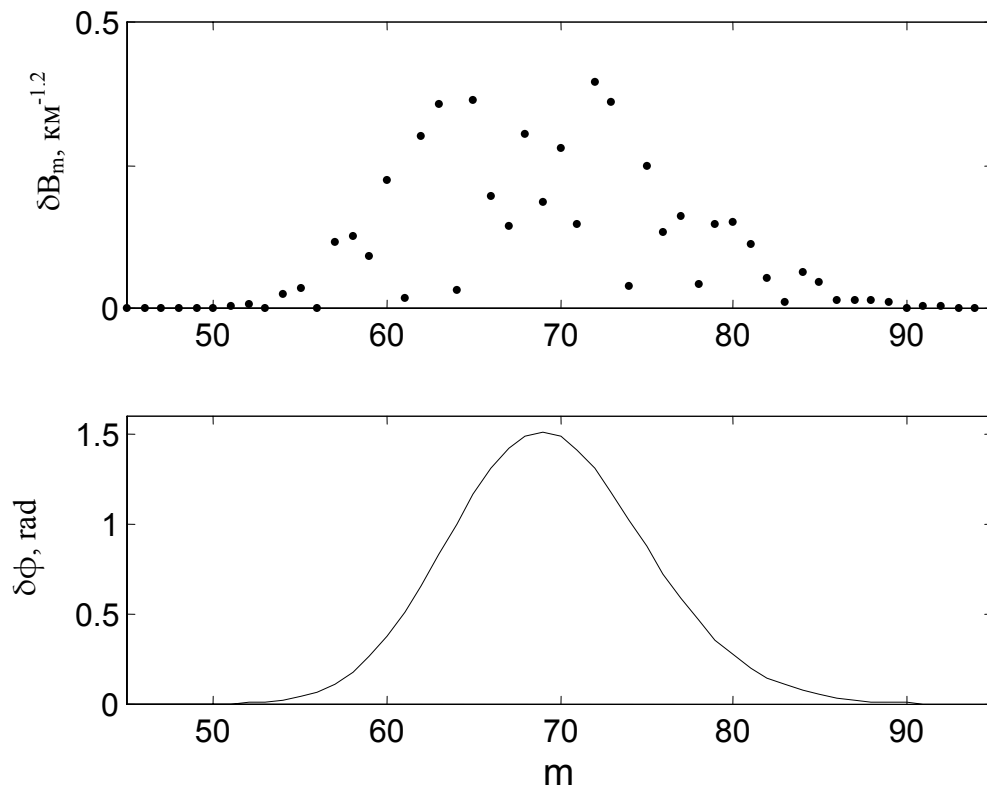


Fig.2. Upper panel: mode amplitude variations due to scattering at an inhomogeneity of the refractive index located in the area 1. Lower panel: variation of the phase of mode rays caused by the inhomogeneity.

$$D/L \ll 1, \tag{5}$$

that requires the smallness of the cycle length D compared to the horizontal scale of the medium range-dependence L . A more exact criterion which account for a dependence on frequency has a form [2]

$$kD^2/L \ll 1. \tag{6}$$

Conditions (5), (6) and similar ones have been obtained by estimating terms in mode coupling equations that are neglected in the adiabatic approximation. Being necessary these conditions are not sufficient and they do not allow one to estimate limiting ranges at which the adiabatic approximation fails. On the other hand, it is clear that at very long ranges the adiabatic approximation cannot be used even the inequalities (5) and (6) are satisfied.

Our method for studying applicability of the adiabatic approximation is based not on the mode coupling equation governing range variations of mode amplitudes [1,2] but on its solution (although approximate) determined by Eqs. (1)-(3). This allows one to look at the problem from a new viewpoint and to obtain an estimates taking into account an accumulation of errors with range. In the parabolic equation approximation our criterion is formulated as follows.

Introduce an auxiliary function $U(r, z) = (1 - n^2(r, z))/2$ and consider the i -th cycle of oscillations of a mode ray corresponding to the m -th mode, that begins at a range point r_i (we assume that the cycle begins at the minimum of the ray path). Then the difference between a real contribution to the eikonal from this cycle and a contribution evaluated in the adiabatic approximation is given by an expression

$$\delta\Phi_i = -k \int_0^{D/2} \left(\rho - D/4 \right) \frac{\partial U(r, z(r))}{\partial r} \Big|_{r=r_i+\rho} d\rho + k \int_{D/2}^D \left(\rho - 3D/4 \right) \frac{\partial U(r, z(r))}{\partial r} \Big|_{r=r_i+\rho} d\rho. \quad (7)$$

When integrating over the first and last incomplete cycles the limits of integration should be changed. In order to find the difference between the exact value of an eikonal for the mode ray and its adiabatic approximation all values of $\delta\Phi_i$ should be summed up. Then the applicability condition for the adiabatic approximation is formulated by an inequality

$$|\delta\Phi| = \left| \sum_i \delta\Phi_i \right| \ll \pi. \quad (8)$$

It is not difficult to show that the condition (8) agrees with the known criteria (5) and (6). But unlike these criteria (8) accounts for accumulation of errors of the adiabatic approximation with range. However, it should be pointed out that our criterion has been obtained under assumption that the geometrical optics approximation is valid. This assumption somewhat restricts the generality of our result.

This work was supported by the Russian Foundation for Basic Research, projects no. 01-05-64394 and 00-02-17409.

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