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CHAOTIC DYNAMICS OF RAYS IN A RANGE-DEPENDENT ACOUSTIC WAVEGUIDE

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A theory has been developed to describe formation of clusters of ray travel times, i.e. arrival times of sound pulses coming to the receiver through different ray paths. This effect observed at long range (over 1000 km) propagation in underwater acoustic waveguides was discovered in the late nineties. Analytical relations estimating main parameters of the clusters have been derived. It is shown that even under conditions of ray chaos clusters formed by steep rays are rather stable. This explains high stability of the early part of the arrival pattern observed in both numerical simulations and field experiments. Stochastic ray dynamics has been studied using the Hamilton formalism taken in terms of canonical variables action-angle. A realistic model of deep water waveguide with internal-wave-induced inhomogeneities of sound speed has been considered. It has been shown that a fluctuating constituent of the action can be approximated by a Wiener random process representing the simplest model of diffusion. An analysis of ray travel times has been carried out in the scope of a stochastic ray theory based on this approximation.

Numerical ray tracing in deep water acoustic waveguides demonstrates that the presence of weak inhomogeneities of refractive index induced by random internal waves whose statistics is determined by the Garrette-Munk spectrum, causes a ray chaos. In a range-independent waveguide or in a waveguide with a smooth range dependence the difference between vertical coordinates of two ray trajectories with close initial conditions grows (on the average) linearly with range. Under conditions of ray chaos a linear growth changes to an exponential one [2,3]: a difference between depths of ray trajectories $\Delta z \propto \exp(\gamma r)$, where r is a distance and γ is a Lyapunov exponent. In typical deep water waveguides $\gamma^{-1} \approx 200$ km. Therefore the description of ray trajectories at ranges on the order of 1000 km requires taking into account the phenomenon of ray chaos. The main difficulty of the problem originates from nonlinearity of stochastic Hamilton equations determining ray dynamics. That is why in order to study characteristics of the field ray structure at very long ranges researchers often apply methods of numerical simulations.

In the present paper we derive a simple approximate analytical description of chaotic ray structure valid at very long ranges. Our approach is based on the Hamiltonian formalism taken in terms of the canonical variables action-angle. To define these variables we shall use the following notation. Neglecting out-of-plane scattering we consider sound propagation in a two-dimensional medium with coordinates z (depth) and r (range). It is assumed that the sound speed field $c(r,z)$ is a superposition of a smooth background profile $c_0(z)$ and a weak perturbation $\delta c(r,z)$ "responsible" for the appearance of the ray chaos. In the standard canonical variables momentum-position, trajectories are governed by the Hamilton equations in an auxiliary mechanical system with the Hamiltonian $H = -\sqrt{c^{-2}(r,z) - p^2}$ [1-3]. Here $p = \sin\chi / c$ is an analog to the mechanical momentum and χ is a ray grazing angle. It is convenient to represent the Hamiltonian H in the form $H = H_0 + V$, where H_0 is an unperturbed Hamiltonian (with c replaced by c_0), and $V \approx \delta c / c_0^2$ is a small perturbation.

The transformation to the new canonical variables action-angle, (I,θ), is performed using standard formulas [2]. We do not present these known relations here and only note that the angle variable θ may be interpreted as a phase of an oscillating ray trajectory. At every cycle of oscillations (say, between two consecutive minima) its value varies by 2π. The action variable I determines the amplitude and period of oscillations. The unperturbed Hamiltonian is independent of r and θ, and its derivative with respect to I, $dH_0(I)/dI=\omega(I)$, represents a spatial frequency of oscillations of the unperturbed trajectory. The Hamilton equations take the form

$$\frac{dI}{dr} = -\frac{\partial V}{\partial \theta}, \qquad \frac{d\theta}{dr} = \omega(I) + \frac{\partial V}{\partial I}. \tag{1}$$

We consider the perturbation V as a random function of arguments I, θ, and r.

It is known that a smooth waveguide with a horizontal scale of medium variations L satisfying the condition

$$\omega L / 2\pi \gg 1, \tag{2}$$

is adiabatic and the action variable I conserves along the ray trajectory. In our problem the condition (2) is not met (otherwise ray paths could not be chaotic), but due to the weakness of perturbation variations of the action variable δI are small compared to a characteristic scale of function $\omega(I)$:

$$\mu = \left| \frac{\delta I}{\omega} \frac{d\omega}{dI} \right| \ll 1. \tag{3}$$

Numerical ray tracing demonstrates that even under condition of ray chaos inequality (3) remains valid up to ranges of a few thousands km [4]. In our theory μ is the main small parameter in the problem. In standard momentum-position variables it is difficult to find this small parameter and to use it for a derivation of a perturbation theory.

An analysis of stochastic ray equations (1) can be significantly simplified if one takes into account that the action variable I weakly varies at cycle length of the ray path $\omega / 2\pi$. Since horizontal scales of the internal wave field, typically, do not exceed $\omega / 2\pi$, the right hand side of the first equation in (1) formally can be treated as a δ-correlated random force. Numerical simulations show that the angle variable θ rapidly randomizes and already at a few hundreds km it may be modeled by a random variable uniformly distributed over the interval $(0,2\pi)$. Then the probability density function (PDF) of the action variable, $W(I,r)$, is governed by the Fokker-Planck equation [4]

$$\frac{\partial W}{\partial r} = \frac{1}{2} \frac{\partial}{\partial I} B \frac{\partial W}{\partial I}. \tag{4}$$

Calculations demonstrate that for a realistic model of a deep water acoustic waveguide the diffusivity B weakly depends on the action variable I and it may be approximated by a constant equal to $1.5 \cdot 10^{-7}$ km. Then the action variable can be written as

$$I = I_s + x(r), \tag{5}$$

where $I_s = I(0)$ is a starting value of this variable, and a $x(r)$ is a Wiener process representing the simplest model of diffusion. The PDF of x at range r is

$$W(x,r) = \frac{1}{\sqrt{2\pi B r}} \exp\left(-\frac{x^2}{2Br} \right). \tag{6}$$

The standard deviation of the action I grows with range in accord with the standard diffusion law

$$\sigma_I = \sqrt{< x^2 >} = \sqrt{Br}. \tag{7}$$

Exploiting the small parameter (3) the angle variable may be approximately expressed via $x(r)$ as

$$\theta = \theta_s + \omega(I_s) r + y, \qquad y(r) = \omega'(I_s) \int_0^r x(r_1) dr_1, \tag{8}$$

where the symbol ω' denotes the derivative of ω with respect to I. Equations (5)-(8) combined with formulas defining the canonical transformation from the action-angle variables to the standard momentum-position variables allow one to find practically any statistical characteristic of the ray structure.

In the present paper we apply this result to describe formation of clusters of chaotic ray arrivals at very long ranges discovered in the late nineties [3]. Under conditions of ray chaos at ranges on the order of 1000 km there are a huge number of eigenrays connecting the source and the receiver. It turns out that travel times of these eigenrays break up into rather compact clusters. Every cluster is formed by eigenrays with the same identifier $\pm J$, where $\pm$ indicates the sign of launch angle of the ray and J is the number of ray turning points. In other words, each cluster is formed by rays with the same topology. It should be emphasized that even though eigenrays forming the cluster have the same endpoints and the same topology their trajectories may significantly deviate at intermediate range points.

Relations (7) and (8) allow one to get estimations characterizing trajectory of rays forming the same cluster. In particular, it is not difficult to estimate the spread of arrival angles (here we mean grazing angles that should not be confused with angle variables) of these rays. The corresponding formula has a form [4]

$$\Delta\chi = \frac{\sqrt{Br/3\omega}}{n\left|\sin\chi_c\right|},$$

where χ_c is a grazing angle corresponding to a central ray in the cluster. This estimates is valid only for steep enough rays with χ_c>5°. In a typical deep water waveguide at range 3000 km it gives $\Delta\chi$~1°.

As it has been indicated already, every cluster can be characterized by the identifier of rays belonging to it. The center of cluster is close to an arrival time of an unperturbed ray (in a waveguide with δc=0) with the same identifier. In Ref. [4] it has been shown that the difference between travel times of a perturbed and unperturbed rays with identifiers $\pm J$ и $\pm(J+2N)$ (respectively) is given by approximate expressions

$$\Delta t = \Delta t_0 + \Delta t_1 + \Delta t_2\,, \tag{9}$$

$$\Delta t_0 = 2\pi N I_0,\ \ \Delta t_1 = \frac{1}{2}\omega'(I_0)\int (I - I_0)^2\,dr,\ \ \Delta t_2 = \int \frac{\delta c}{c_0^2}dr\,. \tag{10}$$

Here I and I_0 are action variables of perturbed and unperturbed rays, respectively, and Δt_2 is evaluated by integration over the perturbed ray path.

The quantity Δt_0 gives the difference between travel times of two unperturbed rays whose trajectories have numbers of cycle that differ by N. In the perturbed waveguide this quantity estimates a temporal interval between centers of neighboring clusters. The two other quantities present in Eqs. (9) and (10), can be interpreted as variations of the ray travel time due to deviation of the ray path caused by perturbation, Δt_1, and an additional time variations due to sound speed inhomogeneities crossed by the ray path, Δt_2. The quantity Δt_2 dominate at short ranges, while at ranges of order 1000 km the main contribution to Δt comes from Δt_1.

Let us emphasize the following important point. In the real deep water waveguide the ray cycle length always increases with the absolute value of the ray launch angle. This means that the derivative ω' is negative for all rays. Therefore, according to the second relation in (11) sound pulses coming to the receiver through perturbed rays arrive earlier than signals coming through unperturbed rays with the same identifiers. In other words, every cluster should be biased toward early times compared to the arrival time of the unperturbed ray with the same identifier. Omitting a somewhat complicate derivation (the difficulties are caused by requirement of selecting rays arriving at the given observation point) we present an estimate for a mean bias of the cluster obtained on the base of Eqs. (5)-(10):

$$<\Delta t_1> = \frac{\omega'}{12c_0}Br^2\,. \tag{11}$$

Another important characteristic of the cluster is its width, i.e. a rms spread of travel times of chaotic eigenrays with the given identifier. It can be shown that this spread is approximately equal to the mean bias of the cluster, i.e. it is also given by Eq. (11).

So, we see that the quantity Δt_1 grows with range as r^2. On the other hand, it is easy to show that $\Delta t_2 \sim r^{1/2}$. This facts qaulitatively explains why Δt_2 dominates at short ranges. Note that at short ranges the perturbation does not yet lead to significant deviations of the ray from its unperturbed path. In this case the integration in the expression for Δt_2 practically goes along the unperturbed path and we arrive at a well-known estimate of travel time variations at small distances [5].

The width of the clusters of arrival times may be used as an estimate of the widening of the sound pulse coming through unperturbed rays due to stochastic micromultipathing. If the duration of an initially emitted pulse τ is less than $<\Delta t_1>$, then (11) can be used as an estimate of the pulse width.

Relations (9)-(11) allow one to obtain the criterion of nonoverlapping (i.e. temporal resolution) of neighboring clusters. As an estimate of difference in travel times of consecutive rays with action variables close to I_0 we can take (on the base of the first relation from (10)) $2\pi I_0/c_0$. Making use of (11) we find that sound pulses formed by different clusters nonoverlap if the parameter

$$R = \frac{2\pi I_0}{|\omega'|Br^2} \tag{12}$$

is greater than unity. This result explains the fact that in the field experiments pulses coming through steep rays are well resolved [6]. The point is that the action variable I_0 grows with the launch angle, i.e. the steeper is a ray, the larger is I_0. Besides, the derivative $|\omega'|$ for steep rays is often smaller than for flat ones.

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