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RESULTS OF MATHEMATICAL MODELING OF THE PROCESS OF GENERATING THE COMPLEX SIGNALS REFLECTED FROM THE SEA SURFACE

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With the aim of investigating the process of generation of signals reflected from the sea surface it was carried out the mathematical modeling in the bounds of the performed experiments. It was calculated the reflected signal, when the complex phase-manipulated impulse was radiated while the rough surface S being illuminated with the harmonic signal exp(jwt). The reflected wave was described by the integral of Helmholts-Kicrhhgoff in approximation of Kicrhhgoff. Signal source was considered to be pointwise and non-directed. Discussed are the obtained (in result of mathematical modeling) dependencies A(t) at different values of dispersion of the wave ordinates, spectra of their fluctuations. Constructed/built are the profiles of the surface waves at which the function A(t) acquires the maximal value; it is studied which parts of the waves make the main input into the reflected signal.

The experiments were carried out at the marine hydrophysical research station of the POI FEB RAS in Vityaz' Bay for 1999-2002, complex signals were applied for the studies of the sound propagation channel. They were discussed in the papers published earlier [5, 6, 7], in which the geometry of the experiments was described in detail and it was given the characteristics of the radiated probing signals with the phase modulation of M-sequence. As a result of processing the records of direct signals obtained at conducting the experiments we have got the dependencies of signals amplitude A(t) on the current time, which studies show well observed periodicity induced by the sea surface wave.

The given paper presents the results of mathematical modeling of the performed experiment with the aim of investigating the generation process of signals reflected from the sea surface.

Let's denote the radiated phase-manipulated signal as P(t).

$$P(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} G(\omega)\exp(j\omega t)d\omega \qquad (1)$$

where $G(\omega) = \int_{-\infty}^{\infty} P(t)\exp(-j\omega t)dt$. As the radiated signal consists of harmonic signals exp(jwt) with the amplitudes $G(\omega)\frac{d\omega}{2\pi}$, then firstly we find the reflected signal $p(t)$ at illuminating the disturbed surface *S* by harmonic signal . $\exp(j\omega t)$. To describe the reflected wave we shall use the integral of Helmholts- Kicrhhgoff in approximation of Kicrhhgoff [1, 2]:

$$p(t) = \frac{1}{4\pi}\int_S R\frac{\partial}{\partial n}[p_S(t)\frac{\exp(-jkR_2)}{R_2}]dS\ , \qquad (2)$$

where *R* - reflection coefficient accepted in the given case to be equal -1; $p_S(t)$ – the pressure of the radiated harmonic signal which could be observed in the absence of the reflected surface *S*; R_2 – distance between the surface element *dS*. and the point of receiving; $\frac{\partial}{\partial n}$ – a derivative by the normal to the surface. The source of signal is considered to be point-wise and non-directed, so

$$p_S(t) = \exp(j\omega t - jkR_1)/R_1 \qquad (3)$$

where R_1 – the distance from the source to the surface element dS. While substituting (3) into (2), in case of small angles of inclination of the sea surface roughness we may consider that the

derivative by the normal to the surface coincides with the derivative $\frac{\partial}{\partial y}$, then
$p(t,\omega) = -\frac{\exp(j\omega t)}{4\pi}\int_S \frac{\partial}{\partial y}\{\frac{\exp[-jk(R_1+R_2)]}{R_1 R_2}\}dS$.

To find the reflected signal $P_S(t)$ at radiation of the complex phase-manipulated impulse $P(t)$, it is necessary to multiply $p(t,\omega)$ to $G(\omega)/2\pi$ and to integrate by the angular frequency *ω*, i.e.

$$P_s(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} p(t,\omega)G(\omega)d\omega \quad (4)$$

At the receiving point it is executed the mutual correlation of the received signal with the standard signal which coincides in form with the radiated one. For that it is necessary to re-multiply the sub-integral expression in (4) to the complex conjugate spectrum $G^*(\omega)$. Then the functions of mutual correlation

$$R(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} p(t,\omega)|G(\omega)|^2 d\omega = -\frac{1}{4\pi}\int_{-\infty}^{\infty}\int_S \frac{\partial}{\partial y}\{\frac{\exp[-jk(R_1+R_2)]}{R_1R_2}\}dS\,|G(\omega)|^2 \exp(j\omega t)\frac{d\omega}{2\pi}.$$

Thus, the problem is reduced to calculating the integral

$$J(t,\omega) = -\frac{1}{4\pi}\int_S \frac{\partial}{\partial y}\{\frac{\exp[-jk(R_1+R_2)]}{R_1R_2}\}dS\ , \quad (5)$$

which is the transferring function of the medium, and due to the motion of the sea surface, it depends not only on the frequency but on time as well.

Let's suggest, that for the time period of the probing impulse illumination of the sea surface area, forming the reflected signal, the phase multiplier $\exp[-jk(R_1+R_2)]$ in integral (5) varies insignificantly. In this case, while carrying out the integrating we may consider the surface to be not changing. Then, integral (5) will depend only on the frequency, and for finding the function *R*(*t*) it should be multiplied to the energy spectrum $|G(\omega)|^2$ of the probing signal, and it should be to performed the reverse Fourier transformation of the obtained product. With such assumption we shall consider the sea surface deformation in time periods equal to one period of radiation of the probing impulse.

Let's consider the question on the area of integrating in expression (5), taking into account that during the period of conducting the experiment, the surface waves were propagating along the stationary route in the direction from the radiator to the hydrophone, i.e. it was taking place the two-dimension problem. At the radiation of the phase-manipulated impulses, the main input into the correlation peak, corresponding to the signal reflected from the sea surface, is made by the scattered signals, delayed relatively to the mirror reflection for the value not exceeding τ_0, where τ_0 – duration of the element of the phase-manipulated impulse. For the used probing signals τ_0 made ≈ 0,001 c. In conditions of the experiment being carried out, in case of τ_0 being of such value, while integrating by x it is possible to limit oneself to the interval of .–80 m ≤ *x* ≤ 80 m. . From the geometry of the experiment . $R_1 = [(x-x_1)^2 + (y-y_1)^2]^{1/2}$, $R_2 = [(x-x_0)^2 + (y-y_0)^2]^{1/2}$ and $\left(\frac{dy}{dx}\right)^2 << 1$, then integral (5) can be presented in the following form suitable for numerical integrating:

$$J(\omega) = a(\omega) + jb(\omega),$$

where $$a(\omega) = \frac{\omega}{4\pi c}\int_{-L}^{L}\frac{1}{R_1R_2}(\frac{y-y_1}{R_1} + \frac{y-y_0}{R_2})\ \sin\frac{\omega}{c}(R_1+R_2)dx \quad (6)$$

$$b(\omega) = \frac{\omega}{4\pi c}\int_{-L}^{L}\frac{1}{R_1R_2}(\frac{y-y_1}{R_1} + \frac{y-y_0}{R_2})\ \cos\frac{\omega}{c}(R_1+R_2)dx$$

$y = y(t,x)$ – the function describing the wave profile at the time moment t.

Surface waves will be described by the function $y(t,x)=\sum_i a_i \sin(\Omega_i t - \frac{\Omega_i^2}{g}x - \varphi_i)$, i.e. by a sum of sinusoidal waves with different amplitudes a_i, frequencies Ω_i and phases φ_i, each of them propagating at proper phase velocity g/Ω_i. Here $\Omega_i = i\Delta\Omega$, $i=1,2,...$; g – acceleration of the gravity force. Phases φ_i are random and regularly distributed in the interval of angles from (0, 2π), and the waves amplitudes a_i are found by the formula $a_i=[2S(\Omega_i)\Delta\Omega]^{1/2}$, where $S(\Omega)$ – the energy spectrum of the sea waves.

There are other mathematical models of the sea wave [3], in which the amplitudes a_i are random values. While describing the sea wave as a stationary random process, all these models are adequate. Still, the model accepted by us, in which the values a_i are deterministic, is the most suitable for the mathematical modeling.

During the experiment, the wave was regular and for the description of its energy spectrum, out of all numerous known formulae [3,4] it may be used the formula of G.A.Firsov, which is written through the parameters of the function of the auto-correlation of the wave measured in the point:

$$S(\Omega)=\frac{4}{\pi}D_\xi \alpha \frac{\Omega_0^2+\alpha^2}{\Omega_0^2+2\alpha^2}\cdot\frac{\Omega^2+\alpha^2}{\Omega^4+2(\alpha^2-\Omega_0^2)\Omega^2+(\alpha^2+\Omega_0^2)^2} \tag{7}$$

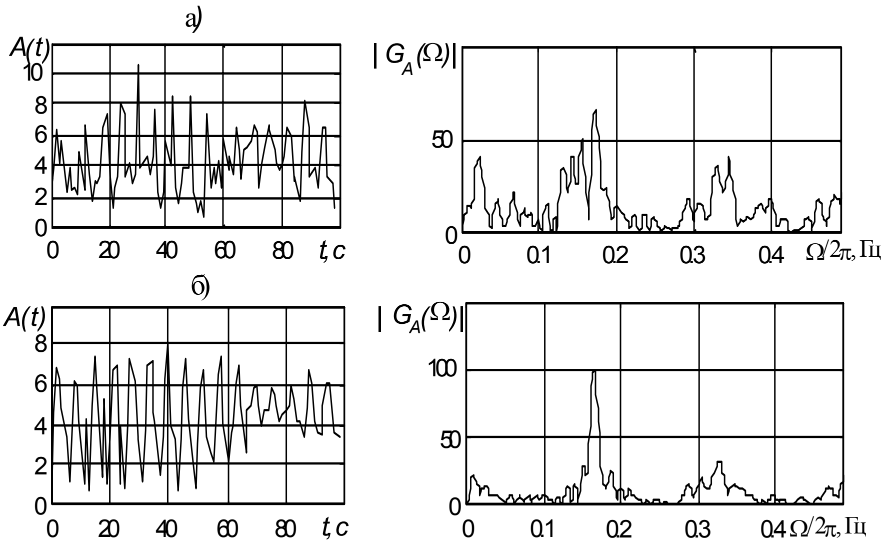

Fig. 1. Modeled dependencies of the correlation peak maximum on time, and the corresponding spectra of fluctuations

Here D_ξ – dispersion of wave ordinates; $1/\alpha$ – the constant of the time of exponential decrease of the auto-correlation function, characterizing the level of non-regularity of the wave; Ω_0 - angular frequency of its oscillations.

In case of the limit-developed wind wave the parameter α is related to the frequency Ω_0 by the empirical ratio $\alpha = 0{,}21\Omega_0$. But if the wave is of regular character, then the numerical coefficient in this ratio will be less.

Mathematical modeling of the experiment was carried out for $\alpha=0{,}02\Omega_0$. Maximum of spectrum (7) was located on the frequency $\Omega_{max}=[(4\alpha^2\Omega_0^2+\Omega_0^4)^{1/2}-\alpha^2]^{1/2}=2\pi/T_э=1{,}03\ c^{-1}$, where $T_э$ – the waves period found experimentally. The wave was described by a sum of sinusoidal waves with the frequencies $\Omega_i=0{,}05\,i\,c^{-1}$, $i=1,2,\ldots,70$. Dispersion D_ξ of the wave ordinates was accepted being equal to 0,044 m^2 and 0,011 m^2.

Upon mathematical modeling the obtained dependencies *A(t)* at $D_\xi=0{,}044$ m^2 and $D_\xi=0{,}011$ m^2, as well as their fluctuations spectra are shown in Fig. 1. We make comparison regular wave the dependence *A(t)* and the fluctuation spectrum with analogous experimental results, were maximum of fluctuations spectrum is located at the frequency of 0,165 Hz (somewhat larger frequency, corresponding to the measured period $T_э$ of the wave), and the width of the spectral peak is 1.5 times larger than the width of the energy spectrum of the wave. At the increase of the wave height the spectrum is getting more complicated (See Fig. 1a): some additional peaks appear in the vicinity of the main frequency of the wave and the second harmonics; frequency ranges occupied by these spectral components are getting wider. In case of the developed wind wave the dependence *A(t)* will be similar to the random process; there will be no periodicity in it, and the fluctuation spectrum will be smearing within the whole studied range of frequencies.

Fig. 2a shows some profiles of the surface waves at which the function *A(t)* acquires the maximal meaning. As the figure shows, the profiles forming the maximal reflected signals are imposed on each other. That shows to the fact that the formation of signals maximal by level takes place almost in equal conditions. With the surface waves propagation these areas are periodically repeated, that results in periodicity of the function *A(t)*. To find out which parts of the waves make the main input into the reflected signal, for one of the profiles shown in Fig. 2a, it has been calculated the dependence of the maximum $A_{\max}$ of the signal in the receiving point on the upper limit of integrating x in integrals (6). The obtained dependence $A_{\max}(x)$ is presented in Fig. 26. It is built in the same coordinates as the profiles of the surface waves in Fig. 2a. The beginning of coordinates is the point of the mirror reflection from the calm sea surface. Intervals of the coordinate x, in which $A_{\max}(x)$ is the most steep, are limiting the surface waves parts making the main input at the formation of the reflected signal.

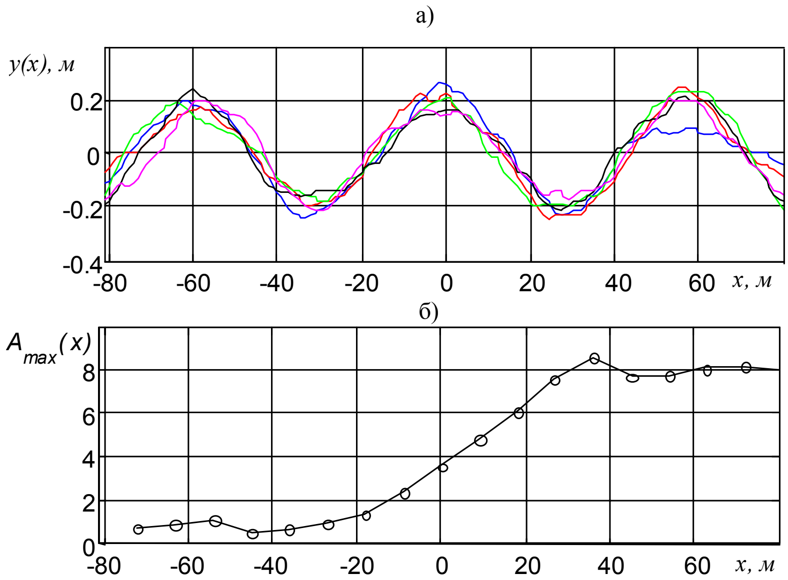

Fig. 2. Profiles of surface waves forming the maximal reflected signals (a) and the dependence of the maximum of the reflected signal on the upper limit of integrating

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