R.M. Khafizov

RELAXATION OF PRESSURE IN A CAVITY SURROUNDED WITN POSOUS BREED, AFTER IT MOLDING

State Pedagogical Institute at Sterlitamak
Russia, 453103 Sterlitamak, pr. Lenina, 37
Tel.: (3473) 43-1056; Fax: (3473) 43-1056
E-mail: tsur1@mail.ru

Molding is one of conventional modes of the definition of tightness of hydraulic systems. It is represented, that this mode also can be used at hydrodynamic trials of slits. The rate of a relaxation of pressure in slits enclosed by porous breeds, after them molding depends from collectors of performances of enclosing porous breed. Therefore, on time of a relaxation of pressure it is possible to judge, for example, magnitude of factor of a permeability of breed around of a slit. Besides adding a gas phase at molding, increasing thus elastic capacity of a medium in a slit, it is possible to achieve, that the characteristic time of a relaxation should be in limits convenient from a point of view of an engineering realization of this mode in practice.

Let's consider a relaxation of pressure at the expense of a filtration of a liquid in enclosing porous space in a cavity, molding by introduction of gas. For exposition of these processes shall note equation of preservation of a mass of a liquid inside cavity, and also equation piezoconductivity and law Darcy for filtration of a liquid [1] as:

$$\frac{d\rho}{dt} = -\frac{n+1}{a}\rho_l u\Big|_{r=a}, \quad \rho = \rho_l\left(1-\alpha_g\right) \tag{1}$$

$$\frac{\partial p'}{\partial t} = æ\frac{1}{r^n}\frac{\partial}{\partial r}\left(r^n\frac{\partial p'}{\partial r}\right), \ u' = -\frac{k}{\mu_l}\frac{\partial p'}{\partial r}, \ (a<r<\infty), \quad \left(æ = \frac{k\rho_l C_l^2}{m\mu_l}\right) \tag{2}$$

Here a – radius of a cavity, ρ_l – denseness of a liquid, α_g – volumetric long of gas in a cavity, μ_l – viscosity of a liquid, m, k – factors of a porosity and permeability, C_l – velocity of a sound for a liquid, æ – factor piezoconductivity, p', u' – distribution of pressure and the velocities of a filtration around of a cavity, $n=0$, 1 and 2 correspond flat one dimensional, radial and spherical tasks. Compression of a liquid were in a cavity and in a porous medium, we shall take into account in an acoustic approximation, and for a behaviour of gas we shall accept the polytropic law, then:

$$p = p_0 + C_l^2\left(\rho_l - \rho_{l0}\right), \ \alpha_g = \alpha_{g0}\left(p_0/p\right)^{1/\gamma} \tag{3}$$

where γ – polytropic exponent, p_0 – initial pressure in a cavity.

Initial and boundary conditions for equation (2) shall note as:

$$p' = p_0' \ (t=0, \quad r>a); \ p' = p(t), \ u' = u \ (t>0, \quad r=a). \tag{4}$$

Applying a principle Duhamel [2], for the equation (2) under conditions (4) the following solution can be obtained:

$$p' - p_0' = \int_0^t \frac{\partial U(r,t-\tau)}{\partial t}\left(p(\tau) - p_0'\right)d\tau,$$

$$U(r,t) = 1 - \Phi\left(\frac{r-a}{2\sqrt{æt}}\right), \ \Phi(\alpha) = \frac{2}{\sqrt{\pi}}\int_0^\alpha \exp(-\xi^2)d\xi, \ (n=0),$$

$$U(r,t) = 1 + \frac{2}{\pi}\int_0^\infty \exp\left(-\frac{æz^2t}{a^2}\right)\frac{J_0\left(\frac{zr}{a}\right)Y_0(z) - J_0(z)Y_0\left(\frac{zr}{a}\right)}{J_0^2(z)+Y_0^2(z)}\frac{dz}{z}, \ (n=1), \tag{5}$$

$$U(r,t) = \frac{a}{r}\left(1 - \Phi\left(\frac{r-a}{2\sqrt{æt}}\right)\right), \ (n=2),$$

where $J_0(z)$ and $Y_0(z)$ – cylindrical functions accordingly of first and second sort of an order zero..

Using these solutions because of equations (1) we shall receive the following integral equation, circumscribing a relaxation of pressure in a cavity

$$\rho_{l0}\alpha_{g0}\left(\left(\frac{p_0}{p}\right)^{1/\gamma}-1\right)-\frac{p-p_0}{C_l^2}\left(1-\alpha_{g0}\left(\frac{p_0}{p}\right)^{1/\gamma}\right)=$$

$$=\frac{(n+1)k\rho_{l0}}{a^2\mu_l}\int_0^t\varphi\left(\frac{æ(t-\tau)}{a^2}\right)\left(p(\tau)-p_0'\right)d\tau, \qquad (6)$$

$$\varphi(S)=\frac{1}{\sqrt{\pi S}},\ (n=0),\ \varphi(S)=\frac{4}{\pi^2}\int_0^\infty\frac{\exp(-Sz^2)}{Y_0^2(z)+J_0^2(z)}\frac{dz}{z},\ (n=1),$$

$$\varphi(S)=\frac{1}{\sqrt{\pi S}}+1,\ (n=2)$$

In case of weak molding $\left(\Delta p_0=p_0-p_0'<<p_0\right)$, by producing a linearization, the equation (6) can be reduced in an aspect:

$$1-\Delta P=\frac{(n+1)\gamma kp_0}{a^2\mu_l(\alpha_{g0}+\gamma\alpha_C(1-\alpha_{g0}))}\int_0^t\varphi\left(\frac{æ(t-\tau)}{a^2}\right)\Delta P(\tau)d\tau \qquad (7)$$

$$\left(\Delta P=\Delta p/\Delta p_0,\ \Delta p=p-p_0',\ \alpha_C=p_0/\rho_{l0}C_l^2\right).$$

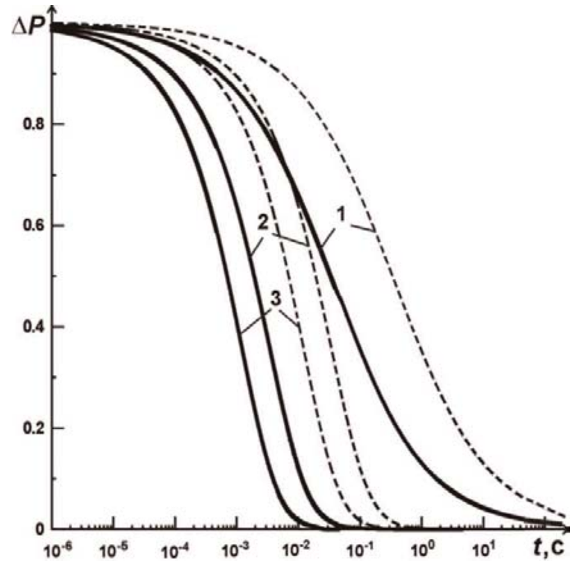

Fig. 1 Dynamics of a relaxation of pressure in a cavity at molding without introduction of gas.

On Fig.1 dynamics of the process of a relaxation of pressure is illustrated at the following parameters of a cavity, porous medium, liquid (water) and gas: $a=0{,}1\text{м},\ m=0{,}1,\ k=10^{-12}\text{м}^2, p_0'=1\text{МПа}, \rho_{l0}=1000\,\text{кг}/\text{м}^3, C_l=1500\,\text{м}/\text{с}, \mu_l=0{,}001\text{Па}\cdot\text{с}, \alpha_{g0}=0$.
The figures on lines correspond to values n. From the graphs follows, that at identical values of a characteristic linear size a. The rate of a relaxation of pressure is force depends on geometry of the task. For a represented example the appropriate times of a relaxation of pressure for a cavity with plane-parallel walls and spherical cavity differ more than on three order. Such force distinction of times of a relaxation is connected, at first, to a diminution of a specific surface of a wall of a cavity, through which the filtration of a liquid in enclosing porous space happens at passage from the spherical task to flat one dimensional, besides for flat one dimensional of the task filtration the current in porous space around of a cavity happens as though in more constrained conditions, than for a case of spherical and radial geometry of the tasks. From a comparison of solid lines with dashed, constructed for $k=10^{-13}\text{м}^2$ .It is visible, that the rate of a relaxation is force depends on magnitude of a permeability, is approximately proportional (modification of this factor on one order reduces in a modification of time of a relaxation, also approximately on one order).

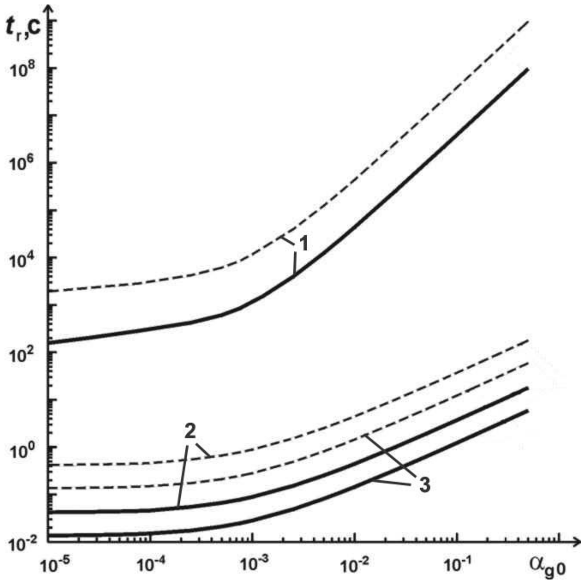

Fig. 2 Assotiations of time of a relaxation of pressure in a cavity from an initial volumetric content of gas entered at molding.

On Fig.2 the assotiation of time t_r. Relaxation of pressure, defined as a phase is represented, during which the surplus of pressure in a cavity will make no more than one interest from initial $\left(\Delta P \leq 10^{-2}\right)$. Values of remaining parameters former. It is not difficult to see, that molding with introduction of gas up to fifty on volume of interests $\left(\alpha_{g0} \approx 0,5\right)$. Reduces in growth of time of a relaxation almost on three order on a comparison with a case, when such molding is produced without introduction of gas.

Thus, molding of a cavity, support of some of gas, increases elastic capacity of a cavity, and it in turn reduces in growth of characteristic time of a relaxation of pressure. Therefore at hydrodynamic trials of open slits molding, with the purpose of the definition of factor of a permeability, for example, the introduction of gas, enables to operate characteristic time of a relaxation.

R E F E R E N C E S

1. Basniev K.S., Kotchina I.N., Maximov V.M. Underground hydrodynamics. Moscow: Nedra, 1993. (In Russian).
2. Tikhonov A.N., Samarsky A.A. The equations of mathematical physics. Moscow: Science, 1972. (In Russian).