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MOTION OF THE DEFORMING STEAM-GAS CAVITIES NEAR THE INTERFACES

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In the development of experimental and theoretical works the expansion and the compression of the deforming cavitation bubbles near the interfaces or under the conditions for their interaction with the adjacent cavities is examined. For fulfilling the calculations during several periods a special mathematical technique is developed, the so-called method of mobile potentials. It is shown that the cumulative stream can be formed not only during the intensive compression, but also during the expansion.

The cavitation noise and cavitation erosion are the most interesting and important for practical purposes among the numerous physical studies of hydrodynamic or acoustic cavitation. A question about the relationship between the contributions of progressive and radial motion and the noise-emission was discussed for many years. In the standard classical formulation of investigating the cavitation noise-emission [1, 2], translational motion not at all is examined. In [3], during the analysis of the unsteady motion of a constant size sphere a conclusion was drawn about the possibility of not considering the translational motion. However, in [4-7] intensive interaction of kinetic energy of radial and translational motion is established, and in [5, 7] a sharp increase in the speed of progressive motion during the intensive compression of cavity is discovered. The speed of the compression even in the empty (vapor) cavity, which moves progressively, noticeably decreases, and the speed of the translational motion with the minimal size of cavity can exceed the speed of radial motion 3-5 times [7]. This result made it possible as shown in [8] to obtain the estimations, according to which the contribution of radial and forward motion with Mach numbers of the order of 1 can be commensurate. However, the enumerated works examined radial and translational motion under maintaining condition by the cavity of spherical form at all stages of expansion and compression. It is shown in [9] that as a result of translational motion the distribution of the coefficient of normal pressure on the surface of cavity proves to be asymmetrical; moreover, during the compression the pressure coefficient in the zone of the "bottom" exceeds the pressure coefficient in the zone of the first critical point 2-3 times. This must lead to the "extrusion" of tail of the surface of cavity and to the loss by it of spherical form, which explains the formation of cumulative stream, investigated in [10, 11]. In [12] additional effects are revealed. It is established that the cumulative stream can be formed not only during the rapid compression in the intensive pressure field, but also in the moderate fields as a result of the accumulation of kinetic energy for several periods of pulsation. The results of the additional calculations are examined below.

Let us examine the dynamics of the pulsatory deforming cavities, which move forward. Translational motion can be caused by the action of pressure gradient in the standing wave, by interaction with the walls, bubbles, etc. At the moment of the time of t=0, fluid pressure at infinity increases by a jump and/or according to the harmonic law $P_\infty^1 = P_\infty + P_m \cdot \sin(\omega \cdot t + \gamma)$, where P_∞ is the pressure at infinity at the moment of time t = 0+, P_m is the amplitude, ω is the angular frequency, γ is the phase of sound pressure. As a result, the cavity begins to pulsate and moves progressively. Let us introduce dimensionless variables.

$$t' = \frac{t}{R_0}\sqrt{\frac{|\Delta|}{\rho}},\ t' = \frac{r}{R_0},\ z' = \frac{z}{R_0},\ w_i = \frac{w_i}{w_{oi}},\ \phi' = \frac{\phi}{R_0}\sqrt{\frac{\rho}{|\Delta|}}, \Delta = (P_\infty^1 - P_\infty^0 + P_m),$$

where R_0 is the initial radius of one of the cavities; $\boldsymbol{w_i}$ is the volume of i th cavity; w_{0i} is the initial volume of i th cavity at the initial moment of time.

In the dimensionless variables (primes omitted) the task is reduced to the determination of the potential $\phi(r,z,t)$, satisfying the conditions

$$\frac{\partial^2\phi}{\partial r^2}+\frac{\partial^2\phi}{\partial z^2}+\frac{1}{r}\frac{\partial\phi}{\partial r}=0,(r,z)\in\Omega \tag{1}$$

$$\frac{\partial\phi}{\partial t}=\kappa+f\cdot(k_i-1)-(v_r^{(i)2}+v_z^{(i)2})/2+\varsigma\cdot[\sin(\omega\cdot t)-1]+\beta_i\cdot(1-w_i^{-\gamma}), \tag{2}$$

$$\lim\phi=0,(r,z)\to\infty,(r,z)\in\Omega \tag{3}$$

$$\frac{\partial\phi}{\partial n}=0,\quad \phi(r,z,0)=0,\ (r,z)\in\Omega\ ,\Gamma;\ i=1,m \tag{4}$$

where Γ_i is the surface of *i* th cavity; ρ is the density of liquid; *P* is the fluid pressure; $P_\Gamma^{(i)}$ is the gas pressure inside of *i* th cavity; σ is the coefficient of surface tension; k_i is the mean curvature of surface of *i* th cavity; *i* is the number of cavities; m is the number of solid interfaces, $\kappa=\Delta/|\Delta|$; $f=2\sigma/(R_0|\Delta|)$ is the coefficient, which characterizes the influence of the forces of surface tension, k_i is the mean curvature of surface Γ_i and of *i* th cavity, $\varsigma=P_m/|\Delta|$ is the coefficient of periodic pressure component, γ is the polytropic gas constant, $\beta_i=P_{\Gamma 0}^{(i)}/|\Delta|$ is the contraction coefficient of gas, $v_r^{(i)}=\partial\phi/\partial r|_{\Gamma_i}$; $v_z^{(i)}=\partial\phi/\partial z|_{\Gamma_i}$ are the components of the velocity vectors of "liquid" particles on the generatrix of *i* th cavity. The method of mobile potentials is developed for the solution of the formulated problem. This made it possible to carry out calculations and analysis of the deformation of cavities not only in the contraction phase, but also for several periods of pulsations, which made it possible to determine the accumulated effects. The idea of method consists of the following. Inside the cavities, on the axis of the symmetry of task, the concentrated hydrodynamic special features are selected. The unknown velocity potential $\phi(r,z,t)$ is replaced by the superposition of the potentials of the special features indicated $\phi=\sum_{i=1}^{m}\sum_{k=1}^{\infty}A_k^{(i)}\Big/\sqrt{(r^2+(z-z_k)^2}$, where $A_k^{(i)}(t),k=1,\infty$ are the intensity of the special features, selected inside of *i* th cavity, $(0,z_k^{(i)})$ are their origin coordinates. The so-called reflection method is adapted for the boundary conditions satisfaction. Then a special numerical procedure is constructed. On the generatrices of the surfaces of cavities, which are the result of cross sectioning by a plane, passing through the axis z, the final system of points is selected. The coordinates of these points are $(R_{l0}^{(i)},Z_{l0}^{(i)})$, *l=(1,M)*, where M is a certain predetermined even number. Inside each cavity (on the axis of symmetry) *N=M/2* of special features are selected. As a result we obtain a system of functions $\phi=\Phi(r,z,A_k^{(i)},z_k^{(i)})$, $i=(1,m),k=(1,N)$. The solution is reduced to the tracking of the motion of "liquid" particles, located at the initial moment of time in the selected points, chosen on the generatrices cavities. The speeds of motion and change in the source strengths are calculated at each time-step.

The obtained system of linear algebraic equations for $A_k^{(j)}$ and $z_k^{(j)}$ is ill-conditioned, and therefore Tikhonov's method [13] of the regularization was used. An interpolation cubic spline method was used to find the mean curvature at the points of the surfaces of cavities. The Hemming's 4th order method with the automatic steps was used for the numerical integration of system of equations. The accuracy of the solution was monitored.

Let us examine the motion of the spherical cavity of a radius R_0 between two solid planes with an increase in the pressure by jump. M=20 discrete points on the generatrix of cavity and N=10 special features were selected. The accuracy of the calculations was ε=0.001. The number of reflections was refined in the solution process. Fig. 1 gives the configurations of generatrix for the time sequence: 1-t=0.0; 2-1.08; 3- 1.45; 4-1.69 for the parameters: L=1.8R$_0$; d=2.5R$_0$; κ=1; β=0; f=0; ω=0; ς=0.

In this case the cavity was located almost at equal distance from two interfaces.

Fig. 2 shows the configurations of the generatrix of cavity at the moments of 1-t=0.0; 2-1.08; 3-1.44; 4-1.542 for the parameters: L=1.2R_0; d=3.5R_0; κ=1, β=0, f=0, ω=0, ς=0. One can see that when distances to the interfaces are commensurate (L ≈ d), during the compression of cavity an annular jet is formed, cavity is extracted and has the tendency of splitting. While the distance to one of the walls increases (L < d) the annular jet is displaced to the pole, opposite to the nearest interface and a cumulative jet is formed.

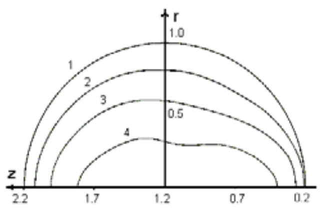

Fig. 1. Deformation of the compressive cavity, located equidistantly from two rigid walls

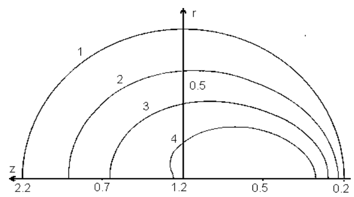

Fig. 2. Deformation of the compressive cavity near one rigid wall

Fig. 3 shows the configurations of the generatrix of cavity in the field of harmonic pressure at the following moments of the time: 1) t = 0.0; 2) 1.08; 3) 2.164; 4) 3.244; 5) 3.274; 6) 3.296 for the values of the parameters of κ=1, ς = 0, β=0.5, f=0, γ=1.401, L=2.5. The forces of surface tension are not considered. The initial distance between the center of cavity and the wall is equal to 2.5R_0, the pressure at infinity increases on $2 \cdot P_{\Gamma 0}$. It is evident that in this case the jet appears at the initial stage of the expansion of the cavity.

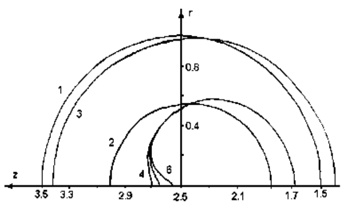

Fig. 3. Deformation of the pulsatory cavity near the rigid wall. Cumulative jet is formed in the expansion stage of the cavity

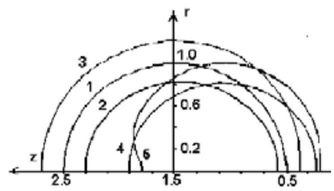

Fig. 4. Deformation of the pulsatory cavity near the rigid wall. Cumulative jet is formed during the compression

The configurations of forming cavities at the moments: 1- t=0.0; 2-0.97; 3-2.06; 4- 4.03; 5-4.446, for the values of the parameters of κ=1, β=2, f=1, ω=4, ς= 0.1, γ=1.0, L=1.5 are shown on Fig. 4. The initial distance between the center of cavity and the interface is - 1.5R_0. Pressure periodically changes with the dimensionless frequency of ω=4 and an amplitude of 0.1. The pressure jump at infinity with $t = 0$ is equal to 0.5$P_{\Gamma 0}$. In this case the jet is formed at the stage of compression. Because of the forces of surface tension, the cavity has rounded form, and jet has the larger cross section.

Calculations show that the development of deformation and the moment of forming the cumulative jet depend substantially on the ambient conditions. With the small amplitudes of excess pressure the cavity approaches a solid boundary and several pulsations can be completed to the moment of forming the stream. During the expansion the cavity does not always restore spherical form, and deformation is accumulated. In the contraction phase in the zone of $R_{\min}$ all particles of liquid near the compressive cavity acquire the directed motion to the wall, but particles from the opposite wall are accelerated more rapidly and preserve their motion to the wall even when cavity begins to grow on the average. That is how the cumulative stream is formed.

Interesting results are obtained during calculations of the speeds of radial and translational motion and speed of cumulative stream during the compression of the empty (steam) cavity: 1) vapor cavity as a result of translational motion does not begin to flap immediately (it does not disappear), but it can pulsate; 2) the maximum speed of cumulative jet exceeds the maximum speed of compression and it is more than the maximum speed of forward motion of cavity without taking into account deformation. The conversion of kinetic energy of compression into the kinetic translational energy does occur and later on, after the deformation, it converts to the kinetic energy of stream. Moreover, as a result of the decrease of added mass and underdamping of momentum, the translational speed grows.

For example, for the empty cavity with a radius of 0.01 mm with drops in excess pressure by 0.1, 1 and 2 atmosphere the compression times are approximately equal $3\cdot10^{-5}$ s, 10^{-5} s, $0,7\cdot10^{-5}$ s and the maximum speeds of cumulative stream reach values respectively: 43 m/s, 136 m/s, 192 m/s and even more. This corresponds to hydraulic shock wave with the pressure to 300 mPa. The accumulated potential energy and energy of external field is converted as a result into the energy of cumulative jet. A Jet pierces the cavity. This leads to the destruction of the cavity. The stream reaches the wall and causes the cavitation erosion. Thus, the pulsatory or compressive cavities effectively emit cavitation noise during the radial pulsations in the uniform field of pressure. In the nonuniform field this emission decreases as a result of translational motion. Translational motion leads to the deformation of cavity, the formation of cumulative jets and the destruction of cavities. Formed "splinters" become nuclei of cavitation. They coagulate, they pulsate intensively, and they are deformed and destroyed. The entire process is repeated. The steadiness does not take place because of stochasticity of nuclei of cavitation distribution in the space. The obtained results are confirmed well by experiments on a study of the ultrasonic cavitation.

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