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**ON THE PROPAGATION OF TRANSVERSE ACOUSTIC WAVES PACKET IN A SOLID**

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*The motion of transverse acoustic waves packet in a solid has been investigated. It is revealed that at the movement of the packet in the place of its location, in the same transverse direction the extension of the solid takes place. After the wave packet coming this extension leads to mass decrease in the volume area occupied by the solid initially, before the packet coming. The total energy of the wave packet $E$ and mass decrease $m_\Delta$ are connected by the relation $E = -km_\Delta c_t^2$ ($k$ - coefficient depending upon the Lame relation coefficients and close to unity, $c_t$ - the speed of transverse acoustic waves in the solid).*

The motion of transverse acoustic waves packet in a solid is being considered. The packet occupies bounded domain in space (fig.1).

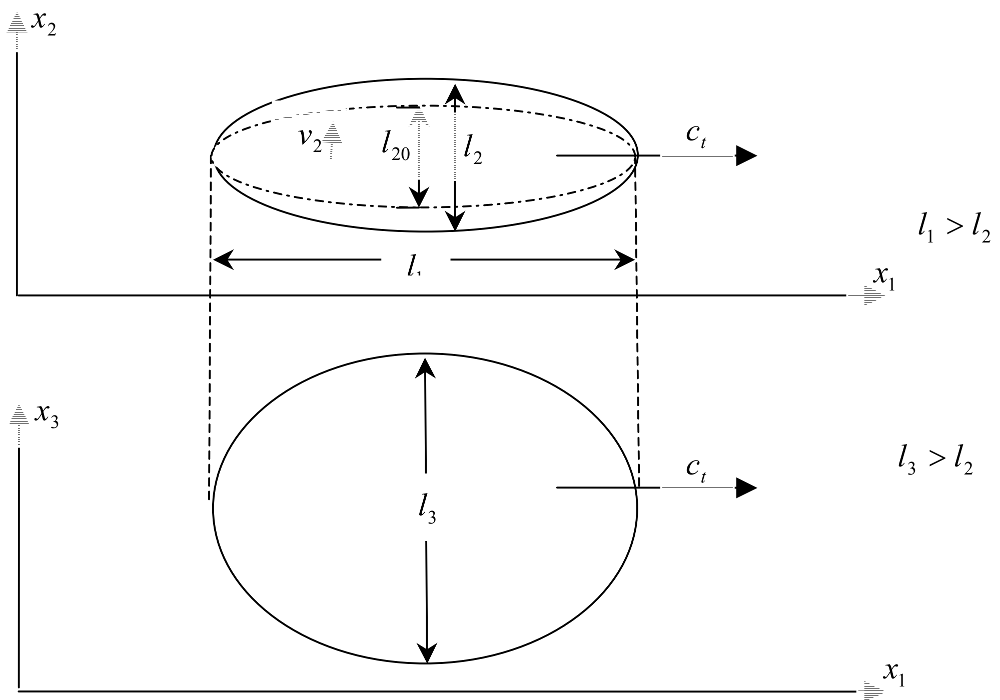


***Fig. 1.*** Projections of the conventional bounds of wave packet moving; $l_{20}$ - initial length of the segment, $l_2$ - the segment length at acoustic waves packet transition.

As is well known [1, 2], common motion equations are derived from statistic equations if the specific sum of internal stress $\frac{\partial \sigma_{ik}}{\partial x_k}$ (sum with respect to k) is equated to the speed of motion quantity change $\rho \frac{dv_i}{dt}$

$$\rho \frac{dv_i}{dt} = \frac{\partial \sigma_{ik}}{\partial x_k}. \tag{1}$$

Here $v_i = \frac{du_i}{dt}$ is the total time derivative from the shift $u_i \equiv x_i - x_{i0}$ $i$ - the position of the point relative to its initial location $x_{i0}$. Stresses and deformations are linearly connected within the classical theory. The dependence for co-ordinate $x_2$, which is considered as transverse, assumes the form [2]

$$\frac{\partial \sigma_{2k}}{\partial x_k} = \mu \Delta u_2 + (\lambda + \mu)\frac{\partial (\nabla \bar{u})}{\partial x_2}, \tag{2}$$

where $\mu$ – shift modulus, $\mu, \lambda$ – Lame coefficients. Discriminating local and convection parts of the speed derivative in (1) and expanding the right part in (2), we obtain the following equation

$$\begin{gathered} \rho\left(\frac{\partial v_2}{\partial t} + v_1\frac{\partial v_2}{\partial x_1} + v_2\frac{\partial v_2}{\partial x_2} + v_3\frac{\partial v_2}{\partial x_3}\right) = \\ = \mu\left(\frac{\partial^2 u_2}{\partial x_1^2} + \frac{\partial^2 u_2}{\partial x_2^2} + \frac{\partial^2 u_2}{\partial x_3^2}\right) + (\lambda + \mu)\left[\frac{\partial^2 u_1}{\partial x_1 \partial x_2} + \frac{\partial^2 u_2}{\partial x_2^2} + \frac{\partial^2 u_3}{\partial x_2 \partial x_3}\right] \end{gathered} \tag{3}$$

Direction $x_1$ is considered as transverse. We are viewing transverse fluctuations (at $x_2$), spreading in the longitudinal direction. We assume, that shifts at $x_1, x_3$ are missing and besides transverse shift derivatives of $x_2$ at the same co-ordinates $x_1, x_3$ are minor (the conditions to accomplish these assumptions are viewed below). Then :

$$v_1 = v_3 = 0,\ u_1 = u_3 = 0,\ \frac{\partial u_2}{\partial x_1} = \frac{\partial u_2}{\partial x_3} = 0\,. \tag{4}$$

According to these conditions the equation (3) acquires the following form

$$\rho\left(\frac{\partial v_2}{\partial t} + \frac{1}{2}\frac{\partial v_2^2}{\partial x_2}\right) = \mu\frac{\partial^2 u_2}{\partial x_1^2} + (\lambda + 2\mu)\frac{\partial^2 u_2}{\partial x_2^2}\,. \tag{5}$$

In this equation addends of the second infinitesimal order are usually neglected, Dalamber Equation connecting speed local derivative (primary flexion of time shift) and primary flexion of the same shift at longitudinal co-ordinate is used instead

$$\rho\frac{\partial^2 u_2}{\partial t^2} = \mu\frac{\partial^2 u_2}{\partial x_1^2}\,. \tag{6}$$

One of the common solutions of the equation appears as a plane wave moving in the direction of the increase of longitudinal co-ordinate $x_1$ with "transverse" speed $c_t = \sqrt{\frac{\mu}{\rho}}$

$$u_2(x_1, t) = f(x_1 - c_t t). \tag{7}$$

Using (7) as the basis we are looking for equation (5) solution as the superposition of the local packet of transverse acoustic narrow-band waves at some frequency $\omega$ and distributed extension wave associated with the packet

$$u_2(x_1, x_2, x_3, t) = \mathrm{Re}\left(\overset{*}{u}_2(y, x_2(y), x_3(y))\exp(i\,y/\lambda)\right) + w_2(y, x_2(y), x_3(y))\,. \tag{8}$$

Here $y = x_1 - c_t t$, $\overset{*}{u}_2$ - is complex amplitude of the wave packet in the transverse direction $x_2$, which changes slowly on the wavelength scale $\lambda = 2\pi c_t / \omega$. It is assumed that the wave length is less than transverse and longitudinal dimensions of the packet, and also that on average the amplitude of the associated extension wave is less than the amplitude of the acoustic wave $<|\dot{u}_2|> \; < \; <|w_2|>$ and the packet is local beyond the bounds of some volume $\overset{*}{u}_2 = 0$.

It is easy to show that the packet presentation (8) is an approximate solution of Dalamber Equation (6) if value $\overset{*}{u}_2$ slowly changes subject to co-ordinates $x_1, x_2, x_3$.

Substituting (8) into (5) some circumstances are to be considered. Since on average by the process area volume $<|\dot{u}_2|> \; < \; <|w_2|>$ and $w_2$ changes slowly in comparison with sound, then by the calculation of squared speed in (5) only the left addend should be considered (8). And vice versa, by the calculation of the second derivative at $x_2$ only the augend (8) should be considered, as by the differentiation of the first addend we obtain quick-variable irregular term. Taking into account that by the substitution (8) the equation of the first addends in (5) is provided by the completion of (6), we obtain the following equation

$$\frac{\partial e_k}{\partial x_2} = (\lambda + 2\mu)\frac{\partial^2 w_2}{\partial x_2^2}, \tag{9}$$

where $e_k = \frac{1}{2}\rho v_2^2$ - is the specific kinetic energy of the process.

Taking integral at $x_2$ we get

$$e_k = (\lambda + 2\mu)\frac{\partial w_2}{\partial x_2}. \tag{10}$$

The integration of the left part of the equation (10) at the wave packet area $V$ gives the total kinetic energy value of the process

$$E_k \equiv \int_V e_k dV. \tag{11}$$

By the same integration on the right initially $x_2$, we receive the change of the initial length of arbitrary segment $\ell_2$ at co-ordinate $x_2$ within volume $V$ (i.e., value $u_2$). By further integration at $x_1, x_3$ we obtain the increase of volume $\Delta V$ relative to its initial value (without the wave packet) (fig. 1)

$$\Delta V = \int_V \frac{\partial w_2}{\partial x_2} dV = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{\partial w_2}{\partial x_2} dx_1 dx_2 dx_3 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} u_2(x_1, x_3) dx_1 dx_3. \tag{12}$$

Value $m_\Delta = -\rho\Delta V$ fits the exportation of mass out of the initial (difference mass [3]). Consequently, the kinetic energy of transverse waves packet and its difference mass are bound with the correlation

$$E_k = -\frac{\lambda + 2\mu}{\rho} m_\Delta = -\frac{\lambda + 2\mu}{\mu} m_\Delta c_t^2 = -m_\Delta c_\ell^2. \tag{13}$$

We used $c_\ell = \sqrt{\frac{\lambda + 2\mu}{\rho}}$ - as the speed of longitudinal acoustic waves [1, 2].

The potential energy of sound fluctuations is equal to the kinetic energy $E_n = E_k$ [1, 2]. Hence, for the total energy $E = E_n + E_k$ the following equation is applied

$$E = -\frac{2(\lambda + 2\mu)}{\rho} m_\Delta = -\frac{2(\lambda + 2\mu)}{\mu} m_\Delta c_t^2 = -k m_\Delta c_t^2 , \qquad (14)$$

where $k = \frac{2(\lambda + 2\mu)}{\mu}$ - is a numerical coefficient, depending upon the correlation of Lame coefficients and close to unity. By $\lambda = 0$ and consequently by Puasson coefficient [1, 2] $\sigma = \lambda /(2(\lambda + \mu)) = 0$ the total energy of the packet equals

$$E = -4 m_\Delta c_t^2 , \qquad (15)$$

where $c_t$ - is the speed of transverse waves.

Relations (13, 15) supplement analogous dependencies between kinetic energy and difference mass for local processes in a liquid [3].

It should be mentioned that by the transition from (3) to (4) and deduction of (11) there was presupposed the absence of accompanying appearing in the substance on the whole (out of volume $V$). This condition can be assumed only when $V$ volume measurements at axes $x_1, x_3$ which are orthogonal to the direction of fluctuations $(x_2)$, is much more than at this direction (fig.1).

The rejection of this condition (the absence of forces out of volume $V$) leads not only to considerable complication of the problem but even to a new statement of it, as in this case the area of waves packet becomes a radiant of expansion – compression waves on wavelength which scale complies with volume measurements and is independent of acoustic wavelength.

It was also presumed that expansion at transverse axis $x_2$ doesn't cause accompanying compression at another transverse $(x_3)$ and longitudinal axes. This condition can be exactly performed only if Puasson coefficient equals zero $\sigma = \lambda /(2(\lambda + \mu))$ [1, 2] ( $\lambda = 0$ ). Taking compression at transverse co-ordinate $x_3$ (according to Puasson coefficient) into account doesn't cause any difficulties and leads to a certain decrease of coefficient $k$ in (14). Unfortunately, it is much more difficult to consider the compression at the longitudinal co-ordinate as in this case appears an extra factor depending on the difference of transverse and longitudinal speeds.

## REFERENCES


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