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DETERMINATION OF CRACK ORIENTATION IN A BOREHOLE VICINITY USING DYNAMICAL PROPERTIES OF TUBE WAVES

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The solution of the problem of external seismic wave field interaction with a borehole, which is crossed by fluid-filled crack, is represented in the report. On the basis of the solution of inverse problem, the procedure of determination of crack orientation, using the hydrophones data of vertical seismic profiling, is proposed.

As the frequency domain, used at vertical seismic profiling (VSP), does not usually exceed several hundreds Hz and a borehole diameter does not exceed 20 cm, then the characteristic wavelengths of elastic waves, both in a borehole fluid, and in external elastic medium are essentially large in comparison with borehole diameter. Hence the problem on the wavefield generation in a fluid-filed borehole, which is crossed by crack, can be considered in the long-wave approximation (in assumption that the linear size of crack and borehole radius are much smaller then the wavelengths). The crack and borehole can be considered as two subsystems, connected one to another by hydraulic connection. The existence of the hydraulic connection leads to fluid cross-flow between a borehole and crack under their deformation by the external seismic field. In this process the equalities of pressures and fluid flows between crack and borehole have to be held at the point of their crossing.

In the long-wave approximation, it is natural to consider the averaged by the borehole cross-section dynamical values, describing an acoustical wavefield in the borehole. Then the propagation of small amplitude waves in the boreholes fluid is described by the relation [1,2]:

$$\frac{\partial^2 P}{\partial t^2} - c_f^2 \frac{\partial^2 P}{\partial z^2} = -\frac{2\rho_f c_f^2}{R}\frac{\partial V_r(r=R,z,t)}{\partial t} \qquad (1)$$

where: - $P(z,t)$ - dynamic perturbation of local pressure near equilibrium value, ρ_f - density of the borehole fluid, c_f - sound velocity, $V_r(r=R,z,t)$ - radial mass velocity of a fluid on the borehole wall. If the fluid flow through the borehole wall is absent, then the radial velocities of the borehole wall and nearest fluid are coincided. In the point of crossing of crack and borehole, the radial velocity of fluid $V_r(r=R,z,t)$ is determined by fluid flow between crack and borehole. This flow can be found by change of volume of the fluid-filled crack in the field of a seismic wave.

In long-wave approximation, the relation between displacement of the borehole wall, change of crack volume and stresses in external seismic wave field can be found on the basis of the solutions of the linear static equations of the elasticity theory. Using this circumstances and the relation (1) it is possible to obtain the following equation:

$$\frac{\partial^2 P}{\partial t^2} - c_{tw}^2 \frac{\partial^2 P}{\partial z^2} = -2\rho_f c_{tw}^2 \cdot \frac{\partial^2}{\partial t^2}\frac{\sigma_{eff}}{E} - \rho_f c_{tw}^2 \frac{\delta(z)\cdot V_0}{\pi R^2}\frac{\partial^2}{\partial t^2}\cdot\left(\frac{P + n_i\sigma_{ik}n_k}{E}\right) \qquad (2)$$

where E - Young's modulus of surrounding elastic medium, V_0 - effective volume of the fluid-filled crack, n_i - normal vector to the crack surface, σ_{ik} - stress tensor of seismic wave in elastic medium, $\sigma_{eff} = Sp\,\sigma_{ik} - (1+\nu)\sigma_{zz}$ - effective external dynamical pressure [3], c_{tw} - tube wave velocity [1, 2], which value is determined by fluid parameters in borehole and by presence of borehole casing. In the non homogeneous wave equation (2), the first term in right part describes the wavefield excitation in the borehole fluid due to change of the borehole cross-section under action of external seismic wave, the second term describes the wave field excitation due to change of crack volume under action of external field. Respectively, the solution of the equation (2) can be represented as a sum of two waves: $P_w(t,z)$ - cogging wave caused due to borehole strain, $P_{tw}(t,z)$ - tube wave generated by fluid flow due to change of volume of fluid-filled crack.

Assuming, that borehole and crack are in the far wave zone, the local interaction of the seismic field with borehole and crack can be considered in the plane wave approximation. Then the displacement vector in the seismic wave can be written as: $\vec{u}(\vec{r},t) = \vec{u} \cdot U(\vec{r}) \cdot f\,(\, t - \vec{e} \cdot \vec{r} \,/\, c\,)$, where: $\vec{u}$ - unit vector of wave polarization, $U(\vec{r})$ - its local amplitude, $\vec{e}$ - unit vector in a direction of wave propagation, c - propagation velocity of wave depending on its type (longitudinal or transversal). After that, it is easy to obtain the components of stress tensor in the seismic wave:

$$n_i \cdot ó_{ik} \cdot n_k = -ì \quad \Big(\big((c_l / c_s)^2 - 2 \big) (\vec{u},\vec{e}) + (\vec{u},\vec{n})(\vec{n},\vec{e}) \Big) \cdot U / c \cdot f'(\hat{\imath}\,)$$

$$s_{eff} = -E \cdot \left(\frac{1}{2} \cdot (c_l / c_s)^2 (\vec{u},\vec{e}) - (\vec{u},\vec{e}_z) \right) \cdot U / c \cdot f'(\hat{\imath}\,)$$

where: m - shear modulus of elastic medium, $\vec{e}_z$ - ort of vertical axis, $\hat{\imath} = t + z / c_w$. Respectively the solutions of the equation (2), describing cogging wave and tube wave have a view:

$$P_w(t,z) = r_f c_{tw}^2 \cdot D_w \frac{U}{c} \cdot f\,'\big(t + z / c_w\big) \quad \text{and} \qquad P_{tw}(z) = \big(D_{tw} - D_w\big) \cdot \int_0^z e^{-(z-x)/t} f\,''(x)\,dx$$

$$D_w = \frac{(c_l / c_s)^2 (\vec{u},\vec{e}) - 2(\vec{u},\vec{e}_z)}{1 - (c_{tw} / c_w)^2}, D_{tw} = \frac{r\, c_s^2}{r_f c_{tw}^2} \Big(\big((c_l / c_s)^2 - 2 \big) (\vec{u},\vec{e}) + (\vec{u},\vec{n})(\vec{n},\vec{e}) \Big), t = \frac{r_f c_{tw}^2}{E} \frac{V_0}{c_{tw} p\, R^2}$$

where: r - density of elastic medium, c_l, c_s - longitudinal and transversal velocities of elastic waves in surrounding medium, $z = t \pm z / c_{tw}$, D_w, D_{tw} - geometrical factors, t - relaxation time of pressure, c_w - visible velocity of seismic wave propagation along the borehole: $c_w = -c / (\vec{e},\vec{e}_z)$. The explicit dependence of amplitudes P_w, P_{tw} of the cogging and the tube waves, which is presented in factors D_w, D_{tw}, on mutual orientations of borehole axis, the normal vector to the crack surface and wave vector of seismic field permits to solve the inverse problem on the restoration of the normal vector to the crack plane by measuring of amplitudes of the cogging and the tube waves. For example, in the VSP case (see fig. 1) when the field in the borehole is excited by external longitudinal seismic waves, the ratio of maximums of amplitudes of tube wave and cogging wave is possible to represent as:

$$\frac{P_{tw}}{P_w} = \left(\frac{ñ c_s^2}{ñ_f c_{tw}^2} \cdot \frac{1 - (c_{tw} / c_w)^2}{(c_l / c_s)^2 - 2 (c_l / c_w)^2} \cdot \Big(\, (c_{tw} / c_l)^2 - 2(1 - \cos^2 È) \Big) - 1 \right) \cdot Q(T,t) \qquad (3)$$

where: $È$ - the seismic wave incidence angle on the crack plane**:** $\cos(È) = (\vec{e},\vec{n})$, and $Q(T,t)$ is the factor, determined only by the pressure relaxation time t and by the duration T of incident seismic wave. All values in expression (3) can be directly measured in geophysical experiment (VSP), excepting for angles $È$, describing the crack orientation. Basing on the relation (3), the following procedure of restoring of the crack plane orientation is suggested. The VSP experimental data from three and more explosions on a surface (see fig. 2) are considered.

The parameters r, r_f, c_l, c_s are considered as known one due to borehole data. The velocities $ñ_w$, c_{tw} and amplitudes ratio P_{tw} / P_w are also considered as known one due to VSP data. Besides, if the frequency spectrums of seismic waves, coming from various explosion points differ insignificantly, it is possible to consider the factor $Q(T,t)$ like a constant value. It can be also achieved with the proper frequency filtration of seismograms, so that the spectrums of a seismic waves in all seismograms would be the same. Further, we assume that in borehole there is a geophone, which one allows to define the directions of longitudinal seismic wave arrivals. Let, for example, there are three series of seismograms from three different explosions, so $\{\vec{e}_1, \vec{e}_2, \vec{e}_3\}$ - local orts of propagation directions of seismic waves are registered. Then the cosines of angles $\cos(\Theta_i) = (\vec{e}_i, \vec{n})$, $i =$ 1, 2, 3 correspond to these directions. Thus, there is a system of three equations $P_{tw} / P_w = f\,(.....,\Theta_i)$, $i = 1,2,3$, with four unknown variables $Q(T,t) = const$, $\{\cos(\Theta_i)\}$, $i = 1,2,3$. Further, to obtain the complete system of equations it is needed to use the normalization condition. Let's consider the vectors $\{\vec{e}_1, \vec{e}_2, \vec{e}_3\}$ as basis of coordinates system, then the normal unit vector $\vec{n}$ can be expanded by this

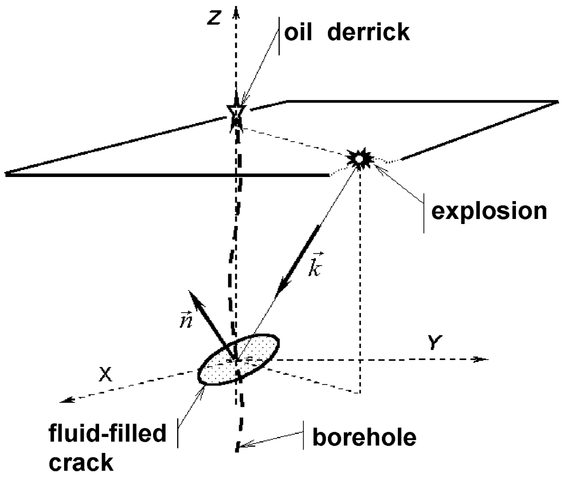

Fig.1 The execution scheme for vertical seismic profiling.
$\vec{n}$ - normal vector to the crack plane.
$\vec{k}$ - wave vector of the incident seismic field.

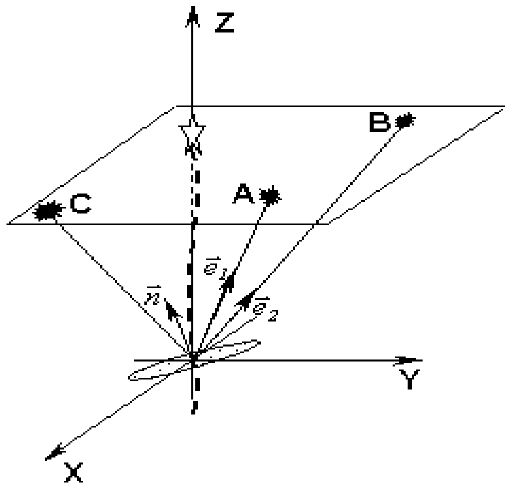

Fig.2 The scheme of experiment for determination of crack plane orientation.

basis and it is possible to obtain the relation $\cos(\grave{E}_j)\cdot a_{ji}^{-1}\cdot\cos(\grave{E}_i) = 1$, which one expresses the requirement of normalization of vector $\vec{n}$ in basis $\{\vec{e}_1, \vec{e}_2, \vec{e}_3\}$, where $\acute{a}_{ji}^{-1}$ - the inverse matrix to $\acute{a}_{ji} = (\vec{e}_i, \vec{e}_j)$. Approximate values $\{\cos(\Theta_i)\}$, $i = 1,2,3$ is possible to find as the solution of the problem on the searching of the function minimum:

$$F(\cos(\grave{E}_1),\cos(\grave{E}_2),\cos(\grave{E}_3)) = (G_1 - G_2)^2 + (G_1 - G_3)^2 + (G_2 - G_3)^2 + (1 - G_0)^2$$

where, $G_0 = \cos(\grave{E}_j)\cdot\acute{a}_{ji}^{-1}\cdot\cos(\grave{E}_i)$, and G_i :

$$G_i = \frac{1}{Q(T,\hat{o})} = \frac{P_w}{P_{tw}}\cdot\left(\frac{\tilde{n}c_s^2}{\tilde{n}_f c_{tw}^2}\cdot\frac{1-(c_{tw}/c_{wi})^2}{(c_l/c_s)^2 - 2(c_l/c_{wi})^2}\cdot\left((c_{tw}/c_l)^2 - 2(1-\cos^2\grave{E}_i)\right) - 1\right)$$

and by c_{wi}, $i = 1,2,3$ - the visible propagation velocities of a seismic waves along borehole axis are noted for i-th explosion point. When the minimum of function $F(\cos(\Theta_i))$ is found, the estimation of a direction $\vec{n}$ can be obtained by the formula: $\vec{n} = \cos(\grave{E}_j)\cdot a_{ij}^{-1}\cdot\vec{e}_i$.

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