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THE RESEARCH OF NONLINEAR ELASTIC PROPERTIES OF MAJOR GENETIC TYPES OF ROCKS

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The method of definition of nonlinear elastic waves from inversion of the measurement data of velocities of elastic waves of longitudinal and cross polarization of the patterns of rocks.

The absence of information about nonlinear properties of rocks complicates interpretation of the data nonlinear seismoacoustics. The new method of definition of the elastic constant third order by inversion of the experimental researches of velocities of elastic waves of compressional and cross polarization of samples of rocks in conditions of high hydrostatic pressure is offered.

Feature of experimental conduction of elastic properties of rocks is their contrast change at transition from stretching to compression. This property of rocks is called different modulation and depends on from degree its mechanical damage.

Therefore use for experimental determinations of nonlinear elastic properties of rocks of known methods [1] meet the certain difficulties. In particular, the most widely widespread technique of definition of the nonlinear elastic constant third order of rocks is based by creation of the axial intense condition for measurement of waves of longitudinal and cross polarization definitely focused to relation of axial loading. At axial compression induced acoustic-elastic anisotropy, caused by disclosing or closing of microcracks depending on their orientation to the main effect stress. Therefore received nonlinear elastic parameters in conditions one-axis of the intense state characterize only given experimental conditions of realization of tests of samples of rocks and their comparison is represented rather problematic. Use of experiments at high hydrostatic compression is more perspective. At hydrostatic compression of samples in the camera of pressure the number of mechanical defects essentially decreases. Amount of microcracks with small format ($a = c/a$ – format of microcracks, c, a – small and long axes spherical microcrack) especially appreciably decreases.

The elastic constant third order, determined in such conditions, of rocks it is possible basically to compare, but necessarily thus it is necessary to specify degree mechanical damage of rock. As such measure it is possible to use size of complex parameter of density of disc-similar cracks $x = \overline{N} a^3$ ($\overline{N}$ — amount of disc-similar cracks of radius a in unit of volume) infinitesimal thickness.

In this paper are considered the results of definition of the elastic constant third order of various genetic types of rocks through inversion in dependence of relation of velocities of elastic waves of longitudinal polarization to velocity of elastic waves of cross polarization at various hydrostatic pressure. For these purposes were used statistically average values of velocities of elastic waves at hydrostatic pressure from 0,2 up to 1,0 GPa, given in work [6]. In this range of pressure there is closing microcracks and their influence on velocities elastic waves is essentially weakened.

For the decision of the put problem of inversion we chosen the simplified matrix model of rock, which consists from isotropic firm skeleton (matrix), which cuts disc-similar by arbitrary oriented cracks infinitesimal thickness. The account of the effective elastic constant third order (à*, b*, ñ*) of matrix model of rock, in view of physical and geometrical nonlinearity, is carried out by the method of conditional moment functions with use of parities(ratio) [2, 3]:

$$a^* = a_0^* + 3g\left[\tfrac{1}{2}a\,(a+b)\,l^* + g\,(a+2b)\,m^*\right], \qquad (1)$$

$$b^* = b_0^* + \frac{1}{2}a\left(b^2-1\right)l_0^* + \tfrac{1}{3}\left(b - a + 9b^2 g\right)m^*\ , \qquad (2)$$

$$c^* = c_0^* + 3\left(b^2-1\right)m^*\ , \qquad (3)$$

where $g = \frac{1}{3}(a - b);$ (4)

$$l^* = a\ l + 2mg\ ; \quad (5)$$

$$m^* = b\ m\ ; \quad (6)$$

$$a = \left[1 + w\left(1 - n^2\right)\left(1 - 2n\right)^{-1}\right]^{-1}\ ; \quad (7)$$

$$b = \left[1 + w\left(1 - n\right)\left(1 - 0{,}2n\right)\left(1 - 0{,}5n\right)^{-1}\right]^{-1}\ ; \quad (8)$$

$$w = 8/3p\ x\ ; \quad (9)$$

l^*, m^* — effective Lame's parameters of rock;

l, m — Lame's parameters of the matrix of rock;

n — Poisson's ratio of the matrix of rock.

Here $a_0^*,\ b_0^*,\ c_0^*$ — effective elastic constant third order of rock in approach(approximation) of physical nonlinearity:

$$a_0^* = a^3 a + a\left(a + b\right)\left(a - b\right)b + \left(a + 2b\right)g^2 c\ , \quad (10)$$

$$b_0^* = b^2\left(ab + b\ c\right)\ , \quad (11)$$

$$c_0^* = b^2\ c\ , \quad (12)$$

where *a, b, c* — elastic constant matrixes of rock without cracks.

The data about nonlinear elastic constant of rock-formation minerals are necessary for numerical accounts of the effective elastic constant third order of rocks, the majority from which represent by low-simmetric crystals of complex structure. For some of them (for example, antigorith, lisardith, hrizotith) are not determined even of the elastic constant second order. At present known the nonlinear elastic constant third order only for two of rock-formation minerals of quartz and calcium.

Certainly, it essentially complicates the decision of the problem, as during inversion it is necessary to determine simultaneously and elastic constant third order of firm skeleton of rock without cracks.

The problem of inversion is formulated as follows: to find of the effective elastic constant second and third order, density of cracks of rocks, if the experimental values of the rations of velocities of elastic waves of longitudinal polarization to velocities of waves of cross polarization are known at various hydrostatic pressure. For its solution the nonlinear method of the least squares [4, 5] and object function was used: $F(\vec{x})$:

$$F(\vec{x}) = \sum_{m=1}^{M}\left[R_{(m)}^{(e)} - R_{(m)}^{(r)}\right]^2\ , \quad (13)$$

where $\vec{x}$ — vector of researched parameters of dimension *N*, which includes of the effective elastic constant second order of firm skeleton (K, G) and their derivative on pressure ($\partial \mathrm{K}/\partial P,\quad \partial G/\partial P$); the elastic constant third order of firm skeleton (a, b, c) and density of cracks (x);

$R_{(m)}^{(e)} = u_p^{(e)}(P)/u_s^{(e)}(P)$; $u_p^{(e)}(P),\ u_s^{(e)}(P)$ – - experimental values of velocities of elastic waves of longitudinal and cross polarization at hydrostatic pressure P;

$R_{(m)}^{(r)} = u_p^{(r)}(P)/u_s^{(r)}(P)$; $u_p^{(r)}(P),\ u_s^{(r)}(P)$ — calculated values of velocities of elastic waves of longitudinal and cross polarization at hydrostatic pressure given from the values of the elastic constant second and third order on r-step of iterations under the formulas of linear approximation of the theory of the large initial deformations received in work [1]:

$$r\,u_P^{*2} = \left(l^* + 2m^*\right) - \frac{P}{3\mathrm{K}^*}\left[7\,l^* + 10m^* + 2\left(3a^* + 5b^* + c^*\right)\right]; \quad (14)$$

$$r\,u_S^{*2} = m^* - \frac{P}{3\mathrm{K}^*}\left[3(\,l^* + 2m^*) + 3b^* + c^*\right]; \quad (15)$$

here l^*, m^*, a^*, b^*, c^* are calculated under the formulas (1) – (3), (5), (6).

For search of global minimum of object function (13) is used quasi-newton's method:

$$x_{K+1} = x_K + l_K H_K P_K \ , \tag{16}$$

l_K — positive multiplier of iterative steps; H_K — positively certain matrix, which is updated during calculations; P_K — direction of slope, which coincides with a direction of antigradient. For recalculation of matrix the method Broyden-Fletcher-Goldfarb-Shenno is used [14]. According to this method the formula for recalculation of matrix H on (K+1)-step of iterations is given:

$$H_{K+1} = H_K + \frac{V_K V_K^T}{V_K^T V_K} - \frac{H_K U_K U_K^T H_K}{U_K^T H_K U_K} \ , \tag{17}$$

where $V_K = H_K P_K$, $U_K = \nabla F(x_{K+1}) - \nabla F(x_K)$;

ò — operation of transponation.

At first as zero approximation H_0 choose the initial matrix. Hence, the first step is defined in direction of fast slope. After calculation of matrix H_K the step of length $l_K = 1$ in direction $d_K = H_K P_K$ is carried out. If thus $F(x_K + l_K d_K) \angle F(x_K)$ then pass to the following iteration. Otherwise is carry out splittings step according to cubic interpolation [14], which is finished only in case of seek such length of step l_K which provides performance of condition:

$$F(x_K + l_K d_K) \angle F(x_K) \ . \tag{18}$$

The procedure of minimization is finished, if is satisfied condition $\|\nabla F(x_K)\| \angle e$. Here e — small enough beforehand given positive value. It means, that the point of local minimum x_K^* is found. The convergence of iterative process to global minimum depends from prominence of object function. In our case it not so in a consequence of nonlinearity of problem. Therefore zero approximation got out as it is possible closer to model, which satisfies to experimental data and physical pithiness of received solutions.

For solution of problem of inversion the experimental values of the attitudes(relations) of velocities of waves of longitudinal polarization to velocities of waves of cross polarization of various rocks high hydrostatic pressure from work were used [6]. For inversion the average values of the ratio u_P / u_S were chosen statistically and the names of petrographic types of rocks are kept which were accepted in work [6]. Reliability of the received estimations of the elastic constant second and third order is confirmed by results of comparison of the found values with the data of independent experimental definitions of the elastic constant third order of mineral of quartz [7] and calcium [8]. For firm skeleton quartzite are received values a^* = -80.80 GPa, b^* = 11.0 GPa, c^* = -64.5 GPa, which are n the satisfactory y consent with average by values in Foygt's approximation for quartz: a = -80.88 GPa, b = 10.95 GPa, c = -64.48 GPa.

The values of nonlinear elastic parameters of skeleton for marbles, consisting mainly from calcite, are equal: a^* = -59.3 GPa, b^* = -30.7 GPa, c^* = -15.4 GPa.. The average isotropic values of elastic constant calcite in Foygt's approximation: a = -59.3 GPa, b = -30.7 GPa, c = -15.4 that corresponds the data of inversion.

The certain tendencies in change of the elastic constant third order a^*, b^* and c^* for rocks are observed depending from structure. In particular, they discover the certain correlation direct link with the contents $S_i O_2$. Nonlinear parameter a^* has direct correlation link with the contents with coefficient of correlation 0.43, and parameters b^* and c^* feedback from factor microcorrelation, accordingly, -0.63 and -0.54. Certainly, density of cracks is one of the most important factors, which influences value of linear and nonlinear elastic properties of rocks. The nonlinear properties of solid skeleton diabases essentially differ from andezites and basalts. The especially significant distinctions, in some times, are observed for values of parameter a^*. Among intrusive of rock — granitoids and diorites — these distinctions is significant smaller. For anortosites the characteristic least value of parameter a^* = -16.2 GPa. The ultra-basic rocks — have values of nonlinear elastic parameters of the same order, as well as gabbronorites.

Among metamophic rocks by the least values of nonlinear parameter a^* are characterized quartzites and amphibolites. The quartzites are differed also by greatest value of parameter b^* = 11.0 GPa.

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