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THE INTERFERENCE OF OPPOSITE LONGITUDINAL ACOUSTIC WAVES IN ISOTROPIC DISSIPATIVE PLATE AND PERIODIC STRUCTURE WITH DEFECTS

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Interference of longitudinal acoustic waves propagating in opposite directions in a homogeneous isotropic absorbing plate and a periodic structure with a defect is considered theoretically. The periodic structure consists of alternating absorbing solid and transparent liquid layers. The defect is modeled by replacing a solid layer by a liquid layer of the same thickness. An expression is obtained to determine the extremums of the wave amplitude transmitted through an absorbing plate depending on the amplitude ratio of the interacting waves. The results of studying a one-dimensional periodic structure demonstrate the possibility to change the transmission spectrum of the pressure wave leaving the structure and also to eliminate the invariance of this spectrum under the interchange of the k-th and (n-k+1)-th layers (where n is the total number of layers in the structure).

The effect of a transmission enhancement in thin metal films as a result of the interaction of electromagnetic waves propagating in opposite directions was studied in many papers [1]. The most interesting results were the dependences of the intensity of a wave transmitted through an absorbing medium on the amplitude and phase of the opposite wave and on the coefficient of light absorption. Drawing an analogy between electromagnetic and elastic waves, authors of [2] demonstrated theoretically the possibility of increasing of transmissions of an absorbing elastic medium due to the interference of opposing longitudinal acoustic waves. This problem is undoubtedly important and interesting from the point of view of investigation of both absorbing media and multilayer structures, in which the interaction of opposing waves occurs as a result of multiple reflections of elastic waves from the boundaries.

We investigate the dependence of the transmission spectrum of a pressure wave transmitted through an absorbing medium on the amplitude and phase of a wave propagating in the opposite direction for two cases. In the first case, a homogeneous plane-parallel plate is considered, and in the second case, a one-dimensional periodic structure consisting of alternating layers of a transparent liquid and an absorbing solid. In assumption that the plate plane is perpendicular to the line segment connecting the sources. Two longitudinal monochromatic pressure waves, which have different amplitudes and a phase difference j between them at the plate boundaries, propagate from the immersion liquid toward the plate normally to the plate surface that can be expressed as follows [3]:

$$p_f = p_f \exp(i(wt - kx)), p_b = p_b \exp(i(wt + kx + j)) \quad . \qquad (1)$$

Here, p_f and p_b are the amplitudes of the direct and opposite pressure waves, respectively; k is the wave number; and w is the cyclic frequency.

Assuming that the waves under study have a plane wave front, we can ignore the formation of shear waves at the liquid-solid interface. Longitudinal acoustic waves are multiply reflected and interfere in the process of their propagation inside the plate. A result of such an interaction in the plate can be described by two opposite waves, each of them being a superposition of single waves propagating in the same direction. In order to simplify the problem, we assume that the amplitudes of pressure waves are sufficiently small and nonlinear effects don't manifest themselves, which in its turn allows us to ignore the time dependence. From the boundary conditions implying equal pressures and particle velocities at the plate boundaries the pressure amplitudes of all interacting waves can be determined. From the practical point of view, the most interesting parameters are the amplitudes of the pressure waves propagating out of the plate, which can be represented in the form of a sum of waves transmitted through the plate and reflected from it [3]. The power flux density S of an acoustic wave can be represented as: $S = p \cdot v = |p|^2 / rc$ [3, 4].

It's necessary to analyze the dependence of the energy carried by the wave on the amplitude of the opposite wave by fixing the total amount of energy supplied to the plate. For this purpose, we introduce the parameter a varying in the range from *0* to $p/4$ and represent the pressure amplitudes in the form $p_f = \cos(a)$ and $p_b = \sin(a)$. Then, we obtain a sequence of amplitudes of the pressure waves from the case of a unidirectional propagation ($a = 0$) to the case of opposing waves with equal amplitudes (at $a = p/4$). We assume that, if the distances from the sources to the closest boundaries of the plate are different, a propagation path-length difference exists between the interacting waves and the phase difference in the studied frequency range is directly proportional to frequency. The transmission coefficient T_E for the energy flux (normalized to the total energy flux supplied to the plate by the waves of one frequency) that is carried by the z-directed wave can be written as:

$$T_E = \frac{16z^2 z_0^2 p_f^2 - 16zz_0(z^2 - z_0^2) p_f p_b \sin(2pn/n_0)\sin(\varphi) + 2(z^2 - z_0^2)^2 p_b^2 (1 - \cos(4pn/n_0))}{(z+z_0)^4 - 2(z+z_0)^2 (z_0 - z)^2 \cos(4pn/n_0) + (z_0 - z)^4}, \qquad (3)$$

where $z = rc$ and $z_0 = r_0 c_0$ are the acoustic impedances of the plate and the immersion liquid, $\varphi = 2pn\Delta l / c_0$, Δl is the path-length difference, and $n_0 = c/d$ is the frequency at which the acoustic wavelength is equal to the plate thickness.

As one can see, the calculated transmission coefficient summarizes three fluxes, namely, the flux arriving from the source p_f and transmitted through the plate, the flux resulting from the interaction of opposing waves, and the flux arriving from the source p_b and reflected by the plate. If the amplitude of the pressure wave generated by one of the opposite sources is equal to zero, the equation (3) describes a reflected flux or a flux transmitted through the plate for the unidirectional case. The phase difference is of key importance for the determination of the value and direction of the opposite flux. If the phase difference between the interacting waves is a multiple of p, the total flux is equal to the sum of the fluxes transmitted through the plate and reflected from it. The peaks of the flux transmitted through the plate in the case of a unidirectional interaction occur when the plate thickness is equal to a whole number of half-waves. The transmission through a transparent plate at the corresponding frequencies ($n = n_0(m+1)/2$, where m is a whole number) doesn't depend on the phase difference between the interacting opposing waves. We'll demonstrate below that this isn't true in the presence of absorption. In the case of an absorbing plate, we represent the sound velocity in the medium in the form of a complex quantity with the imaginary part describing the absorption $c = c/(1 - i/2pQ)$, where $Q = 1/(la)$. One can notice that the dependence $T_E(n)$ for an absorbing plate isn't a monotone and exponentially damped dependence. Taking into account only the first-order terms in the absorption coefficient, we can express the energy transmission coefficient as

$$T_E = \left[\begin{matrix} 16z^2 z_0^2 p_f^2 \exp(-2n/n_0 Q) + Int(a,a) + (z^2 - z_0^2)^2 p_b^2 (\exp(-4n/n_0 Q)) - \\ -2\exp(-2n/n_0 Q)\cos(2pn/n_0) + 1) \end{matrix}\right] / A, \qquad (4)$$

where $A = (z+z_0)^4 - 2(z_0^2 - z^2)^2 \exp(-2n/n_0 Q)\cos(2pn/n_0) + (z - z_0)^4 \exp(-4n/n_0 Q)$,

$$Int(a,a) = 8zz_0(z^2 - z_0^2)(\exp(-n/n_0 Q)\cos(2pn/n_0 + \varphi) - \exp(-3n/n_0 Q)\cos(2pn/n_0 - \varphi)) p_f p_b -$$

$$- \frac{8}{2pQ} zz_0 (z_0^2 + z^2)(\exp(-n/n_0 Q)\sin(2pn/n_0 + \varphi) - \exp(-3n/n_0 Q)\sin(2pn/n_0 - \varphi)) p_f p_b.$$

As one can see from equation (4), the absorption on the whole leads to a transmission decrease. The term proportional to the absorption coefficient is present in the expression describing the interferential interaction. However, it cann't considerably affect the transmission, because it is very small. The change in the transmission has a damping-oscillating character that is determined by the Q -factor and the relationships between the amplitudes and phases of the interacting waves.

It's interesting to study the dependence of ultrasonic transmission on the ratio of the amplitudes of opposing waves with fixed frequency and phase difference. Taking into account the absorption in the plate the condition for the extremums of the amplitude of the transmitted pressure wave can be written as:

$$\mathrm{tg}\, a = \frac{\exp(i2pn/n_0 + n/n_0 Q) - \exp(-i2pn/n_0 - n/n_0 Q)}{4} \times \left(\frac{z}{z_0} \frac{1 + i/2pQ}{1 + 1/4p^2 Q^2} - \frac{z_0}{z}\left(1 - \frac{i}{2pQ}\right)\right) \exp(i\varphi).$$

Let us compare it with the condition for the extremums in the case of a transparent plate, which can be obtained by passing to the limit $Q = \infty$:

$$\mathrm{tg}\, a = \frac{i \sin(2pn/n_0)}{2}\left(\frac{z}{z_0} - \frac{z_0}{z}\right)\exp(i\varphi).$$

One can see from last form that, in the case of an arbitrary ratio of the wavelengths of the interacting waves and the thickness of the plate, the extremums manifest themselves at $\varphi = p(2m+1)/2$, because $\mathrm{tg}\, a$ is real (here, *m* is a whole number). Moreover, if a whole number of half-wavelengths fits into the plate thickness, the phase difference doesn't affect the amplitude of the transmitted wave. In the case of an absorbing plate, all other conditions being the same, a phase shift appears in the indicated dependence in comparison with the case of a transparent plate. The shift value includes a constant component (the term in parenthesis) and a component that depends on the wavelength (frequency) and is essential only for strong absorption. Thus, with the variation of the phase difference and the ratio of amplitudes of the interacting waves, the interaction of opposing waves provides an opportunity to change the transmission spectrum of an absorbing plate.

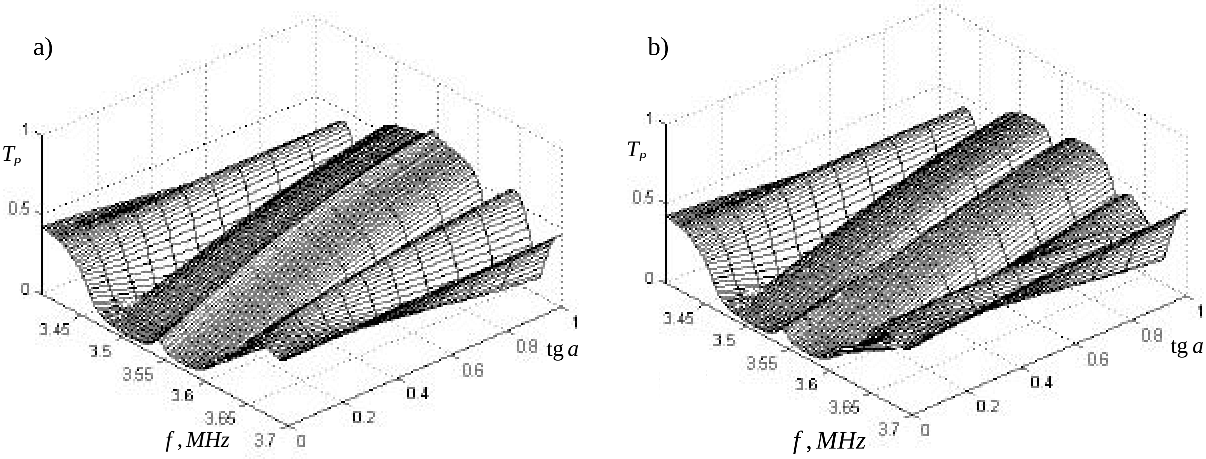
Now let us consider interaction of waves propagating in opposite directions in one-dimensional periodic structures. In the case of unidirectional propagation of ultrasonic waves through a one-dimensional periodic structure, the transmission spectrum consists of transparency and opacity regions. If a defect occurs in the structure (a layer disturbing the structure periodicity), a local maximum arises in the opacity region of the transmission spectrum [5]. Experimental data on the transmission spectra in the case of unidirectional interaction agree well with the calculations. On this basis, it was suggested to apply a nondestructive testing technique grounded on broadband acoustic spectroscopy [6] with a laser source of ultrasound. In solving this problem, it is necessary to stress that the transmission spectra obtained for a direct and inverse positions of a periodic structure with a defect are identical in the case of unidirectional propagation. As it will be demonstrated below, the interaction of opposing acoustic waves eliminates this ambiguity.

We assume that a multilayer structure consists of *N* isotropic layers with known physical constants and thicknesses. Longitudinal acoustic waves propagate toward the structure from opposite directions. The boundary conditions written for each boundary of the multilayer structure yield a set of equations [4]:

$$p_k^+ e^{-i\frac{w}{c_k}x_k} + p_k^- e^{i\frac{w}{c_k}x_k} = p_{k+1}^+ + p_{k+1}^-,\; z_k p_k^+ e^{-i\frac{w}{c_k}x_k} - z_k p_k^- e^{i\frac{w}{c_k}x_k} = z_{k+1}\left[p_{k+1}^+ - p_{k+1}^-\right], \quad (5)$$

where x_k is the thickness of the *k*-th layer; z_k and c_k are the impedance and the sound velocity in the layer located to the left of the boundary; z_{k+1} and c_{k+1} are those in the layer located to the right of it, respectively; p_k^+ and p_k^- are the amplitudes of pressure waves within the *k*-th layer; *k* varies from 0 (the first boundary) to *N* (the last boundary). Thus, we have a set of *2N+2* equations with the same number of unknowns. Solving this set, one can obtain the amplitudes of pressure waves. From the practical point of view, we are interested in the amplitudes of the waves propagating away from the structure. As in the case of a plate, we consider the transmission dependences with a fixed total energy flux in the whole frequency range (we use the parameter a).

The periodic structure under study consisted of ten plexiglas layers (r = 1.12·10^3 kg/m^3, c= 2.65·10^3 m/sec, d = 1.6·10^{-3} m, $Q = 30$) and nine water layers (r = ·10^3 kg/m^3, c = 1.49·10^3 m/sec, d = 10^{-3} m). The modeled defect was a plexiglas layer replaced by a water layer of the same thickness. Absorption in plexiglas was taken into account in the same way as in the case of a plate, and ultrasonic absorption in water was ignored. Figure 1 presents the dependences of the transmission spectra of pressure wave of periodic structures with defects (the seventh and thirteenth layers) for different ratios of the amplitudes of opposing waves with the path-length difference between them equal to 0.2 mm. It is important to note that, in the case of a unidirectional interaction ($a = 0$), the transmission spectra of the structures under consideration are identical and a local peak arises in the opacity region, which corresponds approximately to a frequency of 3.54 MHz in the plot. The interaction of opposing waves eliminates such an ambiguity if $a \neq 0$ the ultrasonic transmission spectra of a periodic structure are



different.

Fig. 1. Transmission spectra of periodic structure with defect 7-th (a) and 13-en (b) layer

The interaction of opposing waves provides an opportunity to model the flux transmitted through a plate in a broad frequency band. We obtained an expression for the extremums of the amplitude of the ultrasonic wave transmitted through an absorbing plate. The results of simulation for one-dimensional periodic structures demonstrated the difference in the ultrasonic transmission spectra of structures with defect layers symmetric with respect to the center, these spectra being identical in the case of a single source. The results obtained can be useful for nondestructive testing.

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