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ISOLATION OF FLEXURALWAVE BY AN ARRAY OF RESONATORS ESTABLISED ON A PLATE

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The scattering of a plane flexural wave in a plate from array of identical resonators (springs with weights) is investigated. The scattered field is presented by the sum of homogeneous and inhomogeneous Bragg spectra. The dispersion equation for frequencies of resonant scattering is obtained. The amplitudes of scattered spectra on resonance frequencies are calculated. At the period of array, smaller $l(1+\sin q)^{-1}$, *where* q *is the incident angle,* l *is the length of flexural wave, only "zero' spectra homogeneous. Then for want of dissipative losses in resonators the incident wave is completely reflected from an array. At the period of an array, greater* $l(1+\sin q)^{-1}$ *, in a scattered field there are also "non-zero" homogeneous spectra. Then the array of resonators will not be the effective reflector of flexural waves. The overall performance of the array of resonators with a dissipation as absorber of flexural waves in a plate is investigated. With this purpose the resonators are attached to the plate in a displacement antinode. It is shown, that at a defined value of a factor of a dissipation the resonators completely occlude an incident wave.*

Resonators are used in practice for flexural-mode vibration isolation in beams and plates [1,2]. The simplest resonator is a weight on spring [3]. When such a resonator is oriented perpendicular to a plate and is attached to it by the spring, flexural waves propagating in the plate are strongly scattered. The scattering length of a single resonator without a dissipation is equal $2p/l$, where l is the length of a flexural wave. An effective means of isolation of flexural waves in plates is the wave-guide isolator [1]. He is created by the grid of identical resonators attached to a plate close-range from each other. It is interesting to consider a problem scattering of a plane flexural wave in a plate from array of resonators with any period at any incident angle.

Let plate lies in a plane xy and to it along a coordinate axis x in points $x = x_s \equiv sL,$ where $s = 0, \pm 1, \pm 2, \ldots,$ the identical resonators with weights m and elastic coefficients $k(1-ie)$ are joined, e is dissipation factor. A harmonic flexural wave is incident on the resonators with displacement

$$w^{(0)}(x,y,t) = exp[i(k_x^0 x - k_y^0 y - wt)], \tag{1}$$

where k_x^0 è $(-k_y^0)$ are projection of a wave vector of the incident wave accordingly on an axis x and y, w is the angular sound frequency. The resonators are exited by the action of the wave (1) and radiate a field $w^{(1)}(x,y,t)$. The total field w in the plate is $w^{(0)} + w^{(1)}$. We denote the displacement of the *s*-th resonator weight by $w_s'(t)$ (attached to a plate in a point $x_s, 0$). The equation of motion of the weight has the form

$$m\frac{d^2 w_s'}{dt^2} = -F_s(t), \tag{2}$$

where the force F_s is given by the equation

$$F_s(t) = k(1-ie)[w_s'(t) - w(x_s, 0, t)] \; . \tag{3}$$

The equation of motion of the plate with the attached resonators can be written in the form

$$r\frac{d^2 w}{dt^2} + GD^2 w = \sum_{s=-\infty}^{s=\infty} F_s(t) d(x - x_s) d(y), \tag{4}$$

ãäå r is the surface density, G is the flexural stiffness, D is the Laplace operator, and $d(y)$ is the delta function. As incident wave $w^{(0)}$ is free wave, on the left of equation (4) is possible to exchange w on $w^{(1)}$.

The structure of a scattered field is determined by the period of a scattering grating (array of resonators), the value $w^{(1)}(x,y,t)exp(-ik_x^0 x)$ is the periodic function on x with period L. On this basis at an incident wave (1) force $F_s(t)$ can be presented in the form

$$F_s(t) = F\, exp[\,i(k_x^0 x_s - wt)],$$

where F is amplitude of force at $s = 0.$ In an unbounded plate the array of dot forces $F_s(t)$ creates a field

$$w^{(1)}(x,y,t) = \sum_{n=-\infty}^{n=\infty} \frac{iF}{4LGk^2}\{\frac{1}{k_y^n} exp(ik_y^n|H-y|) + \frac{i}{a^n} exp(-a^n|H-y|)\} exp[\,i(k_x^n x - wt)], \qquad (5)$$

where

$$k_x^n = k_x^0 + \frac{2p}{L}n,\; k_y^n = \sqrt{k^2 - (k_x^n)^2},\; a^n = \sqrt{k^2 + (k_x^n)^2},\; k = (w^2\, r/G)^{\frac{1}{4}}.$$

In a figured brace the first item is a homogeneous plane wave at $|k_x^n| < k$ and inhomogeneous plane wave at $|k_x^n| > k$, the second item always is inhomogeneous wave.

We select the amplitude F so as to satisfy the relation (3). According to Eq. (2), the displacement of the load is $w_s'(t) = \frac{F_s(t)}{mw^2}$. Substituting $w^{(0)}, w^{(1)}$ and w_s' in the relation (3), we obtain the required force amplitude:

$$F = iw(Y + Y_0)^{-1}, \text{ where } Y_0 = i[\frac{1}{wm} - \frac{w}{k(1-ie)}],$$

$$Y = \frac{-iww^{(1)}(x_s,0,t)}{F_s(t)} = \sum_{n=-\infty}^{n=\infty} \frac{w}{4LGk^2}(\frac{1}{k_y^n} + \frac{i}{a^n}).$$

We obtain the scattered field $w^{(1)}$ from Eq. (5) after substituting F in it. The resonant scattering occur at frequencies defined from an equation $Im(Y + Y_0) = 0$. On the resonance frequency the amplitude of the n-th scattered homogeneous spectrum (scattered homogeneous plane wave) is

$$A_n = \frac{-w}{4LGk^2 k_y^n}\{Y_1 + \frac{ew}{k(1+e^2)}\}^{-1}, \quad Y_1 = Re\,Y = {\sum_s}' \frac{w}{4LGk^2 k_y^s}, \qquad (6)$$

the touch above the sum means, that the summation is made on everything s, at which k_y^s is real. The amplitude of a transmitted flexural wave is $(1 + A_0)$. At the period of the array, smaller $l(1+sin q)^{-1}$, where $l = 2p/k$, $q = arcsin\, k_x^0/k$ is the incident angle, in scattered field (5) only zero spectra is homogeneous. Then from formula (6) we obtain

$$Y_1 = \frac{w}{4LGk^2 k_y^0}, \quad A_0 = -\{1 + e\,\frac{4LGk^2 k_y^0}{k(1+e^2)}\}^{-1},$$

and the amplitude of a transmitted flexural wave is equal approximately to $4eLGk^2 k_y^0/k$. For want of dissipative losses in resonators $(e = 0)$ the incident wave (1) is completely reflected from an array. At the period of an array, greater $l(1+sin q)^{-1}$, in a scattered field (5) there are also non-zero homogeneous spectra. Then the array of resonators will not be the effective reflector of flexural waves.

We investigate an overall performance of the array of resonators with a dissipation as absorber of flexural waves in a plate. The absorption of flexural waves by a single resonator is investigated in the article [4]. Let the plate lies in an upper half-plane xy and is fixed on boundary $y = 0.$ On this boundary incident the harmonic flexural wave (1). The total field in plate, equal sum incident and reflected waves, is

$$W^{(0)}(x,y,t) = i2\{sin(k_y^0 y - j^0) + sin\, j^0\, exp(-a^0 y)\} exp[i(k_x^0 x - wt)],$$

where $sin\, j^0 = \frac{k_y^0}{\sqrt{2}k}$. In this field the displacement antinode are arranged on lines $y_n \approx [p(2n-1) + 2j^0] \big/ 2k_y^0$, where n is any integer. Along any of these lines (for example $y = y_q \equiv H$) at the points $x = x_s \equiv sL$, where $s = 0, \pm 1, \pm 2, ...,$ we shall connect identical resonators with weights m and elastic coefficients $k(1 - ie)$. The resonators are excited under the action of the wave $W^{(0)}$ and radiate a field $w^{(1)}$. The total field w in the plate with resonators is equal to $W^{(0)} + w^{(1)}$. Executing respective calculations, we shall receive following expression for $w^{(1)}$ at $y \geq H:$

$$w^{(1)}(x,y,t) = \sum_{n=-\infty}^{n=\infty} \frac{-2w\, exp(-ik_y^0 H)}{4LGk^2(Y + Y_0)} \{ \frac{1}{k_y^n} [1 - exp(i2k_y^n H - i2j^n)]\, exp[i(k_x^n - wt) + ik_y^n (y - H)] +$$

$$+ i[\frac{1}{a^n} - \frac{2}{k_y^n} sin\, j^n\, exp(ik_y^n H - ij^n - a^n H)]\, exp[i(k_x^n x - wt) - a^n (y - H)]\}, \qquad (7)$$

where

$$Y \approx \sum_{n=-\infty}^{n=\infty} \frac{w}{4LGk^2 k_y^n} \{1 - exp[i2(k_y^n H - j^n)] + i\frac{k_y^n}{a^n}\}, \quad sin\, j^n = \frac{k_y^n}{\sqrt{2}k}.$$

The resonant scattering occur at frequencies, defined from an equation $Im(Y + Y_0) = 0.$ On a resonance frequency the amplitude of the n –th scattered homogeneous spectrum is equal

$$A_n = -\frac{w}{2LGk^2 k_y^n} \{Y_1 + \frac{ew}{k(1 + e^2)}\}^{-1} exp[-i(k_y^0 + k_y^n)H]\{1 - exp[i2(k_y^n H - j^n)]\}, \qquad (8)$$

were

$$Y_1 = Re\, Y = \sum_s{}' \frac{w}{4LGk^2 k_y^s} \{1 - cos[2(k_y^s H - j^s)]\}. \qquad (9)$$

At the period of an array, smaller $l(1 + sin\, q)^{-1}$, in a scattered field (7) only zero spectrum is homogeneous. Then from the formula (8) and (9) we shall receive ratio

$$Y_1 = \frac{w}{2LGk^2 k_y^0}, \quad A_0 = 2\, exp(-i2j^0)[1 + \frac{2eLGk^2 k_y^0}{k(1 + e^2)}]^{-1}.$$

Sum a homogeneous reflected wave with a zero homogeneous scattered spectrum, we shall receive propagating homogeneous wave with amplitude A, equal

$$[-exp(-i2j^0) + A_0] = -exp(-i2j^0)\{1 - 2[1 + \frac{2eLGk^2 k_y^0}{k(1 + e^2)}]^{-1}\}.$$

At a factor of a dissipation e , equal approximately $p/(2LGk^2k_y^0)$, the amplitude A will vanish. It means, that the array of resonators with a dissipation completely occludes an incident wave.

R E F E R E N C E S

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