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SOUND FIELD INSULATION BY A PLANE LATTICE OF SMALL SCATTERERS

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The problem of sound wave scattering by a lattice of small inhomogeneties characterized by impedance is considered. The scattered field is obtained as a sum of scattered fields of monopole and dipole. It is shown, that the intensive monopole scattering appeared only when reactive components of impedances of a scatterers and radiation field compensate each other. The insulation properties of a lattice are studied. It is shown, that a scattering lattice, for which one the spatial period does not exceed half of acoustic wavelength, is effective insulator of a sound. Influence of dipole scattering on sound insulation of a lattice is estimated.

It is known [1,2], that some small inhomogeneties in medium (for example, the gas bubbles in a liquid) intensively scatter sound waves, impinging on them. It should be expected, that the lattice of such small inhomogeneties (scatterers) is effective insulator of a sound field. Sound insulation of a plane lattice of identical fixed spheres characterized by an effective impedance Z_0 is calculated below. The impedance Z_0 is equal to ratio of full radial force acted on a sphere, to volume velocity of this sphere. Radius of a sphere is equal a. It is small as compared to acoustic wavelength.

Let lattice coincides with a plane $z = 0$, the scatterers are placed in points with coordinates $x = qL,\ y = sl$, where L and l are periods of a lattice on axes x and y and q, s are integers. Harmonic sound wave with pressure

$$p^{(0)} = A\,exp[\,i(k_x^0 x + k_y^0 y - k_z^{00} z)], \qquad (1)$$

impinges on a lattice from a half-space $z > 0$. Here k_x^0, k_y^0 and $(-k_z^{00})$ are projections of a wave vector of an incident wave on an axis *x*, *y* and *z*, *A* - wave amplitude. A scattered field *p* from a lattice can be derived in the form of sum of scattered fields of monopole $(p^{(1)})$ and dipole $(p^{(2)})$ fields. The monopole scattering is determined by pulsations of scatterers, the dipole scattering is determined by fluid motion about fixed scatterers [1].

Let's designate through V volume velocity of the scatterer located in a coordinate origin $(x = y = 0)$. The volume velocity of the scatterer located in a point with coordinates $(x = qL, y = sl)$ is equal $V\,exp[\,i(k_x^0 qL + k_y^0 sl)]$. Scattered field of a monopole type is equal to a field created by a lattice of monopoles. It satisfies an equation

$$Dp^{(1)} + k^2 p^{(1)} = iwrV\,exp[\,i(k_x^0 x + k_y^0 y)]d(z)\sum_{q,s} d(x - qL)d(y - sl), \qquad (2)$$

where w is frequency of a sound, $k = w/c$ is wave number, r and c are respectively fluid density and speed of sound, $d(z)$ - delta-function. The solution of an equation (2) can be obtained by a Fourier - transformation method. It looks like

$$p^{(1)} = \sum_{m,n} \frac{krcV}{2Llk_z^{mn}} exp[\,i(k_x^m x + k_y^n y \pm k_z^{mn} z)], \qquad (3)$$

where $k_x^m = k_x^0 + m\frac{2p}{L}$, $k_y^n = k_y^0 + n\frac{2p}{l}$, $k_z^{mn} = \sqrt{k^2 - (k_x^m)^2 - (k_y^n)^2}$,

the top and bottom signs are selected respectively at $z > 0$ and at $z < 0$, the summation is made on all integer *m* and *n*. According to the formula (3), the scattered field $p^{(1)}$ consists of homogeneous and inhomogeneous Bragg spectra (plane waves). Spektrum (m,n) is homogeneous wave at $(k_x^m)^2 + (k_y^n)^2 \le k^2$ and is inhomogeneous wave at $(k_x^m)^2 + (k_y^n)^2 > k^2$.

Volume velocity V can be obtained from impedance conditions on scatterers. The structure of a scattered field is determined by the period of a scattering lattice. A field $(p^{(0)} + p^{(1)})$, multiplied on $exp[-i(k_x^0 x + k_y^0 y)]$, is by a periodic function by x with the period L and periodic function by y with the period l. For this reason it is enough to satisfy with boundary conditions on a sphere at an origin. The full radial force applied to this sphere, is equal to $-\int_S [p^{(0)} + p^{(1)}]_{r=a} dS$,

where $r = \sqrt{x^2 + y^2 + z^2}$, the integrating is made on an sphere S. The followed relation is fulfilled on a pulsating sphere $Z_0 V = -\int_S [p^{(0)} + p^{(1)}]_{r=a} dS$.

It can be converted to

$$(Z_0 + Z)V = -\int_S p^{(0)}_{r=a} dS, \tag{4}$$

where $Z = \frac{1}{V}\int_S p^{(1)}_{r=a} dS$ is radiation impedance of a monopole. Taking into account (3) we can get following expressions for real and imaginary parts of radiation impedance

$$R \equiv Re\,Z \approx {\sum_{m,n}}' \frac{krcS_0}{2Llk_z^{mn}}, \tag{5}$$

$$X \equiv Im\,Z \approx -{\sum_{m,n}}'' \frac{krcS_0}{2Ll|k_z^{mn}|} \int_0^{p/2} J_0(k_{xy}^{mn} a \sin q) exp[-|k_z^{mn}| a \cos q] \sin q dq, \tag{6}$$

where $k_{xy}^{mn} = \sqrt{(k_x^m)^2 + (k_y^n)^2}$, $S_0 = 4pa^2$ is sphere area, $J_0(k_{xy}^{mn} a \sin q)$ are cylindrical functions. In the formula (5) summations are made on all m and n, at which k_z^{mn} are real, in the formula (6) summations are made on all m and n, at which k_z^{mn} are imaginary.

In the equation (4) right member is equal approximately $-AS_0$. From this equation it can be found volume velocity of a monopole

$$V = \frac{-AS_0}{\{(R_0 + R) + i(X_0 + X)\}}, \tag{7}$$

where R_0 and X_0 are respectively real and imaginary parts of an impedance Z_0. Substituting V in the formula (3), we get a scattered field $p^{(1)}$. According to the formulas (3) and (7), the intensive monopoly scattering occurs only at a mutual compensation reactive components of impedances Z_0 and Z, i.e. at fulfillment of an equation

$$X_0 + X = 0. \tag{8}$$

For example, for a gas bubble in a liquid it occurs only at its resonance frequency. At fulfillment of (8) and at the absence of absorption $(R_0 = 0)$ in the scatterer the amplitude of a scattered spectrum (m,n) is equal

$$A_{mn}^{(1)} = -A\{{\sum_{q,s}}' \frac{k_z^{mn}}{k_z^{qs}}\}^{-1}. \tag{9}$$

Let spatial periods of a lattice do not exceed half of acoustic wavelength. Then all scattered spectra, except for a spectrum $(0,0)$, are inhomogeneous. The amplitude of a homogeneous spectrum $(0,0)$ is equal $-A$. Consequently it completely compensates an incident wave (1) in a half-space $z < 0$. Thus, at neglect of dipole scattering the lattice completely insulates a sound. In the case of using customary resonators (Helmholtz resonators, gas bubbles in water etc.) the condition (8) is obeyed only at single - resonant - frequency. Consequently, the lattice will be an effective acoustical shield, which will stops a sound only at this frequency. However if to apply electronic control to each monopole, providing fulfillment of a condition (8) in a broad frequency band, the lattice of scatterers will shield a sound in all this frequency band. It is essential, that the electronic control demanded does not depend on a spatial structure of a sound field (waves arrival direction). The control of each scatterer does not depend on control of other scatterers.

Let's calculate a scattered field of a dipole type. It is connected with movement of fluid relatively to fixed scatterers. Each scatterer is equivalent to a dipole with the moment, equal $-2pa^3$ **v**, where **v -** oscillatory velocity of the "frozen" liquid in bulk of this scatterer. As a first approximation velocity **v** in a point with coordinates $(x = qL, y = sl)$ is equal to $\frac{A}{wr}\mathbf{k0}\exp[i(k_x^0 qL + k_y^0 sl)]$, where **k0 -** a wave vector of an incident wave (1). The lattice of dipoles creates a field

$$p^{(2)} = \sum_{m,n} A_{mn}^{\pm}\, exp[\, i(\, k_x^m x + k_y^n y \pm k_z^{mn} z\,)],$$

where

$$A_{mn}^{\pm} = i\frac{pa^3 A}{Llk_z^{mn}}[\, k_x^m k_x^0 + k_y^n k_y^o \mp k_z^{mn} k_z^{00}\,]. \tag{10}$$

The full scattered field p is equal to $(\, p^{(1)} + p^{(2)}\,)$. The amplitude of a spectrum (m,n) is equal to $(\, A_{mn}^{(1)} + A_{mn}^{\pm}\,)$, where $A_{mn}^{(1)}$, $A_{mn}^{\pm}$ are determined respectively under the formulas (9) and (10).

Let spatial period(terms) of a lattice do not exceed a half of acoustic wavelength. Then the amplitude of a scattered homogeneous spectrum $(0,0)$ is equal in a half-space $z < 0$

$$A_{00}^{(1)} + A_{00}^{-} = \{-1 + i\frac{pa^2}{Ll}\frac{(ka)}{k_z^{00}/k}\}A$$

and the relative amplitude of a wave passed through lattice will be a small value

$$(A + A_{00}^{(1)} + A_{00}^{-})/A = i\frac{pa^2}{Ll}\frac{(ka)}{k_z^{00}/k}.$$

REFERENCES

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2. Morse P.M., Ingard K.U. Theoretical Acoustics. McGraw-Hill, New York, 1968.