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INFLUENCING OF LAYER HETEROGENEITY ON DISPLACEMENT ELLIPSES IN RAYLEIGH TYPE WAVE

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The expressions determining major semi-axes of fragment's displacement ellipses in the Rayleigh type wave distributing in an elastic half-space with a stratified liquid layer are obtained. For this purpose the substitution in boundary conditions of the solutions of wave equations for an isotropic medium and inhomogeneous liquid with an exponential alteration of sound speed and density was made. The applicable expressions for a layer are submitted through cylindrical functions, the order and argument which one is determined by layer thickness and value of heterogeneity gradients. Is rotined, that the stratification of layer parameters changes the form and area of ellipses. The major semi-axes and area of ellipses are augmented as contrasted to by homogeneous layer of the same thickness at descending density and sound speed of a liquid in a direction of layer free boundary. The same characteristics decrease at ascending the applicable parameters of a layer. Thus the displacement ellipses in a half-space change unsignificantly. With growth of layer thickness change in the characteristics of ellipses are augmented.

The inhomogeneous layer on a solid surface meets in many practical cases. Such layer will be derived in near-surface area of water soluble crystals located in a moist air, as a result of sorbate processes and dissolution /1/. The similar processes take place in any solid, contacting with vapours of its solvent, and also in gas sensors, a countermeasure feeler which one is the sorbent film, marked on a solid-state substrate /2/. The layers with vertical gradients of a sound speed exist in water weights of the seas and oceans. Thus the stratification of a sound speed renders essential influencing on pattern of sound fields of a point source and propagation of normal waves in a underwater sound channel /3-9/. The non-uniformity of a layer changes phase and formation speeds of surface acoustic waves (SAW), distributing in bedded structure a solid with a layer /10-12/. Accordingly should change and amplitude of fragment displacement. Thus in a Rayleigh type wave (RTW), for which one the displacement of fragments are elliðtiñly polarized, the non-uniformity of a layer should induce deformation of ellipses. In the present activity influencing stratification of density and sound speed in a layer on ellipses of fragment displacements in a RTW field is studied.

Let RTW propagates along a positive direction of an axis x, the liquid layer takes area $-h < z < 0$, and the isotropic half-space - area $z > 0$. Since a case of the stratified layer can theoretically be investigated only at some private distribution laws of parameters /13 /, the layer is supposed liquid with density and sound speed which is changing with the depth on the laws:

$$\rho = \rho_0 \, exp(\alpha z), c = c_0 \, exp(\beta z), \qquad (1)$$

where ρ_0 and c_0- density and sound speed in a liquid on boundary with a half-space (in a plane z=0). Then expression for sound pressure of a liquid in the case of harmonic relation from x and t at $\beta \neq 0$ looks like /14/:

$$P = [C_1 J\nu (Y) + C_2 N\nu (Y)] \, exp(-\alpha z/2) \, exp\, i(\omega t - kx), \qquad (2)$$

where $J_\nu(Y)$ and $N_\nu(Y)$ – Bessel and Neumann functions, C_1 and C_2- arbitrary constants, k - wave number of RTW, $\nu = \sqrt{\dfrac{4k^2 + \alpha^2}{4\beta^2}}$, $Y = \dfrac{k_0}{|\beta|} exp(-\beta z)$, $k_0 = \dfrac{\omega}{c_0}$.

The horizontal and vertical displacement components in a liquid are determined accordingly by expressions /15/:

$$W_x = \frac{1}{\omega^2 \rho} \frac{\partial P}{\partial x}, \; W_z = \frac{1}{\omega^2 \rho} \frac{\partial P}{\partial z}. \qquad (3)$$

Substituting (2) in (3) and allowing reduced in /12/ ratio between arbitrary constants, after foolproof, but cumbersome conversions, we shall receive:

$$W_x = A\frac{kqk_t^2 D_1}{(k^2+s^2)D} exp(-\alpha z/2) exp\, i(\omega t - kx - \pi/2), \quad (4)$$

$$W_z = A\frac{qk_t^2 D_2}{(k^2+s^2)D} exp(-\alpha z/2) exp\, i(\omega t - kx), \quad (5)$$

where $q=\sqrt{k^2-k_l^2}$, $s=\sqrt{k^2-k_t^2}$, k_l and k_t - wave numbers of longitudinal and shift waves in an isotropic medium, $D=\frac{\alpha}{2}+\nu\beta-\frac{\beta k_0}{|\beta|}\frac{N_v(Y_1)J_{v-1}(Y_0)-J_v(Y_1)N_{v-1}(Y_0)}{J_v(Y_0)N_v(Y_1)-J_v(Y_1)N_v(Y_0)}$,

$D_1=\frac{N_v(Y_1)J_v(Y)-J_v(Y_1)N_v(Y)}{J_v(Y_0)N_v(Y_1)-J_v(Y_1)N_v(Y_0)}$,

$D_2=\left(\frac{\alpha}{2}+\beta\nu\right)D_1+\beta Y\frac{J_v(Y_1)N_{v-1}(Y)-N_v(Y_1)J_{v-1}(Y)}{J_v(Y_0)N_v(Y_1)-J_v(Y_1)N_v(Y_0)}$, $Y_0=\frac{k_0}{|\beta|}$, $Y_1=Y_0\, exp(\beta h)$.

The wave number k contents to a dispersion equation /12/:

$$4k^2qs-(k^2+s^2)^2=\frac{\rho_0 qk_t^4}{\rho D}, \quad (6)$$

where ρ - density of an isotropic medium. Thus in a considered case principal axes of displacement ellipses are determined by the expressions (4) and (5). As to analyze them in an analytical kind it is not obviously possible, their analysis was conducted on the basis of outcomes of numerical calculations, which one were executed for following ratio of parameters of a solid and layer $\frac{\rho}{\rho_0}=2{,}5$; $\frac{k_t}{k_0}=1/3$; $\frac{k_l^2}{k_0^2}=1/27$; $\frac{|\alpha|}{k_0}=\frac{|\beta|}{k_0}=1/6$. Such α .value corresponds to change of layer parameters approximately in 3 times apart equal λ_0 (λ_0 - wavelength in a liquid layer on boundary with a half-space). The calculation of values of ellipse axes was conducted in relative amplitudes $\overline{W}_{x0}=W_{x0}/U_{z0R}$, $\overline{W}_{z0}=W_{z0}/U_{z0R}$, where U_{z0R} - displacement amplitude in the Rayleigh wave in a plane z=0 at h=0 and k=k_R, k_R - solution of an equation (6) with a zero right part. For an isotropic half-space of an ellipse axes are determined also, as in the Rayleigh wave /16/. Vertical coordinate and thickness of a layer also expressed in relative values: $\bar{z}=z/\lambda_0$ and $\bar{h}=h/\lambda_0$. For matching the ellipses for a case of a homogeneous layer $(\alpha=\beta=0)$ with density ρ_0 and sound speed of c_0 are resulted.

In a fig. 1 the ellipses of displacement are adduced at relative thickness of a layer $\bar{h}$ = 0,2; 0,3; 0,4 for cases $\alpha=0$, $\alpha>0$ and $\alpha<0$. It is visible, that at $\bar{h}$ =0,2 non-uniformity of a layer practically do not change ellipses. It is conditioned by that at small thickness of a layer stratification of density and the sound speed cause changes, opposite on the sign, of the RTW characteristics /17/. From a fig. 1 follows, that at $\alpha>0$ because of descending a wave drag of a layer in a direction of free boundary there is an increase of ellipse axes as contrasted to by homogeneous layer and their decreasing at $\alpha<0$. The distinction difference in value of axes increases in a direction of free boundary of a layer and is augmented with increase of layer thickness. On free boundary of a layer (z =-h) the fragment displacement are linearly polarized in bridge axes z, that is conditioned by the applicable boundary conditions and takes place as for homogeneous, and inhomogeneous layer.

The stratification of layer parameters changes also ellipse area. In a fig. 2 the charts of relation of the area from relative vertical coordinate $\bar{z}$ are adduced at different thickness of a layer. At $\alpha>0$

the ellipse area is augmented, and at $\alpha < 0$ decreases as contrasted to with homogeneous layer. The maximum ellipse area at $\alpha = 0$ takes place in middle of a layer (*z* =-*h*/2). At $\alpha > 0$ this maximum displaces in a direction of free boundary of a layer, and at $\alpha < 0$ - in the counter party. It testifies that the non-uniformity of a layer causes reallocating of wave energy inside a layer: in a case $\alpha > 0$ there is "swapping"("transfer") of energy of a surge to free boundary of a layer, and in a case $\alpha < 0$ - in the opposite direction. Thus the ellipses of displacement in a half-space change insignificantly.

Therefore, the non-uniformity of a layer changes the form and area of displacement ellipses of fragments in RTW. These changes are augmented with increase of layer thickness and depend on the sign of gradients of layer parameters. The non-uniformity results also in essential reallocating of wave energy inside a layer and minor between a layer and substrate.

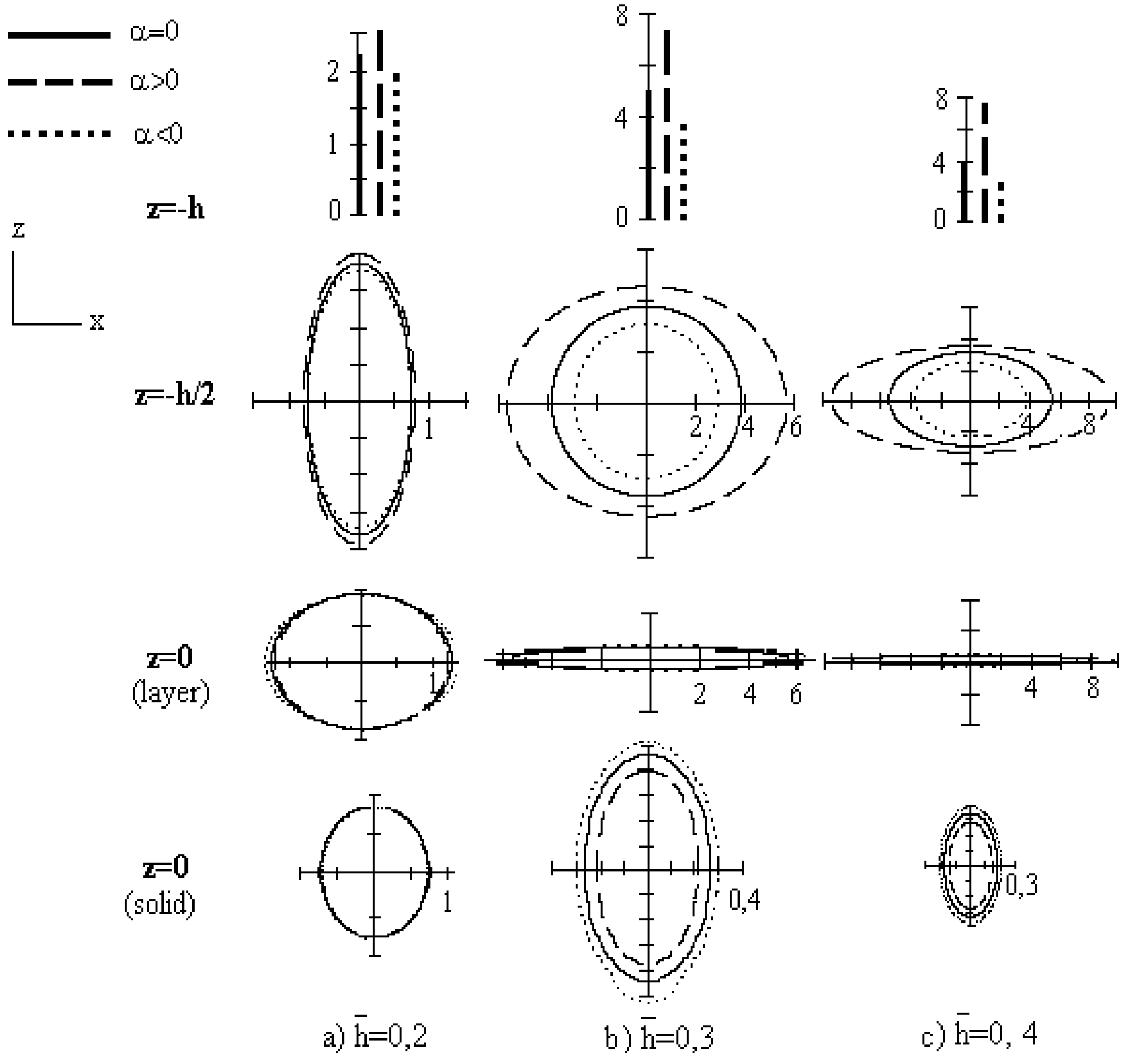

Fig. 1. Ellipses of displacement at different thickness of a layer for cases $\alpha = 0$, $\alpha > 0$ and $\alpha < 0$.

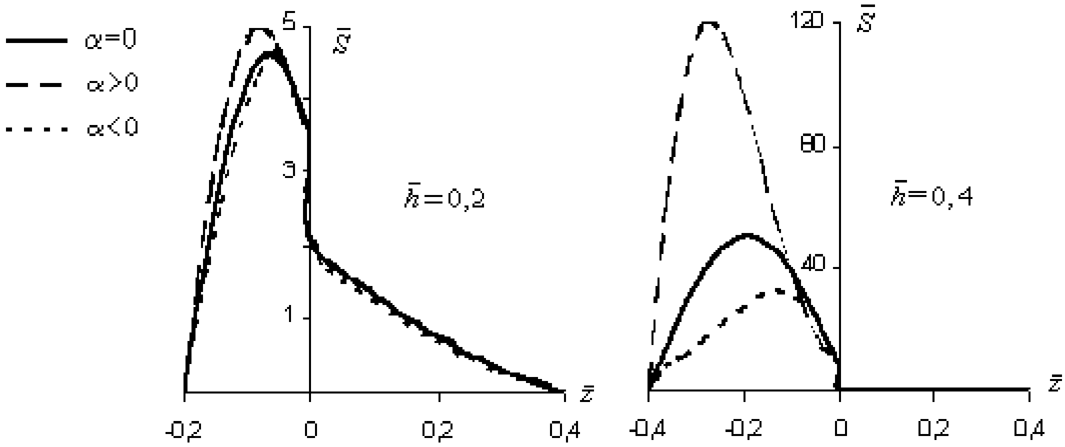

Fig. 2. Relation of the area of an ellipse to relative vertical coordinate $\bar{z}$.

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