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CALCULATION OF A PULSED WAVE FIELD DIFFRACTION ON THE STRONGLY CURVED BOUNDARY OF HALF-SPACE

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The regularization method, which allows to simulate the scattering of pulsed wave fields on curvilinear surfaces with, practically, arbitrary geometry is developed. Mathematically the essence of the method consists in a replacement of exact integral Fredholm's equations for wave field on the boundary by their truncated analogs, in which the contributions of geometrically shadowed areas are eliminated. Nevertheless this method has a deep physical sense and permits to obtain correct solutions when the direct numerical resolving of the mathematically exact equations is an unstable.

There is a problem of discrimination low powerful explosions (<1 kt) and earthquakes at regional distances less then 500 km. The seismic signals from these events are affected by essential influence of geological structure of the Earth, providing rather their complicated waveforms. At the same time the pulsed sources have a wide frequency spectrum and corresponding content of wavelengths, that does not allow to use the only the ray approximation or to neglect by topographic irregularities of free surface in comparison with wavelength. The solution of the similar tasks can be reduced to the solution of the integral Fredholm's equations of the second kind for the source densities distributed on a closed curvilinear surface.

The two-dimensional problem of scattering a scalar wave field on the curvilinear boundary of half-space is considered in the report. This statement from the physical point of view corresponds to an acoustical approximation for signal scattering at the curvilinear boundary of elastic media.

Using the Green's theorems it is possible to obtain the expression for wave field in internal points of medium and through the values of its normal derivative on the curvilinear boundary.

$$U(\vec{r}) = U_0(\vec{r}) + \int_S dS \left\{ U(\vec{r}_S)\frac{\partial}{\partial n_S}G(\vec{r},\vec{r}_S) - G(\vec{r},\vec{r}_S)\frac{\partial}{\partial n_S}U(\vec{r}_S) \right\} \qquad (1),$$

Where the following notations are introduced for the Green,s function of a free space $G(\vec{r},\vec{r}_S)$, and for wavefield at the surface $U_0(\vec{r}\,')$ considered as the source densities.

For the case of the Dirichlet's boundary condition it is possible to obtain from (1) the Fredholm's equation of the second kind for the normal derivative of wave field $V(\vec{r}_S) = \frac{\partial}{\partial n_S}U(\vec{r}_S)$ on the curvilinear surface.

$$V(\vec{r}) = 2V_0(\vec{r}) - 2\int_S dS\, V(\vec{r}_S)\frac{\partial}{\partial n}G(\vec{r},\vec{r}_S) \qquad (2.1)$$

For the case of Neuman's boundary condition ($\left.\frac{\partial}{\partial n_S}U(\vec{r})\right|_{\vec{r}=\vec{r}_S} = 0$) the following Fredholm's equation of the second kind can be derived from (1) for wavefield on the boundary:

$$U(\vec{r}) = 2U_0(\vec{r}) + 2\int_S dS\, U(\vec{r}_S)\frac{\partial}{\partial n_S}G(\vec{r},\vec{r}_S) \qquad (2.2)$$

The integrals by surface in (2) have to be considered in the sense of its principal values.

It is possible to obtain from (1) the diagram of radiation pattern, which describes angular distribution of pulsed wavefield. For the case of the Dirichlet's boundary condition it has a view:

$$W(j)=1+\tfrac{i}{4}\int_S dS\; e^{-ik\left|(\vec{r}_s-\vec{r}_0)\right|\cos(j-j_s)}\; V(\vec{r}_S) \qquad (3.1)$$

Similarly for the case of the Neuman's boundary condition one can be written.

$$W(j)=1+\tfrac{i}{4}\int_S dS\; \cos(j-j_n)e^{-ik\left|(\vec{r}_s-\vec{r}_0)\right|\cos(j-j_s)}\; U(\vec{r}_S) \qquad (3.2)$$

The numerical modeling of a wave field scattering by a curvilinear surface is usually based on integral equations mentioned above. Thus, the problem is reduced to solution of the equations (2). Then, substituting the obtained solution for the field on the surface or its normal derivative into relation (1), it is possible to calculate a wave field in any point of a half-space with the curvilinear boundary or to determine the radiation pattern (3).

The numerical solution of the equations (2) can be obtained by the iteration method or by the method of finite-difference approximation of the integral equations (2), reducing them to a system of the linear algebraic equations for wavefield in the certain boundary grid nodes. For the enough curved surface the contribution of multiply scattered waves becomes to be essential, and the iterative method has a slow convergence and requires the long time for calculations. From the other hand the method of integral sums requires the large machine resources to work with large dimensional matrixes.

For enough smoothed irregularities of the surface the solution of the integral equations does not bring any difficulties. For example, the solution of the problem on the pulsed reflected wavefield, generated by explosion under mountain of gaussian shape (fig.1) with height to width ratio equaled to 0.5, is shown in fig.2.

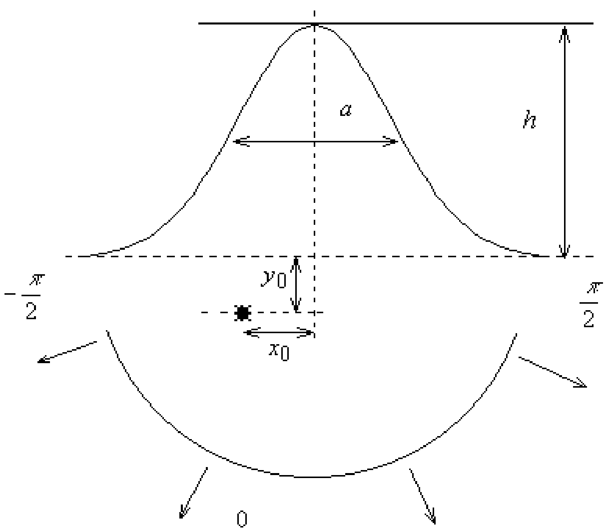

Fig. 1. The problem geometry.

However, in case of the large slopes and curvatures of the surface irregularities in comparison with characteristic wavelength, the described methods became to be unstable and result to incorrect solutions. It occurs because for large slopes and curvatures of irregularities the multiply reflected on the surface waves begin to give essential contribution and there is also the contribution of mutually shadowed points of the surface. This contribution is cancelled in different orders of multiple scattering series corresponding to different orders of iterations [1]. Therefore the iterative process gives an incorrect result. To overcome this difficulty we suggest to eliminate certainly the contribution of mutually shadowed points of the surface that corresponds to the physics of the real wave field propagation.

If, following to [1], to estimate a solution of the problem of plane wave field scattering on an arbitrary curvilinear surface by the stationary phase method in the short-wave approximation, then after the first iteration it is possible to see, that the field is equal to the field of external source and the sum of contributions of all stationary phase points. Further there are two types of stationary phase points: the reflecting one and shadowed one. The contribution of shadowed stationary phase points is cancelled at the next iteration. Therefore at any finite number of iterations it is not possible to achieve the exact solution. This contribution becomes to be important at account of contribution of multiply reflected waves.

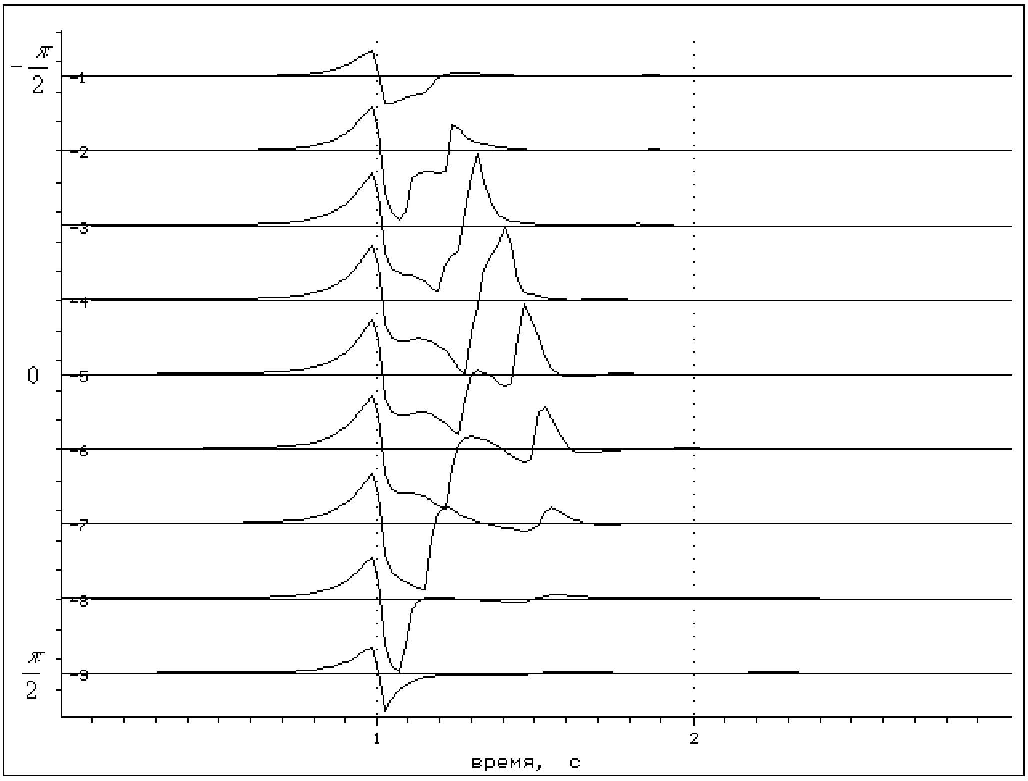

Fig. 2. The pulsed radiation diagram for the explosion under mountain in geometry is shown in fig.1 for $h/a = 0.5$.

The suggested regularization method is based on the approach when at solution of the integral equations the contribution of the source is taken into account only for the geometrically visible from the source areas, and the integration is fulfilled only on the mutually non-shadowed areas of the surface. To take into account the wavefield diffraction the visible areas were extended on a value proportional to wavelength. Thus, the contribution of mutually shadowed points of the surface is eliminated in the integral equation that corresponds to the physical picture of the real wavefield propagation. For more accurate description of the field currents in the light - shadow vicinity the approach suggested by Fock in [2] is used. Because the stationary phase points are closed one to another in the light - shadow vicinity, the wavefield in this area has a local character. It was shown in [2], that for arbitrary surface the field in the light - shadow vicinity is proportional to universal Fock's function, for which there are tables and asymptotic representations.

It is also shown in the report, that represented regularization method can be correctly derived from a mathematical point of view with use of the exact integral equations. The developed approach has allowed us to obtain the correct solutions for surfaces with arbitrary large slopes and curvatures of irregularities. For example, the solution for the problem of explosion under mountain of gaussian shape with height to width ratio equaled to 3 is shown in fig. 3, which can not be calculated using the traditional approaches.

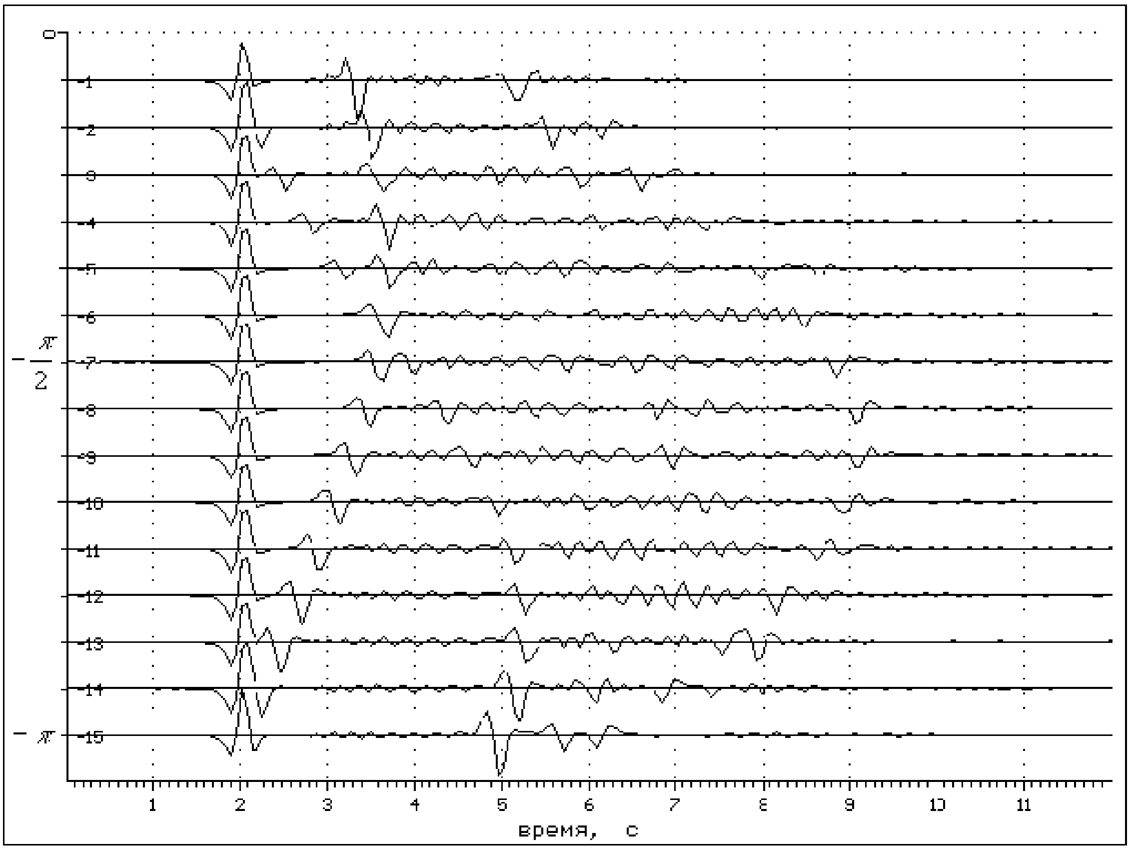

Fig. 3. The pulsed radiation for the explosion under mountain in geometry is shown in fig.1 for $h/a = 3$.

R E F E R E N C E S

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