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# Model of a cavitational cluster in the stator channel of the hydrodinamic – syren type rotary apparatus

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*The model of behavour of a cavitational cluster in the channel of the hydrodinamic – syren type rotary apparatus is constructed. The mathematical model of a cavitational cluster bases on the modified non-stationary Bernoulli equation, which describes the law of stator pulse change , generated in the channel, of the apparatus pulse of acoustic pressure at opening - closing of channels of a rotor and stator of the apparatus, and on the wave equation describing change of pressure in the channel of a stator at the blocked channels.*

The approached mathematical model of dynamics cavitational bubble in the stator channel of the a hydrodynamic -syren type rotary apparatus was offered in work [1]. The specified model of changes cavitational bubble in the rotary device is offered in this work. The method of account of generated acoustic pressure in the channel of a stator of the rotary apparatus as functions of time is developed.

The form and amplitude of pressure pulse depends from kinematic parameters of a liquid flow and wave processes in the stator channel. The period of a cycle' change of pressure in the stator channel can be divided into two characteristic time intervals $\Delta t_1$ and $\Delta t_2$. In the first interval of time there is overlapping the rotor channel to the stator channel, and in second - the stator channel is blocked by a wall of a rotor. The time intervals are determined accordingly as $0 < t_1 \le \frac{a_{\text{ñ}} + a_{\text{ð}}}{\omega \cdot R_{\text{ð}}}$, $\frac{a_{\text{ñ}} + a_{\text{ð}}}{\omega \cdot R_{\text{ð}}} < t_2 \le \frac{a_{\text{ñ}} + b_{\text{ñ}}}{\omega \cdot R_{\text{ð}}}$,

where: $a_{\text{ñ}}$, $a_{\text{ð}}$ - width of the stator or rotor channel, ì; $b_{\text{ñ}}$ - distance between the nearest walls of the next stator channels, ì; $\omega$- angular rotor speed, $c^{-1}$; $R_{\text{ð}}$ - external radius of a rotor, m. The kinematic parameters of a liquid flow in the stator channel for an interval of time $\Delta t_1$ are determined at the decision of the non-stationary Bernoulli equation [2]:

$$\beta \frac{l_{\text{ý}} \cdot dV}{dt} + \frac{V^2}{2}\left(\lambda \cdot \frac{l_{\text{ý}}}{d_{\text{ý}}} + \xi(t) + B(t) \cdot \frac{\mu}{\rho d_{\text{ý}} V} + k_V\right) = \quad (1)$$
$$= \frac{\Delta P}{\rho} + l_{\text{ð}} \omega^2 \left(R_{\text{ð}} - \frac{l_{\text{ð}}}{2}\right) + \omega^2 \left(R_{\text{ð}} - l_{\text{ð}}\right)^2 \cdot (1 - \varphi)$$

where: $k_V = \frac{2 \cdot \left( \Delta P / \rho + l_{\text{ð}} \omega^2 \left(R_{\text{ð}} - l_{\text{ð}}/2\right) + \omega^2 \cdot \left(R_{\text{ð}} - l_{\text{ð}}\right)^2 \cdot (1 - \varphi)\right) \cdot k_1}{V_{max}^2}$; $d_{\text{ý}} = \frac{2 \cdot a_{\text{ñ}} \cdot h}{a_{\text{ñ}} + h}$;

$l_{\text{ý}} = (l_{\text{ð}} + l_{\text{ñ}} + (1 + \tilde{A})\sqrt{\frac{a_{\text{ñ}} \cdot h}{2\pi}}) + \delta$; $k_1$=0.9÷0.99– factors which determine size of pressure reduction difference on the interrupter; $\varphi$=0,0÷1,0- factors of slippage, dependent from Reynolds',Strouhal's criteria, physical parameters of a liquid and boundary conditions; $k_V$ - factor of a high-speed pressure; $V_{max}$ - the maximal value of flow speed of a liquid through the interrupter at overlapping the rotor channel with the stator channel, ì/s; $d_{\text{ý}}$ - an equivalent diameter of the stator channel, ì; $l_{\text{ý}}$ - equivalent length of the apparatus interrupter, ì; $\beta$=0.1÷0.2- factors of the movement quantity determined experimentally; $\Delta P$ - only static difference of pressure, Pà; $t$ - time, s; $V$ - speed, ì/s; $l_{\text{ð}}$ - length of the rotor channel , ì; $l_{\text{ñ}}$ - length of the stator channel, ì; $\xi$- factor of local hydraulic resistance; $\mu$- dynamic viscosity of a liquid,Pà c; $\rho$- density of a liquid, êg/ì$^3$; $\lambda$- factor of resistance to friction; $B$ - factor of hydraulic resistance which takes into account losses of pressure, linearly dependent from speed of a flow; $h$ - height of the stator channel, ì; $\tilde{A}$ - a square root from the attitude of the section area of a target site of the stator channel to the area of section of rotor channel entrance site; $\delta$- size of a clearance between a rotor and stator, m. In the equation (1) the reduction of pressure difference in this interval of time, slippage of a liquid concerning a rotor in absence of vanes in a rotor, centrifugal pressure created by channels of a rotor is taken into account. The pressure generated in the stator channel, is determined by the formula [2]:

$$P_1(t)=\rho\,\frac{dV}{dt}\left(\frac{S_{\max}}{2\pi}\right)^{0.5},\ (2)$$

where $S_{max}=a_{ñ}{}^{x}h$.

The decision of the differential equation (1) is possible by numerical integration. The specification of factor of a high-speed pressure $k_V$ occurs iterating. Method of the equation (1)decision is described in detail in works [3, 4].

After the expiration of a time interval $\Delta t_1$ the wave processes in the stator channel continue to exist. Change of pressure for interval $\Delta t_1$ is defined on the basis of wave equation [5] and enters the name as line Fourie as:

$$P_2(t)=\sum_{n=0}^{\infty}\left[\frac{8\cdot P_{\min}\cdot\exp(-k\cdot t)}{\pi^2\cdot(1+2n)^2}\cdot(-1)^n\cdot\sin\left(\frac{\pi\cdot(1+2n)}{2}\right)\cdot\cos\left(\frac{c}{l_{ñ}}\cdot\frac{\pi}{2}\cdot(1+2n)\cdot t\right)\right],\ (3)$$

Where: $c$ - is recommended to be determined sound speed in gas-liquid conditions ì/s ( sound speed in gas-liquid conditions we should determine using the methods offered in work [6]); $P_{min}$ - the least meaning of generated pressure designed on the differential equation (1) and formula (2), Pà; $k$ - factor of amplitude dissipation of a wave.

For mathematical accounts it is more convenient to choose not border of two time intervals $\Delta t_1$ and $\Delta t_2$, but moment of time $t'$, laying inside an interval of time $\Delta t_1$. The moment of time $t'$ corresponds to the least meaning of generated pressure $P_{min}$, which causes oscillatory processes in the stator channel. Thus, the function describing laws of pressure change on an input in the stator channel has a kind:

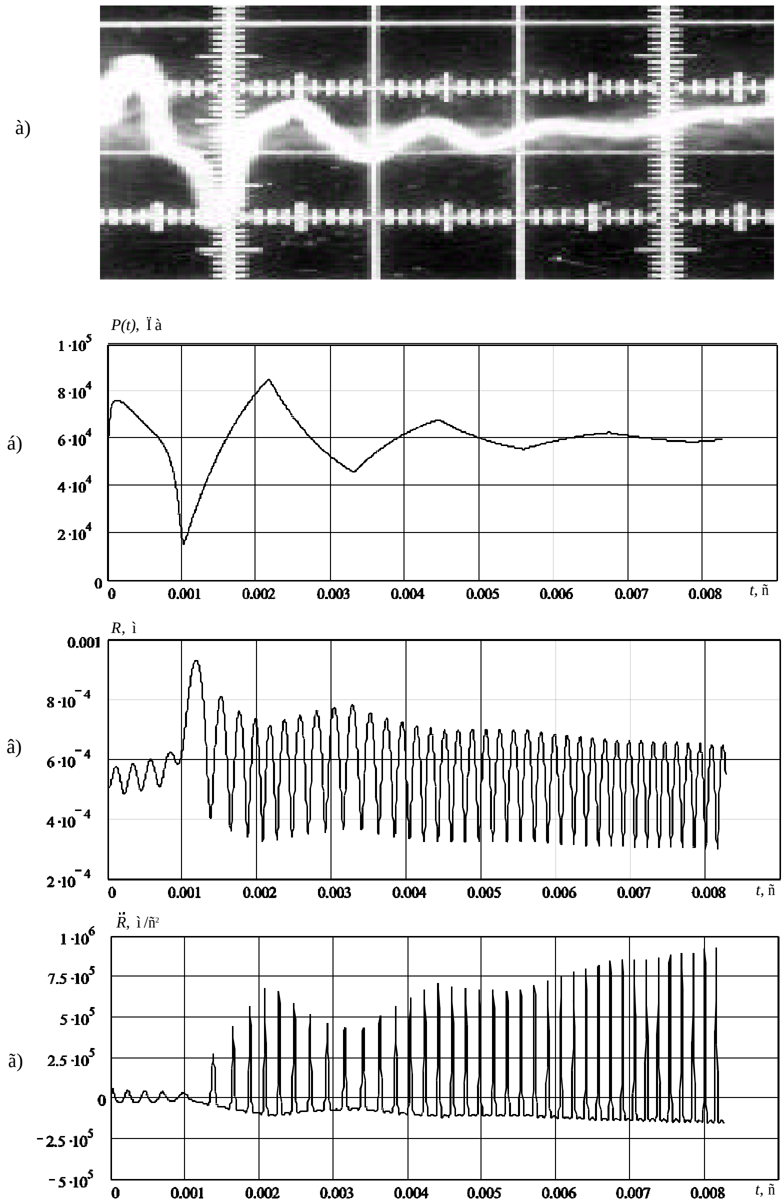
$$P(t)=\begin{cases}P_1(t)\ ,\ 0\le t<t'\ ;\\ P_{\min}\ ,\ t=t'\ ;\\ P_2(t)\ ,\ t'<t\le\dfrac{b_{ð}-a_{ñ}}{\omega\cdot R_{ð}}\ .\end{cases}\qquad(4)$$

Cavitational bubble dynamics is described by Noltingk-Neppiras equation [7]: for cavitational phenomena in stator channel it is written down like this:

$$R\cdot\frac{d^2R}{dt^2}+\frac{2}{3}\cdot\left(\frac{dR}{dt}\right)^2+\frac{1}{\rho}\cdot(P_{ñòàò}-P_{ï}+P(t)+P_{êàâ}+\frac{2\cdot\delta}{R}+\frac{4\cdot\mu}{R}\cdot\frac{dR}{dt}-\left(P_{ñòàò}+\frac{2\cdot\delta}{R_0}\right)\cdot\left(\frac{R_0}{R}\right)^{3\gamma})=0\ ,\ (5)$$

Where: $R_0$, $R$ - initial and current radiuses of cavitational bubble, ì; $Pcòàò$ - static pressure in the stator channel, Pà; $P_{ï}$ – pressure of sated vapour , Pà; $Pkàâ$ - secondary cavitational pressure [8], Ïà; $\sigma$ - a superficial tension, Pà/ì; $\gamma$- a parameter of a polytrack.

The analytical decision of the differential equation (5) is possible only for the elementary cases, therefore equation (5) is solved by numerical methods. On picture 1 we submit dependence of change of acoustic pressure in the stator channel of the rotary device, experimental oscilograme of acoustic pressure change in the stator channel, change of bubble radius and acceleration of a bubble wall.




***Picture1***. Kinematic and dynamic dependences:
à) Experimental oscillogram of change of generated acoustic pressure in the stator channel of the rotary device; b) dependence of change of acoustic pressure designed on (1) - (4); ñ) dependence of radius change of cavitational bubble on (5); d) dependence of acceleration change of a cavitational bubble wall on (5).

## R E F E R E N C E S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