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HOW A GAS BUBBLE PULSATES

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Numerical experiments with gas bubbles, forced to pulsate by the harmonic acoustic wave, have been made. While the wave amplitude increases the subharmonics appears in pulsations spectrum. Further increasing cause nonperiodic pulsations. Bubbles with solid nuclei behave the same way. Only in extremely viscous liquids there are no subharmonics and nonperiodical pulsations. If the bubble is squeezed in a crevice, pulsations pass all these stages as well. Influence of different terms of differential equation on the spectra was cleared up.

Investigations on cavitation noise spectra shows that there is subharmonic with the frequency of one half of the driving wave which appears just before cavitation starts. This phenomenon was also proved by a lot of pulsing bubbles spectra calculations (see, e.g. [1]).

Appearance of oscillations with the frequency of one half of driving force can be explained by the fact that a bubble is restricted in his radius decreasing, but has enough place to expand. When the driving force amplitude increases, the bubble expands more and more; expanding phase becomes longer and longer and at last the compression phase skips over next driving force period [2].

Apart from bubbles filled with gas only, solid particles can be in cavitation zone, too, surrounded by gas layer and also bubbles, squeezed in cracks. To explore the features of gas layer oscillations the numerical calculations have been made for pulsing of spherical bubble with initial radius R_0 , containing solid nucleus with radius R_1.

To describe the oscillations the Herring-Flynn equation was used.

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 + \frac{1}{\rho}\left[\left\{P_a - P_v + \frac{2\sigma}{R}\right\} - \left\{P_a - P_v + \frac{2\sigma}{R_0}\right\}\left\{\frac{R_1^3 - R_0^3}{R^3 - R_0^3}\right\}^{\gamma} + P_m \sin \omega t\right] +$$

$$+ \frac{4\nu\dot{R}}{R^2} + \frac{\dot{R}}{R\,\rho\, c_0}\left[3\gamma\left(P_a - P_v + \frac{2\sigma}{R_0}\right)\left(\frac{R_1^3 - R_0^3}{R^3 - R_0^3}\right)^{\gamma} - \frac{2\sigma}{R}\right] = 0 .$$

From the calculations of bubble radius versus time dependencies we sampled one dot for every period (like [3]), corresponding to the maximum forcing pressure moment. If oscillations are periodical, with the forcing pressure frequency, there is only one curve on the diagram. After one half frequency appearance, graph was bifurcated like plant’s twig.

Diagram of radius oscillations for bubble containing solid nucleus is presented on Fig. 1. Here $R_1 = 0.9R_0$, $R_0 = 10^{-5}$ m. Bubble driven at its resonance frequency and slowly increasing driving pressure amplitude – from p_m = 1 atm to 3 atm during 20000 periods. After certain transient process the bifurcations occurred at 1.4 atm, and at 2.2 atm nonperiodic, chaotic response started.

The same process takes place for very thin gas layer, $R_1 = 0.999R_0$, Fig. 2, except to the earlier chaos beginning. And only in a very viscous fluid, in glycerine, there were neither bifurcations, nor chaos up to the experimentally achievable pressure figures (Fig. 3).

In another two cases – when the bubbles are so small that surface tension play the main role or in the case of viscousity absence – the oscillations pass through the same stages. Bifurcations before, chaos after.

If we shall analyze oscillations of an air bubble with x_0 = 0.001 m height squeezed in a narrow cylindrical crevice of H = 0.1 m depth, r_0 = 0.0001 m radius and take into account only variability of

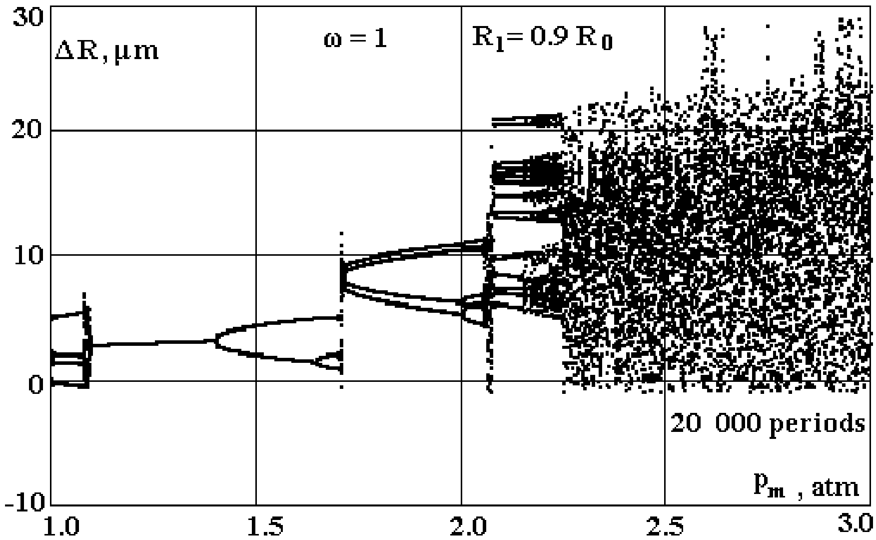

Fig. 1.
Bifurcations and chaos.

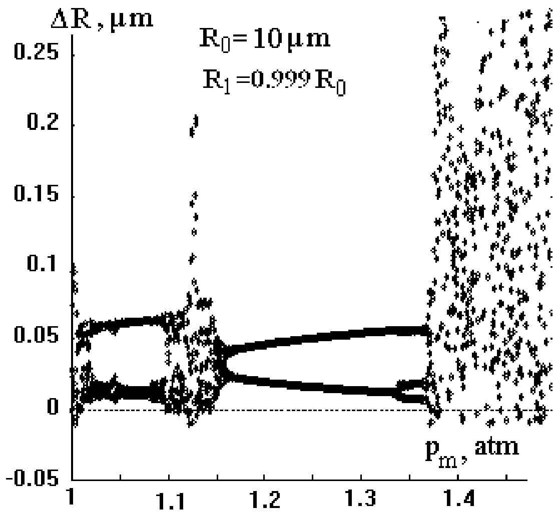

Fig. 2. Thin layer.

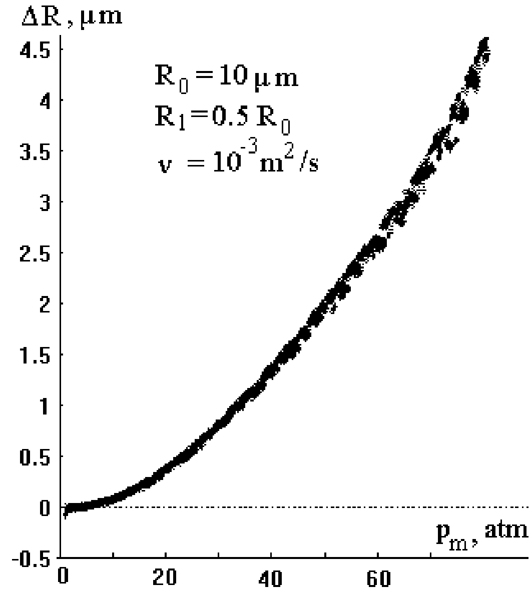

Fig. 3. Viscous liquid.

mass into the crevice and the adiabatic behavior of gas into the bubble, we derive following nonlinear equation

$$(H-x)\ddot{x} - \frac{p_0}{\rho}\left(\frac{x}{x_0}\right)^{-\gamma} + \frac{1}{\rho}\left(p_0 + p_m \sin \omega t\right) = 0 .$$

Three-dimensional spectra for the bubble boundary pulsations in crevice have been calculated. It was turned out that this equation solutions pass through the same stages as solutions for spherical bubbles pulsations. One of these three-dimensional diagrams of the squeezed bubble wall displacement is shown as Fig. 4 versus increasing forcing pressure, which vary from p_m = 0.2 atm to 2.0 atm. Dimensionless forcing pressure frequency f = 1 corresponds to the bubble resonance frequency. Only low-frequency part of the spectra is shown.

It is distinctly seen that at the forcing pressure p_m = 1.2 atm subharmonic with f = 1/2 occurs in the oscillations spectra. Then, at the pressure p_m = 1.4 two subharmonic bifurcation occur with additional frequencies 1/3 and 2/3. These are followed at 1.8 atm by a seemingly chaotic response with nonperiodic oscillations and continuous spectrum.

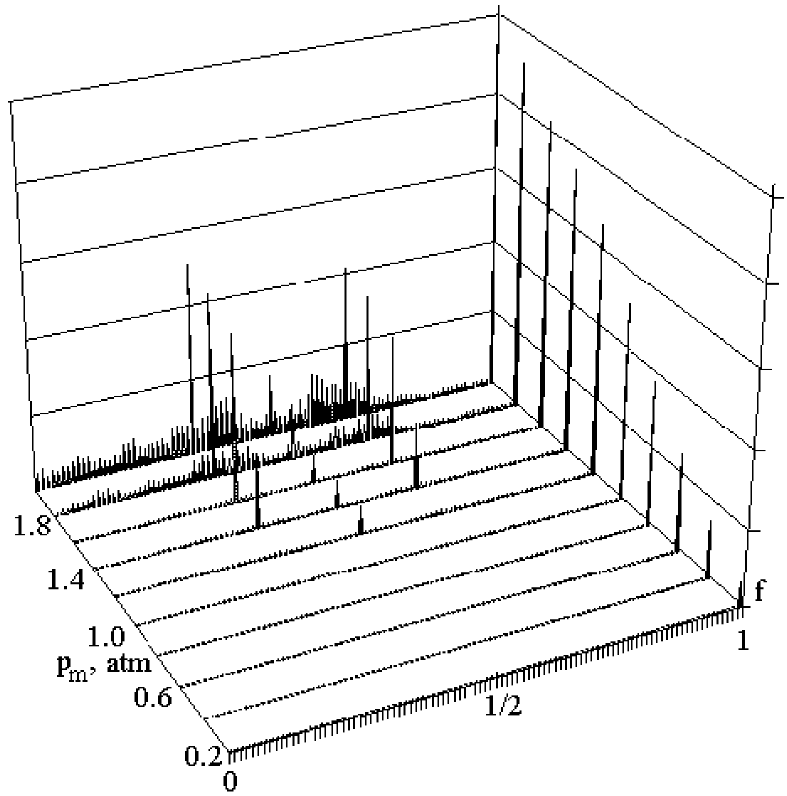

Fig. 4.
Bubble in crevice.

Compared with spherical bubbles oscillations there were numerical values changes, at which subharmonics appears and the spectrum becomes continuous one and oscillations – nonperiodic.

There are two nonlinear terms in the equation of oscillations. To make clear which term causes such type of solutions we replace the second term by linear one, i.e. we assume *g* = 1. Result is shown on Fig. 5.

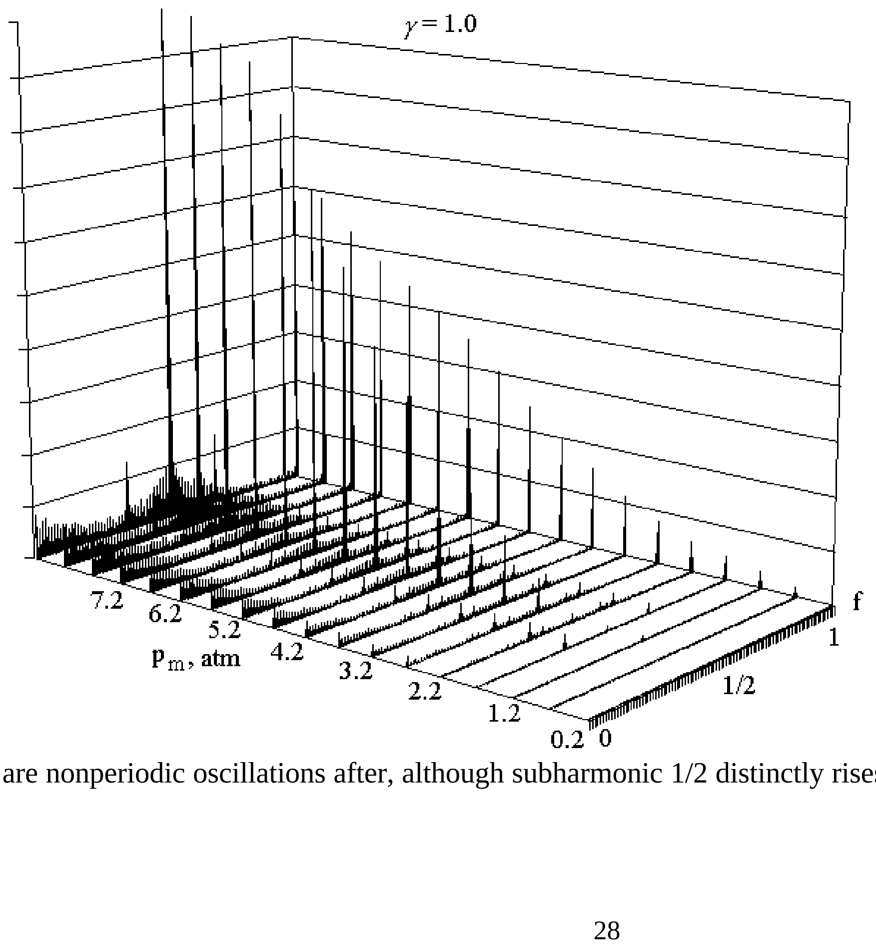

Fig. 5.
Linear second term.

Exactly as on the previous diagram at the pressure 1.2 atm subharmonic 1/2 occur. Then – 1/3 and 2/3. There are nonperiodic oscillations after, although subharmonic 1/2 distinctly rises above the noise spectrum.

However the most interesting result came in after moving away the first nonlinear term of equation $(x \cdot \ddot{x})$, which responds for the variable mass of liquid in crevice. We assumed that the mass of the liquid was constant and oscillations nonlinearity in this case were caused by gas adiabatic nonlinearity only.

Spectra for the case are shown on Fig. 6 for acoustic wave pressure values 0.2 atm to 10.2 atm. Exactly as in the previous calculations at the forcing pressure 1.2 atm a subharmonic 1/2 occurs. After that the spectrum becomes a continuous one but it consists mainly of low frequency noises.

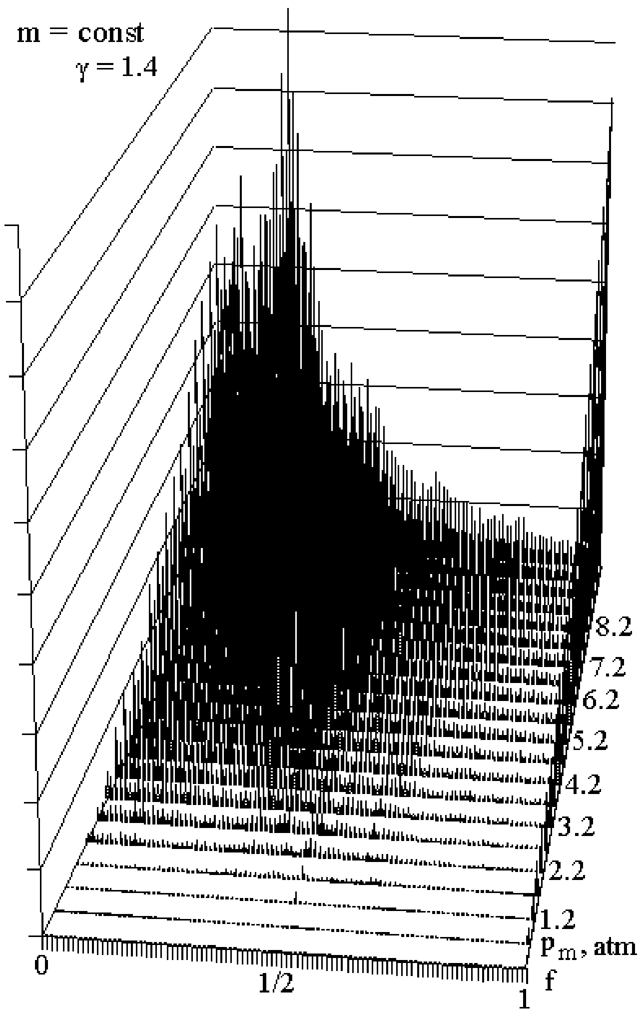

Fig. 6.
The case of the constant mass of the liquid. Nonlinearity caused by gas adiabatic only.

As a result of our numerical experiments we can resume that in water or even in two order more viscous liquids, with bubbles of different shape, which pulses under harmonic forcing pressure, it is subharmonic 1/2 that occurs in spectrum just before the cavitation noise with continuous spectrum.

R E F E R E N C E S

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