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THE DROPLET MOTION IN A CONIC CAPILLARY UNDER THE ACTION OF VIBRATION

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The translation drop movement in a conic capillary under action of vibration is studied. This problem is interesting in connection with the analysis of efficiency of acoustic vibrating influence on a porous medium with the aim of permeability increasing. In approximation of thin drop the equation of its motion is obtained. This one was solved by numerical methods. It is shown that in the presence of gravity force acting along the conic axis the drop begin to move. From the physical point of view this effect is caused by the existence of contact angle hysteresis. The different examples of droplet motion are examined.

In the report the drift motion of separate drops in conic capillaries under action of vibration is studied. Earlier this problem was examined for a cylindrical capillary [1]. The present paper represents its generalization on a conic case.

Formulation of a problem and basic approximations. Let in a cone with an angle a at distance R_0 from top there is a droplet. Let's R is current distance from the top of the cone and r is its cross section radius (r_0 corresponds to an initial place of the droplet center). We shall consider that angle a is small, so

$$tg\,\frac{a}{2} = \frac{r_0}{R_0} = \frac{r}{R} << 1. \tag{1}$$

Such restriction enables to take the assumption of "thin" droplet that is to consider that

$$R_+ - R_- << R, \quad r_+ \approx r_- . \tag{2}$$

Here R_+, r_+ and R_-, r_- are values corresponding to the right and the left edges of the droplet. During motion the thickness of the droplet will be vary but under the conditions (1), (2) such changes will be insignificant. The given approximation is broken if the droplet gets in the area of the cone top.

Let's assume that the vibrations of capillary walls occur in direction of its axis with velocity u varied according to the law

$$u = U \cos wt , \tag{3}$$

where w is the frequency of vibrations, U is some constant, ($\frac{U}{w}$ is equal to amplitude of displacement) and t is the time. The axis of a cone can be directed arbitrarily. If it is not horizontal the force of weight influences on motion of the droplet. A projection of this force to an axis of a cone we shall designate G.

In consequence of attachment condition the particles of the droplets adjoining with walls of the capillary is moving with velocity u. But the droplet (its center of mass) can move with other velocity v. It is directed along the cone axis and is positive if the droplet remotes from the top. Equation for droplet's motion has the form:

$$m\frac{dv}{dt} = F_{cap} + F_{vis} + G , \tag{4}$$

m is droplet mass and F_{cap} and F_{vis} are the forces caused by the action of capillarity and viscosity.

In an equilibrium case the value of capillary force acting on meniscus of a liquid is equal to $2p\,r\cos q$, here s is coefficient of a surface tension and q is contact angle [2]. In the case of droplet there are two menisci. The meniscus forces are directed to the different sides and for a thin droplet (almost equal radii $r_\pm$ for left and right of its edges) and at absence of vibration these ones counterbalance each other. The presence of vibration can break balance of forces. The affair is that if menisci will begin to move relative to capillary walls the contact angle size at of coming and of receding ones will be various (so-called hysteresis of contact angle). In a fig. 1 it is given an example of experimental dependence of angle hysteresis for system water - firm paraffin [3]. As it is seen the contact angles depend on velocity s of meniscus motion relative to a wall which is equal

$$s = v - u\,. \qquad (5)$$

This dependence has a site ambiguity at s = 0. At $s \neq 0$ contact angle is already one-meaning function of the velocity and gets constant meanings q_+^* and q_-^*. The expression for total capillary force acting on both menisci has the next form $F_{cap} = 2ps\, r(\cos q_+ - \cos q_-)$.

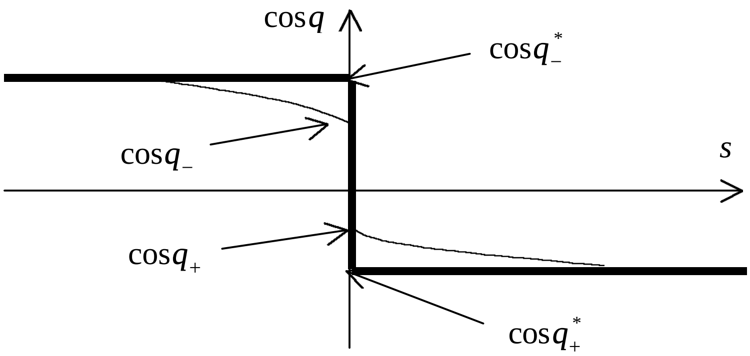

Fig.1 Contact angle hysteresis curve

The value of this force is determined as by quantity so and by sign of s. For simplicity we shall consider that the contact angles reach the limiting meanings very quickly and the resulting force is constant on the module but it varies the direction at change of sign s (that corresponds to Z-characteristic of hysteresis curve). Thus, the final expression for the force is represented as follows

$$F_{cap} = 2ps\, r(\cos q_+^* - \cos q_-^*) sign\ s = 2psbr\ sign\ s\,, \qquad (6)$$

here b is some negative number determined by properties of hysteresis curve.

The velocity of flow in a capillary is very small therefore viscous forces can be neglected in comparison with capillary. Let's take into account this circumstance and we shall substitute expressions (3), (5), (6) in the equation (4). In a result we shall obtain

$$m\frac{ds}{dt} = mUw \sin wt + 2ps\, rb\ \ sign\, s + G\,. \qquad (7)$$

In this equation the size r varies in time during droplet's motion so it depends on s. Locally the droplet moves through cylindrical capillary with variable cross section radius. The connection r and s is easily found from the following expression

$$\frac{r}{r_0} = \frac{R_0 + \int_0^t s dt}{R_0}\,.$$

Finally the equation (7) has the next form

$$m\frac{ds}{dt} = mUw\ sinwt + 2ps\, r_0 b\ \left(1 + \frac{\int_0^t s dt}{R_0}\right) sign\, s + G\,. \qquad (8)$$

This basic equation will be solved later.

The analysis of droplet motion. Let's enter dimensionless value of drop displacement h and time t :

$$h = \frac{R_0 + \int_0^t s dt}{r_0}, t = wt$$

Then the equation (8) will be copied so

$$\frac{d^2h}{dt^2} = a\sin t - |b|(1+eh) sign\frac{dh}{dt} + g\ , \qquad (9)$$

where the designations are used

$$a = \frac{U}{r_0 w},\quad b = \frac{2pbs}{mw^2},\quad e = \frac{r_0}{R_0},\quad g = \frac{G}{mr_0 w^2}\ .$$

Parameter a determines the intensity of vibrating influence on a capillary, b determines the mean of capillary force (b is negative), g - gravitational force and e is the parameter of obliquity.

We shall begin from the most simple situation, when $g = 0$ and $e = 0$ (cylindrical capillary). At the chosen meanings of parameters the equation (9) accepts the form

$$\frac{d^2h}{dt^2} = a\sin t - |b| sign\frac{dh}{dt}\ . \qquad (10)$$

The last term in (10) represents in the essence the friction force of the rest. It acts only when the droplet moves. But this one begins to move if the amplitude of vibrating action exceeds $|b|$. At enough intensive vibration ($a > |b|$) during a part of one period the droplet will be motionless, then it shall move, then it shall stand in an extreme position, and at last it shall return on an initial place. Obviously as a whole the vibration will not result to its displacement. Absolutely other picture of the motion will be observed if some static force g acts along the axis of the cylinder. In this case drop has an averaged motion in a direction of this force [1].

We shall return now to consideration of droplet motion in a conic capillary. The equation (9) was investigated numerically. The initial place of droplet in these calculations is determined by the forces acting to it.

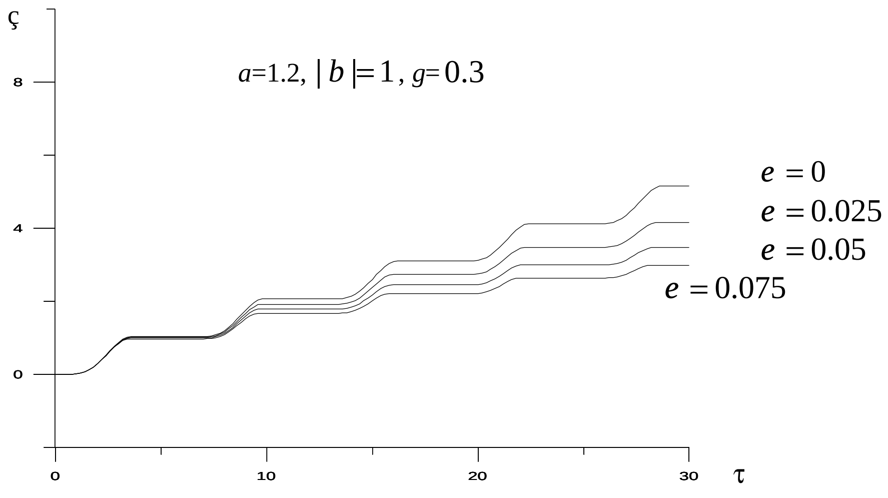

Fig.2 Droplet motion displacement as a time function in the case of the coincidence of static force direction with the cone top side

The equation (9) was solved numerically under the next parameter meanings: a=1.2, $|b|$=1, g=0.3 and g= - 0.3. For positive g the gravity force projection is directed along the conic axis and for negative g oppositely. On figures 2,3 the graphs of displacement dependence from the time are represented for different meanings of conicity e . From this figures it is clear that the increasing of e leads to larger drop's displacement if the static force directs to the conic top and to lesser displacement in inverse case.

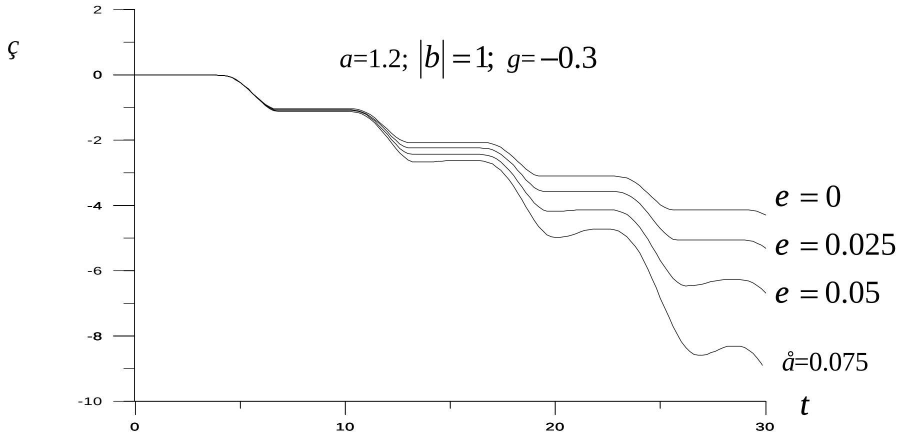

Fig.3 Droplet motion displacement as a time function in the case of counter direction of static force and the cone top side

R E F E R E N C E S

1. Averbach V.S., Vlasov S.N., Zaslavsky Yu.M. Movement of a drop in a capillary under action of static and acoustic fields // Radiophysics, 2000. Ò.XLIII. N2. P. 155-161.
2. Adamson A. Physical chemistry of surfaces. M., 1979 (in Russian).
3. Zhelezny B.V. An experimental research of a dynamic hysteresis of a contact angle // Doclady AS USSR, 1972. V.207. N3. P. 647-650. (in Russian).