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MODELS OF VIBRATIONS GENERATION WITHIN BIOLOGICAL TISSUE BY A SURFACE SOURCE

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The "models with the pressure source of vibrations", based on the Lamb's problem approach, have been offered recently for the description of frequency dependencies of mechanical impedance of layered biological tissues. Similar models are offered in this paper for calculation of displacements within tissues, produced by the round source of oscillatory pressure on the surface. Theoretical expressions for displacements are written for three-layered semi-space with bound or slipping layers and numerical calculations are carried out with parameters corresponding to the system "skin-fatty layer - muscle – liver".

INTRODUCTION

Methods of nonuniformity visualization of biological soft tissues, based on measurements of vibration displacements, produced within tissues by a shear wave (elastography [1]), have been actively developing recently. At the same time, only few theoretical works are known [2-8], based on the approaches of dynamic theory of elasticity and seismology, which study low-frequency processes in tissues. Analysis of shear displacements within tissue sample of limited size has been conducted in the work [2] at the excitement of vibratory modes in the sample. Tissues are simulated here as a viscoelastic medium with inclusions, relating to tumors. The surface displacements on various distances from the source of vibration and dispersion relations for the surface waves have been studied in the works [3, 4]. Tissues are simulated here as a uniform viscoelastic semi-space or as a uniform layer, bound to the hard semi-space with bone properties. The peculiarity of the approach of these works is using the approximate boundary conditions in the contact region of vibration source with tissues, taken ordinarily when formulating the Lamb's problem. The frequency dependencies of mechanical impedance of gelatin layer and of different tissues have been described in the works [5, 6], where more strict boundary conditions in the contact region have been taken, greatly complicating the models. Tissues are simulated here as a uniform viscoelastic layer, bound to the rigid base or as a set of layers (from one to three) bound with each other and with the base. Similar models of tissues have been used in the works [7, 8], where description of mechanical impedance has been developed using the approximate boundary conditions. Models of this type (PSV-models – from "Pressure Source of Vibrations") are offered below for calculations of displacements field within tissues, produced by a surface source. Tissues are simulated as a three-layered viscoelastic semi-space with bound or slipping layers (Fig. 1).

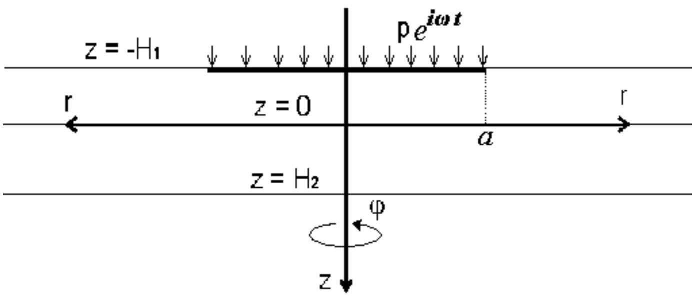


Fig.1. Vibration excitement scheme.

MODELS

Expressions for components of displacement vector in axially symmetric case in the *i*-th tissue layer can be written similarly to the solution of the Lamb's problem for the uniform semi-space by the transition to potentials in the general equations of dynamics of linear elastic solid and by taking the general solution of resulting equations applying the direct and inverse Hunkel transforms:

$$U_{z,i}(r,z)=\int_0^\infty W_{z,i}^0(k,z)kJ_0(kr)dk,$$

$$U_{r,i}(r,z)=\int_0^\infty W_{r,i}^1(k,z)kJ_1(kr)dk\,, \quad (1)$$

where

$$W_{z,i}^0(k,z)=-k^2e^{k_{ti}z}A_i-k^2e^{-k_{ti}z}B_i+k_{li}e^{k_{li}z}C_i-k_{li}e^{-k_{li}z}D_i,$$

$$W_{r,i}^1(k,z)=kk_{ti}e^{k_{ti}z}A_i-kk_{ti}e^{-k_{ti}z}B_i-ke^{k_{li}z}C_i-ke^{-k_{li}z}D_i\,. \quad (2)$$

Upper indexes here correspond to the order of Hunkel transform; k - parameter of that transform; J_0 and J_1 – Bessel functions; parameters $k_{ti}=\sqrt{k^2-k_{ti}^2}$ and $k_{li}=\sqrt{k^2-k_{li}^2}$ are defined by the wave numbers of shear and longitudinal waves $k_{ti}=w/c_{ti}$, $k_{li}=w/c_{li}$, where $c_{ti}=\sqrt{m_i/r_i}$, and $c_{li}=\sqrt{(l_i+2m_i)/r_i}$ are the velocities of these waves, defined by the tissue density r_i and Lamé constants l_i and m_i.

Expressions for the Hunkel images of stresses in the medium could be obtained from the expressions for displacements images (2) taking into account the strain definitions and Hooke's law:

$$s_{zz,i}^0(k,z)=-2m_ik^2k_{ti}e^{k_{ti}z}A_i+2m_ik^2k_{ti}e^{-k_{ti}z}B_i+m_i(k^2+k_{ti}^2)e^{k_{li}z}C_i+m_i(k^2+k_{ti}^2)e^{-k_{li}z}D_i,$$

$$s_{rz,i}^1(k,z)=m_ik(k^2+k_{ti}^2)e^{k_{ti}z}A_i+m_ik(k^2+k_{ti}^2)e^{-k_{ti}z}B_i-2m_ikk_{li}e^{k_{li}z}C_i+2m_ikk_{li}e^{-k_{li}z}D_i. \quad (3)$$

On the external border of three-layered semi-space ($z=-H_1$) conditions are taken of uniform pressure distribution in the region $r\le a$ and of absence of shear stresses on the whole surface:

$$s_{zz,1}^0(k,-H_1)=-\frac{pa}{k}J_1(ka)\,,$$

$$s_{rz,1}^1(k,-H_1)=0\,, \quad (4)$$

On the internal borders ($z_1=0$ and $z_2=H_2$) the full adhesion between the layers is taken (i = 1,2)

$$s_{zz,i}(k,z_i)=s_{zz,i+1}(k,z_i),\quad s_{rz,i}(k,z_i)=s_{rz,i+1}(k,z_i)\,,$$

$$W_{z,i}(k,z_i)=W_{z,i+1}(k,z_i),\quad W_{r,i}(k,z_i)=W_{r,i+1}(k,z_i)\,, \quad (5)$$

and, in other case, the continuity of only normal components of stresses and displacements is taken as well as the absence of shear stresses (slippage of layers)

$$s_{zz,i}(k,z_i)=s_{zz,i+1}(k,z_i),\quad s_{rz,i}(k,z_i)=s_{rz,i+1}(k,z_i)=0,$$

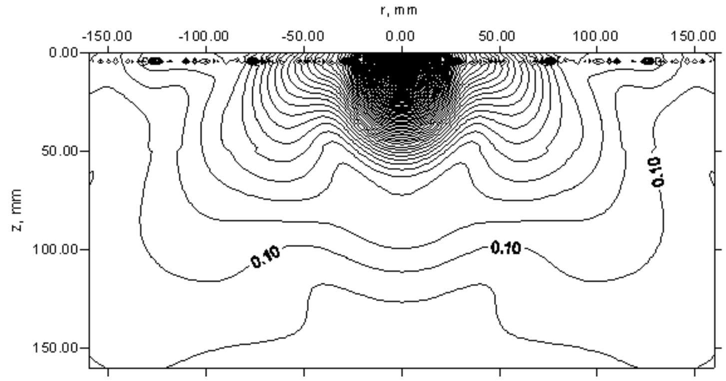
$$W_{z,i}(k,z_i)=W_{z,i+1}(k,z_i). \quad (6)$$

For the infinite third layer, the condition of solution limitation is taken

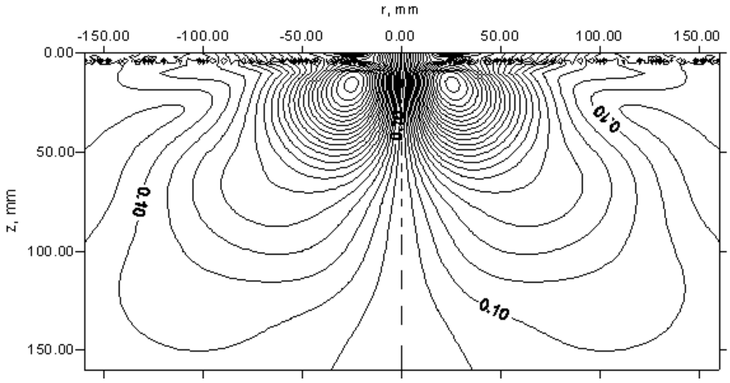
$$A_3=C_3=0\,. \quad (7)$$

Viscous properties of tissues are taken into account by accepting $m_i=m_{0i}+jwh_i$, where m_{0i} are static shear elasticity modules, and h_i are shear viscosity modules. Viscous properties of tissues in bulk deformations are disregarded.

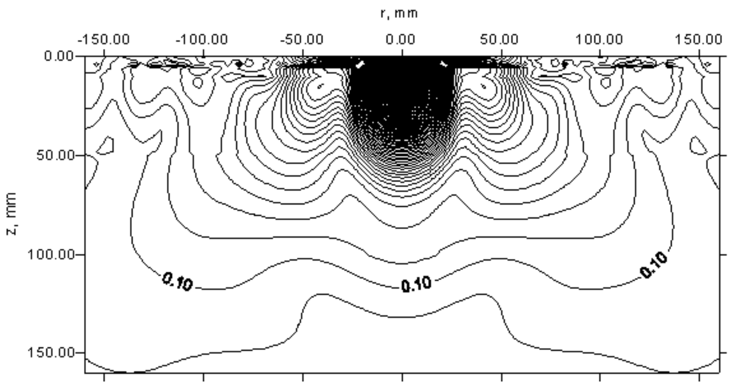
a) $U_z(r,z)$, mm



b) $U_r(r,z)$, mm



b) $U_z(r,z)$, mm



b) $U_r(r,z)$, mm

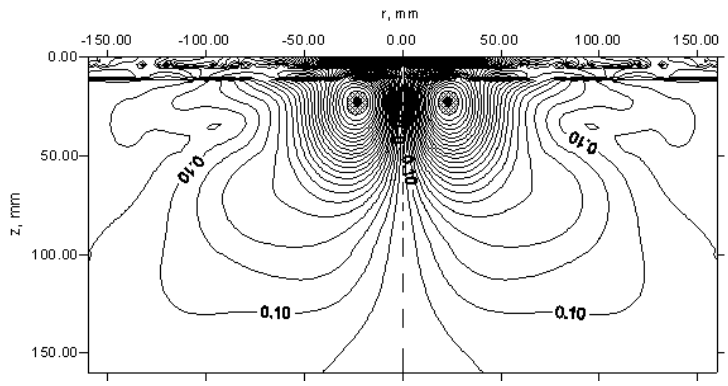


Fig. 2. Displacements under vibration source in the three-layered semi-space with bound (a), (b) or slipping (c), (d) layers. The level lines are drawn through *0.05 mm*.

CALCULATION RESULTS

Calculations of displacements within tissues can be carried out directly by formulas (1) and (2), where unknown functions A_i, B_i, C_i, D_i $(i = 1,2,3)$ have to be found from the system of linear algebraic equations, obtained by substitution of expressions (2), (3) in the boundary conditions (4) - (7) on all external and internal borders of layers. For the three-layered semi-space with bound or with slipping layers the 10-th order systems are got, which have not listed here because of their huge size.

Ensuring correspondence of models to the system "skin-fatty layer - muscle - liver", the following values of parameters have been taken: $H_1 = 5\ mm$, $H_2 = 10\ mm$, $m_{10} = 1\ kPa$, $m_{20} = 4\ kPa$, $m_{30} = 1.7\ kPa$, $h_1 = 6\ Pa \cdot s$, $h_2 = 3\ Pa \cdot s$, $h_3 = 2\ Pa \cdot s$, $c_{l1} = 1450\ m/s$, $c_{l2} = 1570\ m/s$, $c_{l3} = 1574\ m/s$, $r_1 = 930\ kg/m^3$, $r_2 = 1040\ kg/m^3$, $r_3 = 1080\ kg/m^3$. The source size as well as amplitude and frequency of its vibration are taken such as to be easily realized in practice: $a = 25\ mm$, $p = 1\ kPa$, $f = 20\ Hz$. Calculations have been carried out by the *MathCAD* means in the region $r \le 160\ mm$, $z \le 160\ mm$ with 2 *mm* steps on *r* and *z*. Upper limits of integration in (1), different in the different calculation points, have been chosen on the base of preliminary studies of integrands there. At the choice of calculation points some distance has been ensured from the borders of layers, because of integrands getting undamped and integrals getting nonconvergent on the borders and in their vicinity. The examples of the calculation results are given on Fig.2. One can see that displacement amplitudes remain large enough to be used in sonoelastography up to depth 150 *mm* and more. Distortions of the displacements field by the upper layers are limited by depths *20 – 30 mm* and appeared to be more essential in the slipping layers case, especially for $U_r(r, z)$.

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