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SPATIALLY – UNCHANGED (SELF-TRAPPED) SOUND BEAM IN NONLINEAR ACOUSTICS

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The capability of spatially - unchanged (self-trapped) propagation of sound beams under compensation of a diffraction divergence by a nonlinear non-inertial refraction in the media with power nonlinearity has been demonstrated theoretically. Obtained unchanged along longitudinal axis solutions of a nonlinear equation of beam acoustics (the Khokhlov-Zabolotskaya equation) describes the distinctive continuous wave profiles of a beam and amplitude distributions, auto-localized in a transverse direction.

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The possibility of a mutual compensation of a diffraction divergence and nonlinear refraction for implementation of self-trapped (auto-localized on transversal coordinates) mode of propagation of wave beams has been demonstrated a long time ago in nonlinear optics and plasma physics [1,2]. However, in spite of the fact that a diffraction divergence and nonlinear refraction are also typical of a nonlinear acoustics of sound beams [3], the problem of capability of self-trapped propagation of sound beams at the expense of non-inertial (non-thermal) nonlinear effects until now has not been posed and accordingly the positive or negative answer to the capability of its solving was not given. Probing we shall conduct the investigation of this problem within the framework of a generalized Khokhlov-Zabolotskaya equation [4] for the description of propagation of acoustical beams in non-dissipated media with the arbitrary power law of non-linearity, which one for non-dimensional values looks like:

$$\frac{\partial^2 V}{\partial z\cdot\partial t}=\frac{N}{4}\Delta_{\perp}V+(-1)^{n+1}\frac{\partial}{\partial t}\left(V^n\cdot\frac{\partial V}{\partial t}\right). \qquad (1)$$

Here $V=u/u_0$ (u, u_0 - oscillatory velocity and its peak value accordingly), $N=\frac{Z_{sh}}{Z_d}=\frac{c_0^{n+1}/(|e|\cdot w\cdot u_0^n)}{wa^2/(2c_0)}$ - Khokhlov's parameter, defining a ratio of nonlinear and diffraction effects, Z_{sh} - length of derivation of a discontinuity, Z_d - diffraction length, C_0 - equilibrium velocity of a sound, e - parameter of nonlinearity of the medium, a^2 - distinctive area of cross-section of a beam, w - distinctive frequency.

For traditional nonlinear acoustics with quadratic nonlinearity $(n=1,\ e>0)$ and for others odd n the sign " + " before the last term in an equation (1) is determined by consideration of waves traveling to the right from a source; at symmetrical initial wave profile concerning t -axis the effects defined by a ratio of a diffraction and non-linearity, for odd n do not depend on this sign (replacement $V\to -V$ demonstrates it). At even n (for example, for cubic nonlinearity, where $n=2$) this sign determines the nonlinear media with essentially different properties: the sign "+" corresponds to the defocusing media, in which the nonlinearity magnifies the local velocity of propagation of perturbation, and the sign " - " corresponds to the self-focusing media, in which the compensation of a diffraction divergence can take place.

The existence of self-trapped mode of propagation of sound beams will be determined by presence of the physically justified solution of an equation (1) in stationary along Z case. This solution just is simply dictated by pointed compensation under condition of $N/4\sim1$ (precise values of parameter N will be obtained below). Firstly, the required solution should describe the wave profiles with strong nonlinear distortions (concerning a sine-wave profile), but not having discontinuities. These are the **nonlinearly -**

continuous profiles. For such type of profiles the nonlinear high-frequency attenuation will be absent and the profiles of such type can give the stationary along Z solutions of the equation (1). Secondly, this required solution should be localized on transversal coordinates. Therewith the essential distinctions between the required stationary solutions will be determined also by parity or oddness of a nonlinearity index n in (1).

In the beginning we shall consider "slotted" beams, for which the possible self-trapped mode of propagation is determined by the following equation

$$\frac{N}{4} \cdot \frac{\P^{2} V}{\P y^{2}} + (-1)^{n+1} \frac{\P}{\P t}\left(V^{n} \cdot \frac{\P V}{\P t} \right) = 0 \cdot \qquad (2)$$

Simpler for the analysis and search of the "eligible" solution of an equation (2) is the case of even n. The requirement of the zero total area of positive and negative half-waves of a wave profile in the solution of (1) and (2) on its period [4], and also the independence of result of a manifestation of nonlinearity from a sign of V shows symmetry of the required solution concerning t - axis at apparent symmetry on y - axis concerning center. The structurally simple partial solution, that allows the separation of variables, satisfies this condition:

$$V(t, y) = T(t) \cdot Y(y), \qquad (3)$$

which at substitution in (2) leads to following dependences satisfying above-listed requirements

$$\pm t + 2pk = \sqrt{\frac{n+2}{2 l^{2}} \cdot T_0^{n}} \cdot \int_{\tilde{T}}^{1} \frac{\tilde{T}^{n} d\tilde{T}}{\sqrt{1-\tilde{T}^{n+2}}} \qquad (\tilde{T} = T/T_0 ;\ 0 \le \tilde{T} \le 1) \qquad (4)$$

$$\pm y = \sqrt{\frac{N}{8} \cdot \frac{n+2}{l^{2} Y_0^{n}}} \cdot \int_{\tilde{Y}}^{1} \frac{d\tilde{Y}}{\sqrt{1-\tilde{Y}^{n+2}}} \qquad (\tilde{Y} = Y/Y_0 ;\ 0 \le \tilde{Y} \le 1) \qquad (5)$$

where T_0 and Y_0 - normalizing "peak" values of appropriate quantities, l^{2} - separation constant, n - even. The functions, inverse of the obtained solutions (4), (5), give required relations $\tilde{T}(t)$ and $\tilde{Y}(y)$. The first of them determines a reference kind asymptotically of universal non-linearly - continuous profile for even n (including for cubic nonlinearity). The required normalizing relation $\tilde{T}(p) = -1$ (for what the multiplying factor before an integral in (4) should be equaled 2,6 at $n = 2$) provides normalized period $2p$ in an equation (1). Dependence $\tilde{Y}(y)$, giving localization of a beam lengthwise of y -axis and playing the role of amplitude, is defined only on an interval $|y| \le y_0$, where it decays from 1 up to 0; outside of the prescribed interval on y the significance of $\tilde{Y}$ are defined as zero. Such modified solution (non-zero inside an interval $|y| \le y_0$ and zero outside of it) saves the continuity of function $\tilde{Y}(y)$ in boundary points, and with it the continuity of the second derivative of this function by virtue of the unique connection between them. It is vital to ensure the absence of singular terms at the substitution of modified solution into the equation (2). Account must be taken that variable y was normalized to beam width, consequently there are bound to be $y_0 = 1$, for this purpose the multiplier before the integral in (5) must be equal to 0,77 at $n = 2$. If the values T_0, Y_0 are expressed in terms of the obtained number significance of multiplier before the integral and product of T_0 and Y_0 is equated to 1 (this is meant that $V_{\max} \equiv Y_0 \cdot T_0 = 1$), so will be $N = 0.35$ for $n = 2$. Under these values of the parameters the self-trapping of sound beams in cubic nonlinear medium is realized. The resulting wave field $V(y,t) = Y(y) \cdot T(t)$ in this self-trapped mode of sound beam is plotted in Fig.1 (the amplitude distribution is shown only along the positive half-axis of y).

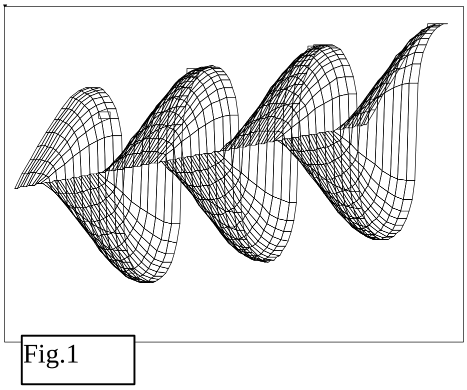

Fig.1

The situations with odd n in the equation (2) are more difficult for the analysis and search of the necessary solution since the symmetry concerning t - axis disappears and the solution in the form of (3) does not provide the requirements their continuity. In these cases it is necessary to search solutions that will be particular for each odd value of n and will satisfy the necessary requirements (see above).

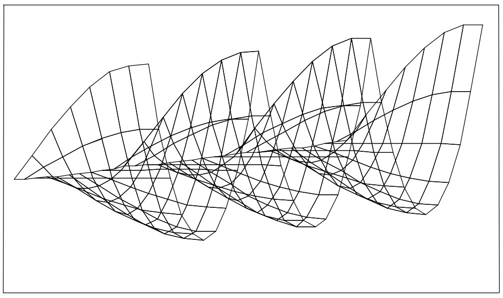

Fig.2

Let's consider a case of a conventional nonlinear acoustics with $n = 1$. For this most relevant and interesting case the eligible solution (new in comparison with introduced in the handbook [5]) has the form

$$V(t,y) = Y_2(y) \cdot (t - 2\pi k + \gamma \cdot z)^2 - (Y_0(y) - Y_0(1)), \tag{6}$$

where the functions $Y_2(y)$ and $Y_0(y)$ must be determined from the following equations, as seen from substitution (6) into (1),

$$Y_2'' + \frac{24}{N} Y_2^2 = 0 \tag{7}$$

$$Y_0'' + \frac{8}{N} Y_2 \cdot Y_0 = 0 \tag{8}$$

The retrieved solution (6) - (8) represents the periodically recurring on t symmetrical chunks of a parabola. They are characterized by a varied along y coordinate steepness of branches of parabola (is determined by values of function $Y_2(y)$) and displacement of a parabola on a vertical concerning an t -axis (is determined by values of function $Y_0(y)$). The functions $Y_2(y)$ and $Y_0(y)$ in aggregate implicitly determine the distribution of "amplitude" of received **arched waves.** For beam localization these functions should abate to zero with an increase in $|y|$ symmetrically relatively $y = 0$. This will do zero amplitude of arched wave at beam boundary. The solutions of equations (7), (8) satisfy to these conditions and as a result the wave field is forming. This field is plotted in Fig**. 2** (for clarity the amplifier distribution is shown only along positive half-axis of y). This wave field is apt to ensure the self-trapped mode of propagation of a slotted beam in the quadratic-nonlinear medium. As evident from the normalizing condition this mode can be realized under significance $N \cong 1{,}22$. It is also noted that additional phase (temporal) shift dependent on longitudinal z-coordinate needs to enter into the solution (6), that does not disturbing the permanency of wave field structure under propagation along this axis.

For axial - symmetrical beams the operator $\frac{\partial^2}{\partial y^2}$ is replaced by operator $\frac{1}{r} \cdot \frac{\partial}{\partial r}\left(r \cdot \frac{\partial}{\partial r} \right)$ in the equations (2), (7), (8) and corresponding solutions in its behaviour are similar those, which were obtained for the slotted beams (see Fig.1 and Fig.2), but with another values of distinctive parameters.

For example, the values $N = 0{,}31$ and $N = 0{,}35$ are required for possible) self-trapped propagation of axial - symmetrical beams accordingly in cubic and quadratic- nonlinear media.

Notice that the obtained analytically characteristic shapes of nonlinearly-continuous wave profiles are supported by the some of earlier performed numerical solving [3,6,7] of the Khokhlov-Zabolotskaya equations where similar profiles were obtained (irrespective of self-trapping problem) for some values of reference parameters, but special role of such waves was not considered in previous investigations.

The conditions for self-trapped propagation of sound beams can be obtained experimentally by previous focusing of a beam because wanted wave profiles are form in focal region under certain initial parameters.

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