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MOTION OF DROPLETS WETTING THE WALL OF CAPILLARY UNDER VIBRATION

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In this report an attention would pay to study of features in behavior of glycerin droplet placed in capillary that arise under action of static forces – own weight and low frequency vibration. Probably this peculiarities are similar to that which known for motion of material particles placed on oscillating surface by presence of solid friction between them [1]. The analysis of droplet capillary system dynamics is carried out due to the investigation by the problem of acoustical action on the porous medium with partial liquid saturation because this simplification should be helped to study in detail the main physical mechanisms of effective mutual interaction between two subsystems – skeleton and fluid. Our experiments show the possibility to find capillary constants that is difficult obtained by optical observations.

By the wave propagation the capillary walls execute periodic longitudinal motions of the amplitude Y. Due to the capillary forces the walls act on the droplet by the force F that described by the Laplacian formula for the excess pressure provided by the droplet from the menisci. In the first write the expression for capillary force on the liquid pillar bounded of one meniscus: $F = \pm 2\pi\sigma r\cos\theta$, where positive value of force by suggestion corresponds to z-axis direction , and cosθ - cosinus of wetting angle θ between touch line on meniscus surface and the wall with the side occupied by the liquid. Signs ± mean the opposite direction of surface tension force on the pillar that is either left or right with regard of the meniscus. Cosθ - in static equilibrium state has a positive mean (cosθ>0) in the case of a wall wetting by a liquid and negative mean (cosθ<0) in the case of non wetting. Due to the motion of the droplet relatively walls there is a change of the angle θ of each meniscus limiting the droplet. The dependence of the angle θ on the meniscus motion velocity for a glycerin -glass pair was determined in [2]. There is a distinguish feature: the presence of an ambiguous (hysteresis) part of the curve θ(s) in the vicinity of zero value $s = u - u_0$, where u,u_0-the velocity of droplet mass center and wall motion. The angle θ can vary from θ_- to θ_+ at s=0 depending on prehistory of the process. At s≠0 the meniscus angle is unambiguously related to its motion velocity and achieves very rapidly, with the characteristic velocity w, the limiting values θ_{-0} or θ_{+0}.

The equation of a droplet motion has a next form:

$$m\frac{du}{dt} = F_l(s) + F_r + G = -2\pi r\sigma(\cos\theta_l - \cos\theta_r) + G, \qquad (1)$$

where m-droplet mass, F_l and F_r - forces acting by the side of left and right menisci, G – static force induced for example by the weight action, θ_l, θ_r - wetting angles of left and right menisci which have complex dependency from s due to hysteresis its part. The analysis corresponds to the case of a capillary with a constant cross-section for a droplet having the mass value $m=\pi r^2 L\rho$, where ρ- is a liquid density. If take in account the relation between u and s the equation of motion could be obtained:

$$m\frac{ds}{dt} + m\frac{du_0}{dt} = F_l(s) + F_r(s) + G. \qquad (2)$$

For the fast switching of the vibration when it holds:

$$u_0 = 0, t < 0, \qquad u_0 = \omega Y\sin\omega t, t > 0 \qquad (3)$$

the next description would be used for the capillary force:

$$F_l + F_r = 2\pi r\sigma[-\cos\theta_+ + \cos\theta_- - (-\cos\theta_{+0} + \cos\theta_{-0} + \cos\theta_+ - \cos\theta_-)th\frac{s}{w}], s < 0, \qquad (4)$$

$$F_l + F_r = 2\pi r\sigma[\cos\theta_+ - \cos\theta_- + (\cos\theta_{+0} - \cos\theta_{-0} - \cos\theta_+ + \cos\theta_-)th\frac{s}{w}], s > 0. \qquad (5)$$

By the notation: $\cos q_{-0} - \cos q_{-} - \cos q_{+0} + \cos q_{+} = 2a$, and using the experimental data of α ≅ 0.25 and $\cos q_{-} - \cos q_{+} = 1$, that holds accordingly [2] for example in the case of glycerin-glass pair the equation of motion for t>0 could be presented in the form: $\frac{ds}{dt} = -w^2 Y \cos wt + \frac{G}{m} + \frac{2psr}{m}(\pm 1 - 2ath\frac{s}{w})$, (6) where sign plus is taken by s<0, sign minus is taken by s>0.

The equation (8) describes the material point motion under the static and oscillating forces and also in addition to solid friction force that represented by term of ±2πσr/m. The expression – th(s/w) is similar of usually viscosity friction and involved at this equation. Let us make the exchange of variables s,t by substitution of dimensionless variables – S,τ:

$$g = \frac{Yw^2 m}{2rps}, t = \frac{2rps}{mw}t, \Omega = \frac{wmw}{2rps}, f = \frac{G}{2rps}, S = \frac{s}{w}. \qquad (7)$$

By new notation and introduction of "y" coordinate: $y = \int S dt$, the equation (6) could be presented in the form:

$$\frac{d^2 y}{dt^2} + 2ath(\frac{dy}{dt}) = -g\cos\Omega t + f \pm 1, \qquad (8)$$

By a variable force g = 0 the droplet remains at former place and also the same result would be by a static force is less than solid friction force:|f|<1. There is a threshold value of static force $G_{\text{ïîð}} = 2rps$ by an exceeding of which the droplet mass center begins to move. Threshold force could be expressed by the pressure gradient $\frac{dP}{dz_{\text{ïîð}}} = \frac{2s}{rL}$. If it holds the condition: 1<|f|<1+2α - static force is grater than a threshold value but less than value – G < $G_{\text{ïîð}}$+4πασr, so on rather during time intervals droplet moves translating uniformly with the velocity $S = arth\frac{|f|-1}{2a}$, that in dimension variables has a form $u = w \cdot arth\frac{G - 2rps}{4raps}$. If the static force exceeds of the value $2rps(1+2a)$ the droplet reaches over the some little time the velocity u>>w, after which it moves with acceleration: $\frac{du}{dt} = \frac{G - 2rps(1+2a)}{m}$.

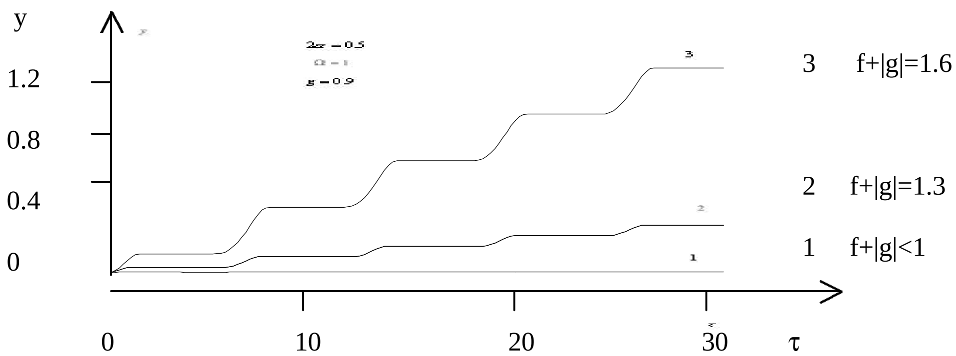

Fig.1

In the presence of oscillating force the solutions of equation (10) were investigated numerically. Note that the droplet center dependence on the time is presented in the coordinate system related with the wall of capillary and for the transition to laboratory system it is necessary to mind of oscillating (periodic) motion of coordinate origin. The oscillating force changes essentially a kind of the droplet motion. Under the condition $f + |g| < 1$ or $G < G_{\text{ïîð}} - w^2 Ym$ the droplet remains at former place. In the case $1 < f + |g|$ or $G_{\text{ïîð}} < G + w^2 Ym$ the droplet begins a motion. At small excesses over the threshold $f + |g| - 1 \le 1$ the droplet moves mainly at time instance when the force is $|-g\cos\Omega t + f| > 1$. This is illustrated by curve 1,2,3 on Fig.1. Thus the droplet motion is stepwise. An average displacement of the droplet mass center during the oscillation period is

essentially less than the wall displacement amplitude in the acoustical field. In the range $1 < f < 1 + 2a$, where the droplet moves uniformly because of static forces the appearance of variable action due to acoustical field induces the increasing its translation motion velocity. At lower frequency for the achievement of equal relative addition of velocity it is required of lower field amplitude. Evidently under condition of existing flows and by presence of static pressure gradient the acoustical action causes only slight increasing of percolation velocity.
Some experiments were carried out to demonstrate the effects of the droplet motion threshold decrease in the presence of an acoustical action. Rather detail test of influence of several low frequency vibration on translation motion velocity was performed. A laboratory setup represented a vibromodel (VEB) with a glass capillary fixed vertically on its operating plarform, so that the gravity was a static force. A cylindrical capillary glass tube with the channel diameter 1 mm and the length of several tens of centimeters was used. Each measurement session began with introduction of a glycerine droplet of the length 5-7 mm into the capillary from the upper end at a short distance from the edge. The droplet was introduced with an injector to conserve dry the inner surface of the capillary - without a moist film leaving a trace behind the moving droplet due to glass moistening by liquid. The level of vibration was controlled using a measuring accelerometer (KB-10) mounted on the operating platform. Measurements were taken at the frequencies 20, 30, 40, 50 and 70 Hz, while the vibroload was within the limits 1-30 m/s^2.

Let us describe the experimental results qualitatively. According to the theoretical notions in the absence of vibroaction the droplet remained in place in the capillary for a long time during tens of hours. Under the action of vibrations a translational motion in down direction with some velocity occurred beginning with a definite amplitude. Cessation of the vibroaction caused the droplet motion stop. Quantitative data as dependences of the droplet motion velocity on the vibroaction amplitude at listed above frequencies are given in Fig.2.

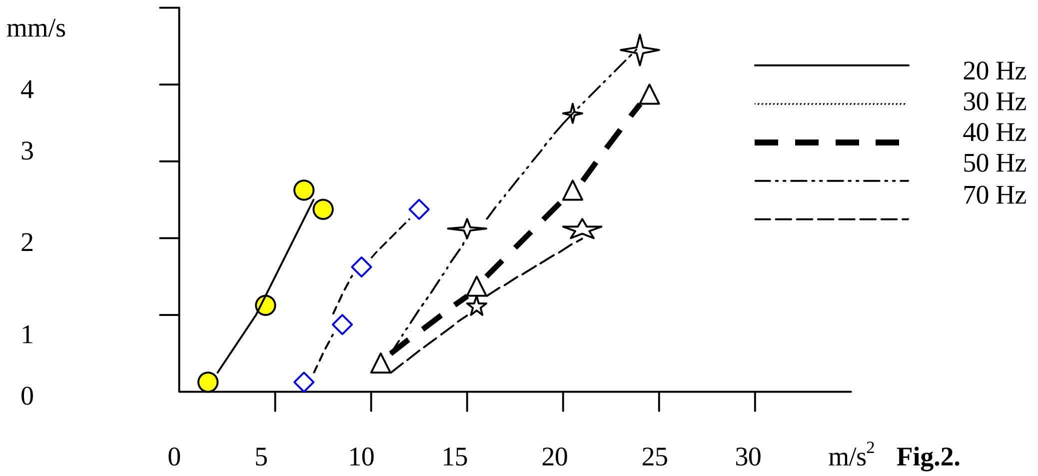

Fig.2.

The threshould of the droplet motion onset could be seen from this figure. This threshould value of the acoustical action can be calculated employing the expression $(Yw^2)_{ïîð} = \frac{2s}{rLr}(\cos q_- - \cos q_+) - g$ (where g=9.8m/s^2). In other hand value of difference of static wetting angle cosine could be obtained (hysteresis part of full dependence). So from Fig.2 the estimations were made and value of cosine difference is equal to 0.3..0.4. It satisfactorily aggree with data of another authors [2].
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