D. E. Leikin

NON-PERTURBATIVE THEORY OF COHERENT SCATTERING IN MULTIMODE WAVEGUIDES

N.N.Andreyev Acoustics Institute
4 Shvernik St., Moscow 117036, Russia
Tel.: +7 (095) 1269835; Fax: +7 (095) 1268411;
E-mail: leikin@orc.ru

Abstract: Coherent scattering is rather unusual diffraction phenomenon which may be described as a situation when the ingoing wave experience no diffraction passing over a perturbation to the refractive index of the medium. Physically, it means that the inhomogeneity becomes «invisible» because of spatial coherence between incident and scattered waves. In this report, a brief account of the phenomenon is presented. Novel monitoring techniques based on non-perturbative theory of coherent scattering are proposed. Feasibility of the new methods is discussed.

To outline the principal ideas of the theory of coherent scattering and its possible practical applications let us consider the problem of sound propagation in range dependent oceanic environment. Assume that energy losses due to absorption and backscattering are negligible. Such model is valid in many a problem of low-frequency underwater sound propagation in the ocean.

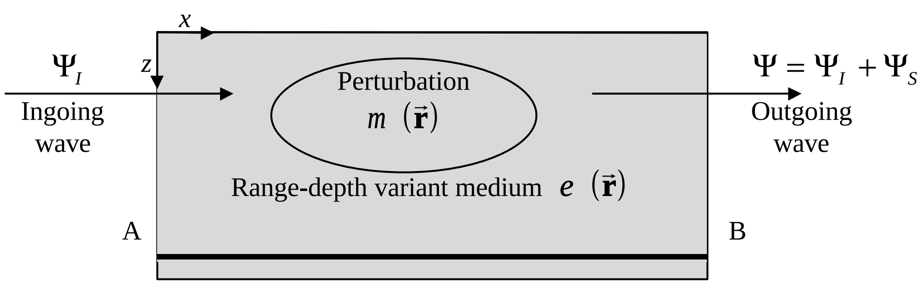


Fig.1. Geometry for the problem of wave propagation. $m\ (x,z)$ is a perturbation to inhomogeneous background stratification $e\ (x,z)$.

Acoustic field is described by the Helmholtz equation

$$\Delta\Psi + k^2\left[e\,(\mathbf{r}) + s m\,(\mathbf{r})\right]\Psi = 0 \tag{1}$$

where $k = w / c$ is a reference wave number, e stands for the background stratification of the underwater channel, and m describes additional («foreground») perturbation to the refractive index, s being its magnitude.

This classification is useful to distinguish long-lived sound speed structures, e.g., the seasonal stratification, from inhomogeneities displaying noticeable temporal variability over the time of experiment, such as internal waves and turbulence. The exact solution to Eq.(1) may be written as (See Fig.1)

$$\Psi = \Psi_I + \Psi_S \tag{2}$$

where Ψ_S is the wave scattered by the perturbation m, and Ψ_I denotes the ingoing wave satisfying the unperturbed wave equation

$$\Delta\Psi_I + k^2 e\,(\mathbf{r})\Psi_I = 0 \tag{3}$$

supplemented by the same boundary conditions as Eq.(1). The scattered wave Ψ_S is usually sought of the form of a perturbation series. Generally, all terms in this series expansion essentially contribute to the overall effect. In the forward scattering approximation , however, the sum of the perturbation series can be found exactly in the case when the scattered waves are spatially coherent with the ingoing field:

$$\Psi = e^{ia}\Psi_I \quad , \quad \text{i.e.,} \quad \Psi_S = \left(e^{ia} - 1\right)\Psi_I \tag{4}$$

with a constant scattering phase $a = const$. Virtually, Eq.(4) implies that the ingoing wave Ψ_I experience no diffraction passing over the inhomogeneity m. Hereinafter, such waves Ψ_I are referred to as coherent waves.

In this report we shall touch upon there main issues:

- do the coherent waves exist in the ocean and what their main features are,
- is there any way to excite coherent waves in the absence of prior knowledge of both the background and the foreground structure of sound speed inhomogeneities, and
- what practical implication coherent waves may have.

I. COHERENT WAVES

Equations (1-4) have a set of non-trivial solutions $\Psi^{(a)}$ corresponding to different scattering phases a. Each solution can be obtained using experimental data on modal spectrum of the ingoing and outgoing waves. The main properties of the coherent waves may be described by the following invariants:

- The intensity of coherent wave is identical to that of the ingoing wave outside the region occupied by the «foreground» inhomogeneity $|\Psi(\mathbf{r})|^2 \equiv |\Psi_I(\mathbf{r})|^2, \quad \mathbf{r} \notin \text{support}(m)$. (5)
- The intensity of coherent wave is independent of the magnitude s of the perturbation:

$$\frac{d}{ds}|\Psi(\mathbf{r})|^2 \equiv inv = 0\,, \quad \mathbf{r} \notin \text{support}(m)\,. \tag{6}$$

- Total intensity of coherent waves is range invariant $J = pk^{-1}\sum_a |\Psi^{(a)}(\mathbf{r})|^2 \equiv pk^{-1}\sum_n U_n^2(z) = inv$ (7)

and is functionally independent of both e and m: $\dfrac{dJ}{de(\mathbf{r})} \equiv inv = 0$, $\dfrac{dJ}{dm(\mathbf{r})} \equiv inv = 0$.

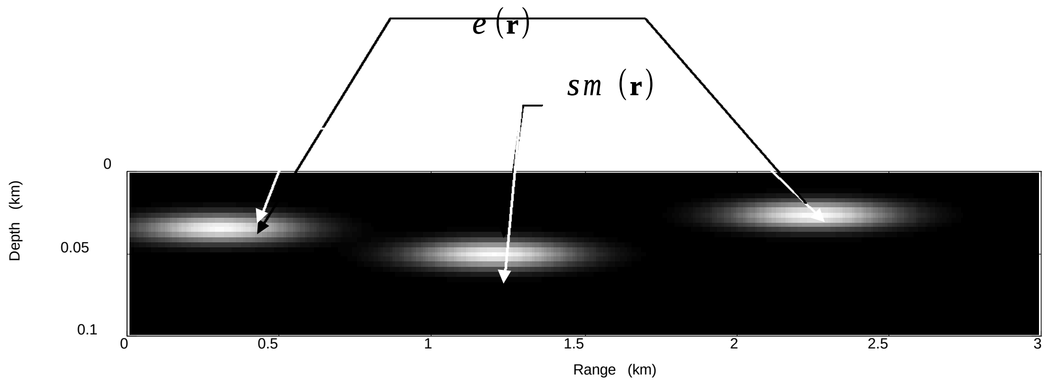


Fig.2. The background inhomogeneities e and the perturbation sm
Channel stratification includes downward positive gradient
0.1 s^{-1} of the speed of sound (not shown in this picture).

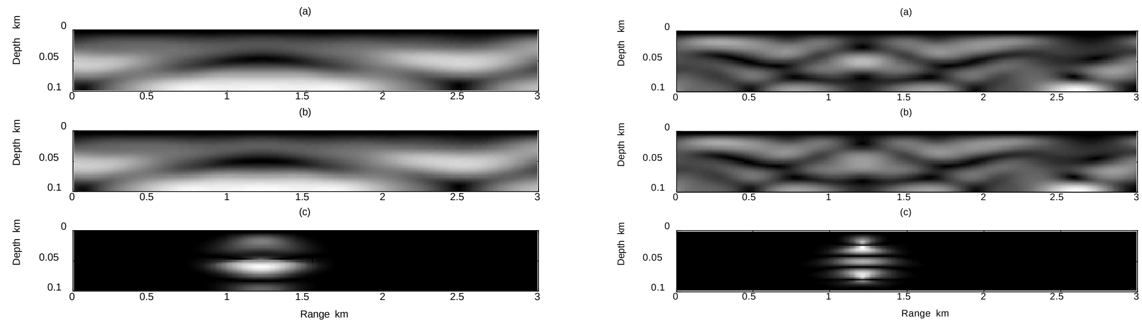
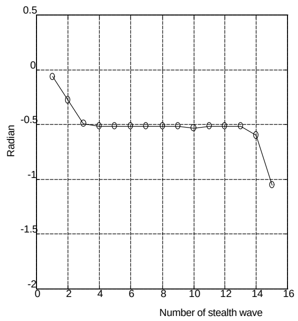


Fig.3. The intensity of coherent wave Ψ (a), Ψ_I (b), and their difference $h(\mathbf{r}) = |\Psi - \Psi_I|$ (c) for coherent waves *N=1* (on the left) and *N=15* (on the right).



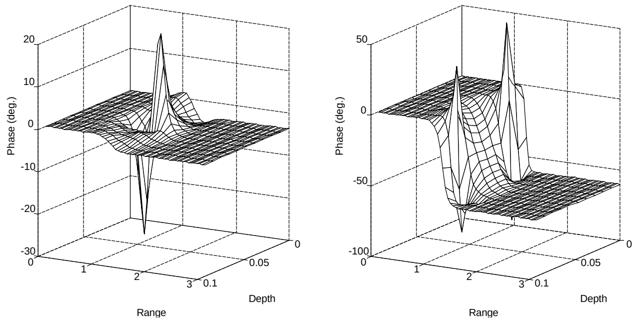


Fig.4. Scattering phases a of coherent waves

Fig.5. Phase difference $\Delta f = \arg\left(\Psi^{(a_N)}(\mathbf{r})\right) - \arg\left(\Psi_I^{(a_N)}(\mathbf{r})\right)$ for coherent waves *N=1* (left panel), and *N=15* (right panel).

Here, U_n are modal functions calculated in a reference waveguide $U_p'' + k^2 n^2(z) U_p = k_p^2 U_p$ with a predetermined profile *n*(*z*). The stronger resemblance to actual stratification the sound speed profile *n*(*z*) has, the less number of almost degenerate solutions $\Psi^{(a)}$ there will be. In the limiting case of *n*(*z*) exactly matching actual local profile, the number of coherent waves $\Psi^{(a)}$ will equal the number of propagating modes.) Figures (3-5) illustrate interference patterns and scattering phases of coherent waves in the channel shown in Fig. 2. Calculations were carried out for $w = 110\ \mathrm{Hz}$, and maximum sound speed variations due to e and sm up to 30 m/s.

II. REMOTE SENSING

Phases a in Eq.(4) can be measured experimentally and are given by

$$a = \frac{s\,k}{2} \int_A^B dx \int I_a\, m(x,z) dz \tag{8}$$

where $I_a = \left|\Psi^{(a)}(x,z)\right|^2$ is the intensity of the coherent wave corresponding to phase a. This equation suggests a scheme for tomographic inversion. Suppose that scattering phases a were measured at range *B*. Equation (8) implies that quantities a may be thought of as projections of unknown inhomogeneity $sm(\mathbf{r})$ onto a subspace spanned by the set of functions $\{I_a\}$. If these functions were known it would be possible to restore the $m(\mathbf{r})$ by a backprojecting process. As we have seen earlier, the intensity of ingoing coherent wave everywhere equals that of the full wave except for the region occupied by the perturbation (See Eq.(5) and Fig.3). Therefore, for the first approximation one may use $I_a^I = \left|\Psi_I^{(a)}(x,z)\right|^2$ instead of I_a. The next step will be to substitute recovered inhomogeneity m into the governing wave equation (1) and to calculate next-step approximation for I_a. This leads to a simple recursive procedure. In practice, however, even the first approximation may yield satisfactory accuracy.

Using the phenomenon of coherent scattering one can also suggest a precision technique intended for temperature monitoring in the absence of prior information on acoustic properties of the medium. Assume that phases a are occasionally measured in the region under control. Let perturbation $sm(\mathbf{r})$ in Eq.(1) describe change, which the refractive index had undergone since the previous measurement. The simplest method to detect spatial average of $sm(\mathbf{r})$ may be based on the fact that total phase of coherent waves $\Phi = \sum a$ is functionally independent of the background stratification $e(\mathbf{r})$ of the medium and linearly depends on the magnitude s of the perturbation m :

$$\Phi = \frac{k^2}{2p} \int_A^B \int J(z) s m(z,x) dz dx , \qquad (9)$$

$$\frac{d\Phi}{de(\mathbf{r})} \equiv inv = 0 ; \quad \frac{d^2\Phi}{ds^2} \equiv inv = 0 .$$

where k is predetermined, $J(z)$ calculated, and Φ measured. Equation (9) determines a practical scheme for monitoring temporal trend in the weight-average of the refractive index (temperature) of the medium. The most important feature of this method is that no prior information on the background medium stratification needs to be incorporated into the signal processing algorithm.