A.A.Pudovkin

The detection error probability estimation for the signal, which is described by disorder stochastic model

N.N.Andreev Acoustic Institute, North branch.
3, Pochtovaia st., Severomorsk, Murmansc region, 184600 Russia
Tel: (81537) 7-8875; Fax: (81537) 7-7725;
E-mail: sfakin@murmansk.rosmail.com

One ought to find signal model, which not contradictory with observations forecasts its values for false alarm or missing error estimation. Such qualified model of the signal, which is conducted through the natural reservoir waveguide, has colored noise component, which is connected with the multiplicative clutter due to corruption during the propagation. Such qualified model significantly complicated optimal treatment algorithm. The error probability ratios were deduced for the situation where treatment algorithm is constructed on simpler model than true model is and the autoregressive stochastic clutter model is known. There are examples of calculation.

The false alarm and missing probability are usually used for detection task solving description. It is difficult to estimate those probabilities itself, but there are rather developed method for its Chernoff lower boundary. The likelihood logarithm moments generating function is used in formulas which may be used for error probability calculation. The signal model defines likelihood. Let us use filter with impulse response b(t,λ) as the waveguide model, where argument t described time selective waveguide properties and argument λ described frequency selective ones. Let us $s_p(t)$ will be denotation for signal in conditions which are connected with the index p value.

The eigenfunctions $\Phi_{Lq}(\lambda)$ series with eigenvalues b_{Lq} decomposition is true if signals with the different lead time values are independent.

$$b(t,\lambda)=b(t)\sum_{q=1}^{m} \sqrt{b_{Lq}}\Phi_{Lq}(\lambda).$$

Now signal on the waveguide output may be described by m independent components

$$s_{pLq}(t)=\int \Phi_{Lq}(\lambda)s_p(t-\lambda)d\lambda,\quad s_p(t)=b(t)\sum_{q=1}^{m} \sqrt{b_{Lq}}s_{pLq}(t),$$

in this case in every lead time waveguide eigenfunction $\Phi_{Lq}(\lambda)$ bandpass the Doppler properties which are described by process b(t) may be individual. The Doppler waveguide properties are most efficiently described by autoregressive model because of possibility Kalman filtration usege.

The array “sow” point source in disperse waveguide under the set of angles, each of which corresponds to one of eigenvectors of covariance matrix. So to collect all point source energy in waveguide one ought to use set of eigenvectors, it’s properties are defined by covariance function. The waveguide propagating signal covariance function are allegedly filtered by waveguide eigenfunction. There are additional decay in covariance function of Doppler waveguide propagating signal, which monotonically depends on lag value and is described by Doppler waveguide character. Both transversal and longitudinal lag impact is the significant difference from deterministic model. The total covariance matrix in time and frequency spread channels is formed by weighting sum of partial ones, the weights are equal to eigenvalues. One likes to simplify models which are used in treatment algorithm construction forasmuch as above mentioned models complexity. It is very important to control characteristics decay forasmuch as mentioned simplicity.

Let us analyze sub optimal [1] receiver characteristics. The moments generating functions was required for different interactive situations of three hypothesis, which are connected with the index p value, p=0,1,2:

$H_0(K_0,m_0)$ - noise signal is received with simpler characteristics than is true,
$H_1(K_1,m_1)$ - useful signal is received with simpler characteristics than is true,
$H_2(K_2,m_2)$ - true description of noise and/or useful signal.
K_p means correlation function and m_p - theoretical average, both for process

$$b(t) \sum_{q=1}^{m} \sqrt{b_{Lq}} s_{pLq}(t).$$

The set task peculiarity is that two different description of hypothesis H_1 and H_2 are under consideration when there is sub optimal treatment, and third description H_0 is involved when there is colored and possibly anisotropic noise. Nevertheless it is possible to find eigenfunction system [1], in which eigenvalues are independent on both hypothesis of any pair. One can believe that eigenfunctions $\Phi_{Lq}(\lambda)$ series becomes Fourier series, if considered time interval between initial T_{in} and end T_{fin} of treatment is mach greater than correlation interval. One can get for moment generation function after integrating variable full square allocation in exponent, integration and alike coercion

$$\mu(s)=(s/2)\ln |K_0| / |K_1| -(1/2)\ln\left|sK_2K_1^{-1}-sK_2K_0^{-1}+I\right|+$$
$$+\left[-(s/2)m_1^T K_1^{-1}m_1+(s/2)m_0^T K_0^{-1}m_0-(1/2)m_2^T K_2^{-1}m_2\right]-$$
$$\left[-(s/2)K_1^{-T}m_1+(s/2)K_0^{-T}m_0-(1/2)K_2^{-T}m_2\right]^T\left[-(s/2)K_1^{-1}+(s/2)K_0^{-1}-(1/2)K_2^{-1}\right]^{-1}$$
$$*\left[-(s/2)K_1^{-T}m_1+(s/2)K_0^{-T}m_0-(1/2)K_2^{-T}m_2\right].$$

Let us give interpretive execution to $K_i^{-1}K_j^{-1}K_k^{-1}=K_p^{-1}$. Let us use inverse kernel concept [1], which is defined for K_p^{-1} in temporal region as follows:

$$K_p^{-1}=(2/N_j)[\delta(v-w)-h_p(v,w|T_{fin}-T_{in})],\ T_{in}<v,w<T_{fin}$$

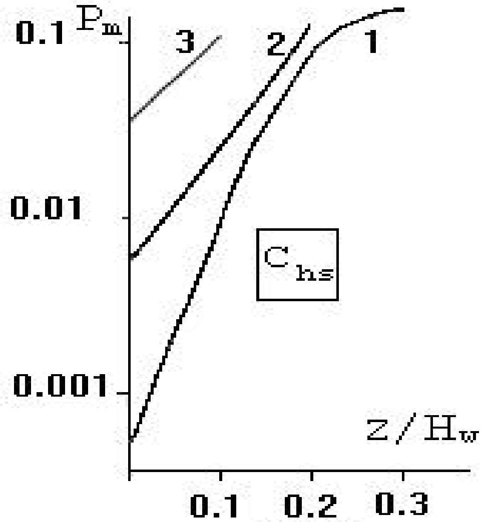
where h_p – is an impulse response, which is accorded with the correlation matrix K_p [1]. With the usege of inverse kernel in component that contained theoretical mean one can write convenient for temporal treatment form after differentiation on T_{fin} and T_{in} analogously [1]

$$m_i^T K_p^{-1} m_j=(2/N_p)\int_{T_{in}}^{T_{fin}} [m_i(t)-\int_0^t h_p(t,u|T_{fin}-T_{in})*m_j(u)du]$$

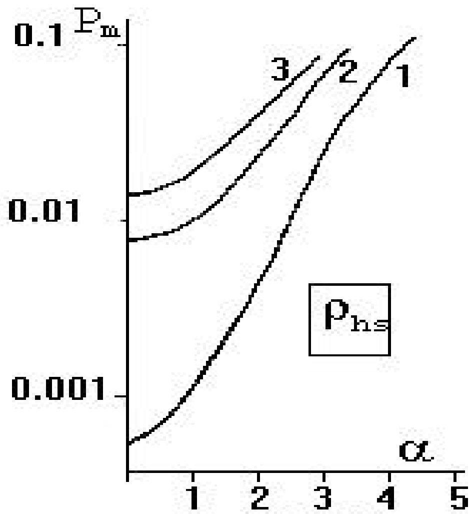
$$[m_j(\tau)-\int_0^t h_p(t,v|T_{fin}-T_{in})*m_i(v)dv]dt.$$

Above used inverse kernel concept makes it plain [1] that for zero mean signal moments generation function is designated by filtration errors sum of three process. First two processes, according to [1], are designated by H_0 and H_1 hypothesis. The last is combined process. In the contrary to [1] it consist of three components. The first is unity white noise. The second is the difference of hypothesis Í$_2$ process and filtered process, where filter is optimal for Í$_1$ hypothesis but excited Í$_2$ hypothesis signal, the difference is normalized to Í$_1$ white noise (in other words: whitened Í$_1$ hypothesis filter excited Í$_2$ hypothesis signal). The third is analogous to second, where Í$_1$ hypothesis role plaid hypothesis Í$_0$. The monotonic character is well known for signal parameters estimation error and observed signal value prediction error. Analogously , the signal detection error probability is described by the stochastic signal value forecasting error as well. Such processes correspond to signals, which models was used for likelihood as well as whitening filters characteristics. If signals have coherent components than corresponding filters are used during its treatment. Those circumstances determine the fundamental role of consensual optimal filter and connected with it mistakes. Those filtration errors is the model forecasting observed signal values errors. Theirs characteristics are necessary for forecastings both estimation error and detection error probability. Coincidence of model forecasting error with experimentally observed ones or residuals determine not contradictory models.

As example of usege deduced ratios let us consider qualitative illustrations. The signal on one receiving element isn't exact signal copy on the other one if waveguide destroys the signal. They say that there are coherent and incoherent parts of signal. Let us suppose that waveguide parameters exactly known, signal is within the bandpass of one eigenfunction, noise signal energy is twice the additive corrupted noise, receiving linear aperture has 20 receivers half wavelength spread, treatment interval is equal to correlation interval. For the false alarm probability $P_{fa}=10^{-2}$ the missing probability P_m varies (linearly for logarithm gauge) from 9,63% for entirely coherent signal to 23,2% for signal with 70% coherent part. For the 50 receivers aperture and the false alarm probability $P_{fa}=10^{-4}$ with the same others parameters missing probability P_m varies from 2,13% to 14,3%.



Pic.1.



Pic.2.

Next let us consider the situation when waveguide doesn't destroy the signal, but one use wrong waveguide parameters values. There are illustrations for P_m, when $P_{fa}=10^{-4}$ on pictures. The horizontal linear bottom array received bandpass noise like signal, distance between array and source is 250 waveguide thickness (H_w), source depth is $0{,}42H_w$, source course is 30° to array normal. There are focal course overshooting on abscissa axis, in pic.1 overshooting on source depth (z), in pic.2 overshooting on source course to array normal in horizontal play (α, degree). True waveguide parameters corresponds to number 1. Waveguide parameters enlargement corresponds to number 2, longitudinal wave speed c_{hs} in half space enlarged 1.5 times in pic.1, half space material density (ρ_{hs}) enlarged 1.2 times in pic.2. Waveguide parameters decrease corresponds to number 3, c_{hs} decrease 1.5 times in pic.1, ρ_{hs} decrease 1.25 times in pic.2. As mentioned above, error probability depends on models of experimentally observed signals fluctuations over forecasting values. True model rejection probability ought to be scheduled because of necessity fluctuations descriptions. Otherwise it is impossible to make a decision of experimental observations and used model not contradictoriness, or in other words of adequacy degree for situation used in signal detection error probability calculations and aquatory occur one. The model simplification result in error decision probability increase, which may be estimated only when not contradictory model is known.

REFERENCES

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