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VERTICAL CYLINDRICAL ARRAY IN PEKERIS WAVEGUIDE

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The field expansion of vertical cylindrical array in rigid screen in terms of Pekeris waveguide normal mode was obtained. The numerical analysis of the impedance of array emission and its components responsible for radiation into waveguide and half-space was carried out.

Numbers of paper are devoted to investigation of the sound field of directional projector working in a waveguide. Among them noteworthy are the articles [1], [2] presenting a detailed analysis of power and field characteristics of the directional projector in waveguide with ideal boundaries and those articles [3], [4] which give the generalization of the theory for real waveguides. However they do not involve results of the numerical analysis.

This paper demonstrates the discrete representation of the field of vertical cylindrical array in a rigid screen as expansion in terms of complete system of the Pekeris waveguide normal modes including properly normal, quasinormal (generalized) and leaking waves. The corresponding boundary problem can be represented as $\Delta j_1 + k_1^2 j_1 = 0;$ (1)

$$z = 0,\ \ j_1 = 0,\ z = h,\ \frac{\partial j_1}{\partial z} + i r_{12} k_{32} j_1 = 0;$$
$$r = a,\ \ -\frac{\partial j_1}{\partial r} = v_0 f(z), \qquad (2)$$

where $j_1(r,z)$ - velocity potential in the liquid layer having the thickness h, $r_{12} = r_1 / r_2$; r_1, r_2 - density of the layer and lower half-space respectively, $k_1 = w / C_1$; $k_{32} = \sqrt{k_2^2 - x^2}$; $k_2 = w / C_2$; C_1, C_2 - sound speed in the liquid layer and half-space respectively, w - circular frequency, x - propagation constant, v_0, $f(z)$ - particle velocity amplitude and distribution function giving on the side surface of projector.

Eigenfunctions of the boundary problem (1) at time dependence as $\exp(iwt)$ take the form:

$j_{1n}(r,z) = j_{1n}(z) H_0^{(2)}(x_n r)$,

$j_{1n}(z) = \sin(k_{31,n} z)$ - cross-section function, $k_{31} = \sqrt{k_1^2 - x^2}$, and propagation constants x_n satisfy the dispersion equation:

$$\cos(k_{31} h) + i r_{12} \frac{k_{32}}{k_{31}} \sin(k_{31} h) = 0. \qquad (3)$$

Let us select four subsets among solutions of the equation (3): subset $n(1)$ of the solutions $x_n^{(1)}$ corresponding to properly normal waves for which $\mathrm{Im}\, x_n^{(1)} = 0$, $\mathrm{Re}\, k_{32,n}^{(1)} = 0$, $\mathrm{Im}\, k_{32,n}^{(1)} < 0$, subset $n(2)$ of the solutions $x_n^{(2)}$ corresponding to quasinormal (generalized) waves for which $\mathrm{Im}\, x_n^{(2)} = 0$, $\mathrm{Re}\, k_{32,n}^{(2)} = 0$, $\mathrm{Im}\, k_{32,n}^{(2)} > 0$, subset $n(3)$ of the solutions $x_n^{(3)}$ corresponding to leaking waves for which $\mathrm{Re}\, x_n^{(3)} > 0$, $\mathrm{Im}\, x_n^{(3)} < 0$, $\mathrm{Re}\, k_{32,n}^{(3)} > 0$, $\mathrm{Im}\, k_{32,n}^{(3)} > 0$, subset $\bar{n}(3)$ of the reverse leaking waves for which $x_n^{(\bar{3})} = -(x_n^{(3)})^*$, $k_{32,n}^{(\bar{3})} = -(k_{32,n}^{(3)})^*$.

The problem of expansion of an arbitrary function $f(z)$ into a series on eigenfunctions of the problem (1) essentially is reduced to definition of the range and conditions of the generalized orthogonality for a complete set of the eigenfunctions pointed above. Let us prolong the field represented by functions $j_{1n}(z)$ continuously on the pressure to lower half-space with pseudoeuclidean metric and form two systems of the functions describing horizontal flux of power

$$p(z)=\begin{cases} p_1(z)=\sin(k_{31}z); & z\in(0,h), \\ p_2(z)=\sin(k_{31}h)e^{-ik_{32}(z-h)}; & (z-h)\in(0,\pm ih), \end{cases}$$

$$v(z)=\begin{cases} v_1(z)=\sin(k_{31}z); & z\in(0,h), \\ v_2(z)=r_{12}\sin(k_{31}h)e^{-ik_{32}(z-h)}; & (z-h)\in(0,\pm ih). \end{cases}$$

Allow two ranges of definition for coordinate z, which intersection is the segment $z\in(0,h)$: $R^{\pm}=z_h\cup z_h^{\pm}$; $z_h\in(0,h)$, $z_h^{\pm}\in(h,h\pm ih)$.

The relations of generalized orthogonality for eigenfunctions of sets $n(3)$, $n(2)$ in R^- and sets $\bar{n}(3)$, $\bar{n}(2)$, $n(1)$ in R^+ have a canonical form:

$$(p_n,v_m)^- = \int_{R^-} p_n(z,x_n^-)v_v^*(z,x_n^{-*})dz=\frac{h}{2}E_n d_{nm},$$
$$(p_n,v_m)^+ = \int_{R^+} p_n(z,x_n^+)v_v^*(z,x_n^{+*})dz=\frac{h}{2}E_n^* d_{nm}, \tag{4}$$

where x_n^- - solutions corresponding to sets the $n(3)$, $n(2)$; x_n^+ - solutions corresponding to the sets $\bar{n}(3)$, $\bar{n}(2)$, $n(1)$; d_{nm} - Kronecker delta, $E_n=1-\dfrac{\sin(2k_{31,n}h)}{2k_{31,n}h}-ir_{12}\dfrac{\sin^2(k_{31,n}h)}{k_{32,n}h}$.

Assume now the function $f(z)$ in (2) in the form: $f(z)=\begin{cases}1, & z\in(z_0-l,\ z_0+l), \\ 0, & z\,\overline{\in}\,(z_0-l,\ z_0+l),\end{cases}$

where z_0 - coordinate of a center of projector with length $2l$.
The desired for expansion of boundary function can be sought as two representations

$$z\in R^-,\ f(z)=\sum_{n^-}a_n^- v_n(z,x_n^-),\ n^-=n(3)\cup n(2);$$
$$z\in R^+,\ f(z)=\sum_{n^+}a_n^+ v_n(z,x_n^+),\ n^+=\bar{n}(3)\cup\bar{n}(2)\cup n(1); \tag{5}$$

The relations of orthogonality (4) allow to define coefficients of expansions (5)

$$a_n^{\pm}=\frac{4l}{h}v_0\frac{j_{1n}(z_0,x_n^{\pm})}{E_n}\Phi_n,\ \Phi_n=\frac{\sin(k_{31,n}l)}{k_{31,n}l}.$$

Take a half-sum of expansion (5) as the desired expansion of real function on the interval $(0,h)$

$$f(z)=\frac{1}{2}\left\{\sum_{n=1}^{N^-}a_n^- j_{1n}(z,x_n^-)+\sum_{n=1}^{N^+}a_n^+ j_{1n}(z,x_n^+)\right\}+\mathrm{Re}\sum_{n=1}^{\infty}a_n^{(3)}j_{1n}(z,x_n^{(3)}), \tag{6}$$

where N^-, N^+ - number of quasinormal (generalized) and normal waves respectively, and only the leaking waves of set $n(3)$ are summarized in the last addend of expression (6).

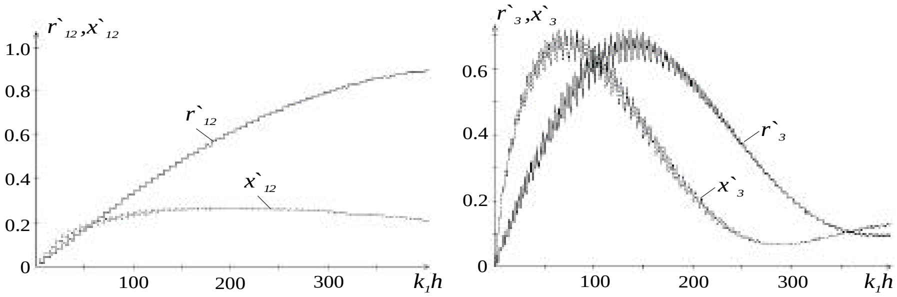
Let us represent the field in the waveguide as similar sum of all normal waves of $n(1)$, $n(2)$, $\bar{n}(2)$, $n(3)$ - types satisfying the radiation conditions $(dx_n/dw>0,\ \mathrm{Re}\,k_{32,n}\geq 0)$. Allowing (4) and (6) we take desired expansion

$$j_1(r,z)=\frac{2l}{h}v_0\sum_n\frac{j_{1n}(z_0)j_{1n}(z)}{x_nE_nH_1^{(2)}(x_na)}\Phi_n e_n H_0^{(2)}(x_nr), \tag{7}$$

where $e_n=\begin{cases}1-\ \text{for waves } n(1),\ n(2),\ \bar{n}(2)-\text{types}, \\ 1+\dfrac{a_n^*}{a_n}r_{12}\dfrac{|\sin(k_{31,n}h)|^2}{E_{1n}\,\mathrm{Im}(k_{32,n}h)}\ -\text{for waves } n(3)-\text{type},\end{cases}$ $\quad E_{1n}=1-\dfrac{\sin(2k_{31,n}h)}{2k_{31,n}h}$.

Let us calculate the radiation impedance $Z_R = 4\pi a l \rho_1 c_1 Z'_R$ as an energy characteristic of the projector $Z'_R = 2k_1 l \sum_n \frac{\sin^2(k_{31,n} z_0)}{\xi_n h E_n} \Phi_n^2 \varepsilon_n \frac{iH_0^{(2)}(\xi_n r)}{H_1^{(2)}(\xi_n a)} = Z'_{12} + Z'_3$,

where $Z'_{12} = r'_{12} + ix'_{12}$, $Z'_3 = r'_3 + ix'_3$ - components responsible for radiation into waveguide and half-space. A frequency dependence of the components of Z'_{12}, Z'_3 is illustrated in Fig. 1 and Fig. 2. In calculations it is assumed that $\rho_{12} = 1/1.6$, $C_{12} = 1.5/1.7$.



a) b)

Fig. 1. (a) Frequency dependence of the impedance components z'_{12}, $l/h = 10^{-2}$, $a = l$;

(b) frequency dependence of the impedance components z'_3, $l/h = 10^{-2}$, $a = l$.

The discrete structure of frequency dependences completely corresponds to a discrete structure of normal waves. In the limiting case $k_1 h \to \infty$ the routine relations usual for radiation theory take place such as $r'_{12} \to 1$, $x'_{12} \to 0$, and also physically clear relations $r'_3 \to 0$, $x'_3 \to 0$ describing the process of redistribution of the radiated power between waveguide and the half-space.

a) b)

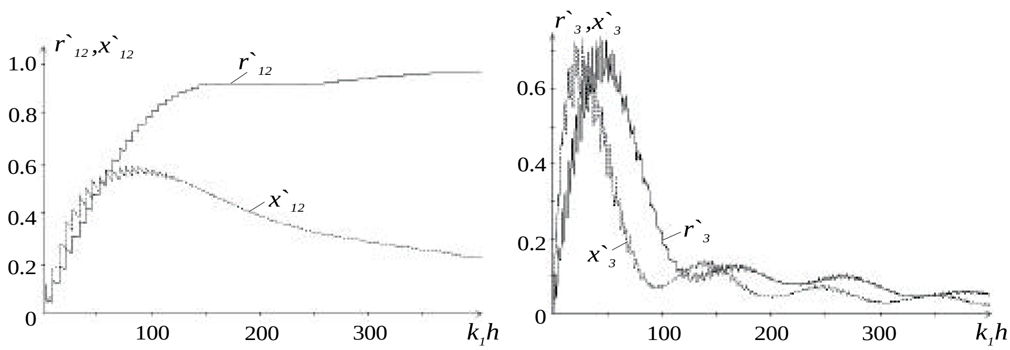


Fig. 2. (a) Frequency dependence of the impedance components z'_{12}, $l/h = 3 \cdot 10^{-2}$, $a = l$;

(b) frequency dependence of the impedance components z'_3, $l/h = 3 \cdot 10^{-2}$, $a = l_3$.

The global maxima for component r'_3 well correspond to Fresnel's law in accord with which the radiation is maximum, when the odd number of half-wavelengths is placed on the array aperture.

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