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RESONANCE THEORY OF ACOUSTIC WAVES INTERACTING WITH ELASTIC LAYERED BOTTOM

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The exact expression for the reflection coefficient has been obtained with the Thomson-Haskell technique for the bottom model consisting of an elastic homogeneous layer overlying an elastic half-space. This expression was transformed to more convenient for the resonance decomposition form. Inspection of the exact expression for the reflection coefficient shows that the resonance behaviour will exhibit when the real part of the denominator vanishes. The characteristic equations in resonance conditions were obtained. The roots of the characteristic equations were found and the analytical expressions for angular and frequency resonance positions relatively the compressional and shear wave velocities were derived. The frequency and angular resonance positions were determined both using the resonance theory and the exact computations. The analytical expression for the resonance reflection coefficient was obtained. Comparison between the resonance approach and the exact computation was performed. The excellent agreement was obtained in the vicinity of the frequency and angular resonances.

EXACT REFLECTION COEFFICIENT

The physical model used in the study of the frequency-angular resonances consist from elastic layer covering an elastic half-space. In the layer, including the substrate, effects of the attenuation are taken into account by assuming shear and compressional wave velocities are complex. It is requires the complex wave numbers. The solution of the matrix equations system for the reflection coefficient V can be obtained by the Kramer's rule. The exact expression for the complex reflection coefficient may be written using Thomso-Haskell [1] technique:

$$V=(Q^{-1}_{21}(D_{21}D_{33}-D_{23}D_{31})-Q^{-1}_{22}(D_{43}D_{31}-D_{41}D_{33}))/(Q^{-1}_{11}(D_{21}D_{33}-D_{23}D_{31})-Q^{-1}_{12}(D_{43}D_{31}-D_{41}D_{33})). \qquad (1)$$

The exact expression for the reflection coefficient don't reduced due to it awkwardness. This expression may be written in the compact form, where the real and imaginary parts of the reflection coefficient are selected:

$$V=\frac{E^2-B^2+C^2-G^2}{E^2+2EB+B^2+C^2+2CG+G^2}+\frac{i(2CB+2GE)}{E^2+2EB+B^2+C^2+2CG+G^2}. \qquad (2)$$

Here the functions E, B, C and G connected both the material parameters of media and the angle-frequency-thickness variables $\delta=\alpha d$ and $\eta=\beta d$. The reflection coefficient has a complex structure consisting from the regular sequences of mimima and maxima. These features connected with the reflection coefficient resonances.

RESONANCE FORMALISM

Preliminary analysis of the exact Eq.2 for the reflection coefficient was performed for the elastic two-layers model. So, the exact expression for the reflection coefficient (2) was transformed to more suitable form for the resonance decomposition by selecting of the real and imaginary parts in the denominator of the reflection coefficient:

$$V=\frac{(E^2-B^2+C^2-G^2)^2+(2CB+2GE)^2}{(E^2+2EB+B^2+C^2+2CG+G^2)(E^2-B^2+C^2-G^2-i(2CB+2GE))} \qquad (3)$$

The resonance behaviour exhibited when the real part of the denominator of the reflection coefficient vanishes, i.e., when the characteristic equations are satisfied:

$$E^2-B^2+C^2-G^2=0, \qquad (4)$$

$$(E+B)^2+(C-G)^2=0. \qquad (5)$$

After variables separation the characteristic equation (4) may be written in the form:

$$K_1\cos(\delta)\cos(\eta)+K_2\cos(\delta)^2\cos(\eta)^2+K_3\sin(\delta)\sin(\eta)+K_4\cos(\delta)^2+K_5\cos(\eta)^2+K_6\cos(\delta)\cos(\eta)\sin(\delta)\sin(\eta)+K_7=0, \qquad (6)$$

here the constants K_1 - K_7 are connected only with the material parameters of the layered elastic media. In the case of the elastic layer covering the elastic half-space the characteristic equation (6) may be solved separately for δ and η variables. Solutions of this equation will be determine the positions of the resonances for the compressional and the shear wave on the angular-frequency plane. Solutions for δ and η may be obtained in the form of the Decart-Eyler's solutions by solving two additional 4-th order equations:

$$(-K_1\cos(\eta) + K_2\cos(\eta)^2 + K_4 + K_5\cos(\eta)^2 + K_7)X^4 + (2K_3\sin(\eta) - 2K_6\cos(\eta)\sin(\eta))X^3 + (-2K_2\cos(\eta)^2 - 2K_4 + 2K_5\cos(\eta)^2 + 2K_7)X^2 + (2K_3\sin(\eta) + 2K_6\cos(\eta)\sin(\eta))X + K_1\cos(\eta) + K_2\cos(\eta)^2 + K_4 + K_5\cos(\eta)^2 + K_7 = 0, \quad (7)$$

$$(-K_1\cos(\delta) + K_2\cos(\delta)^2 + K_4\cos(\delta)^2 + K_5 + K_7)X^4 + (2K_3\sin(\delta) - 2K_6\cos(\delta)\sin(\delta))X^3 + (-2K_2\cos(\delta)^2 + 2K_4\cos(\delta)^2 - 2K_5 + 2K_7)X^2 + (2K_3\sin(\delta) + 2K_6\cos(\delta)\sin(\delta))X + K_1\cos(\delta) + K_2\cos(\delta)^2 + K_4\cos(\delta)^2 + K_5 + K_7 = 0, \quad (8)$$

or the 4-th order equation in the general form: $X^4 + a X^3 + b X^2 + c X + d = 0$. Then roots of the Eq. (7,8) may be found as: $\delta = 2\mathrm{arctg}(X)$, $\eta = 2\mathrm{arctg}(X)$. (9)

The period of the arctangent is π, so the resonance positions δ and η for compressional and shear wave velocities will be determined: $\delta_n = \delta + 2\pi n$, $\eta_n = \eta + 2\pi n$, (10)

here n=0,1,2,… is the resonance number, X_1 - X_4 are the roots of the 4-th order general equation were obtained analytically. Simultaneously, the exact angle-frequency-thickness variables δ' and η' can be written as the functions of the frequency and angular variables, relatively the compressional and shear wave velocities:

$$\delta' = \frac{2pFd\cos(q_l)}{c_l}, \quad \eta' = \frac{2pFd\cos(q_t)}{c_t}. \quad (11)$$

The resonances of the reflection coefficient will exhibit when the difference between the exact angle-frequency-thickness variables δ' or η' and the resonance variables δ_n or η_n aspires to zero: $|\delta'-\delta_n|=0$, $|\eta'-\eta_n|=0$. Its are the resonance conditions for the reflection coefficient, when the reflection coefficient has the resonance minima. The Figure 1 shows both the exact computation of the reflection coefficient at the fixed incidence angle $\theta=61^o$ and the difference between δ' and δ_n. The variables δ' and η' are the functions of the frequency and the incidence angle. So, holding the angular and frequency variables fixed, the positions of the frequency and angular resonances will be written for the compressional and shear waves separately. Comparison between the resonance positions obtained by the numerical method from the exact reflection coefficient and the resonance positions derived analytically from the characteristic equations (7,8) shown on the frequency-incidence angle plane for the first resonances (Fig.1 b). The dependence of the resonance positions versus the compressional wave velocities in the layer is clear observed. The resonances associated with δ were calculated for three different compressional wave velocities in the sediment. The exact computation of the resonance positions shown in the (Fig. 1 b) as solid line and the resonance approach computation selected by markers.

a) b)

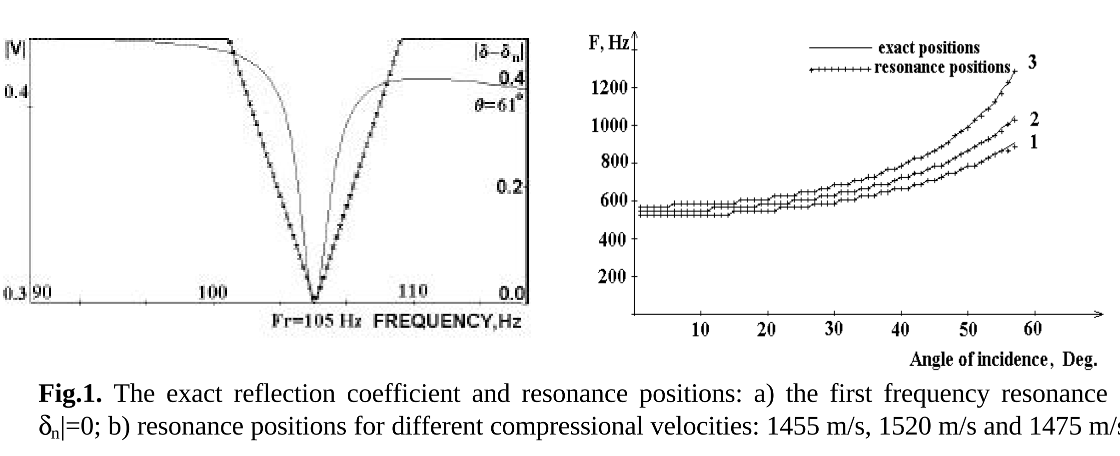

Fig.1. The exact reflection coefficient and resonance positions: a) the first frequency resonance $|\delta'-\delta_n|=0$; b) resonance positions for different compressional velocities: 1455 m/s, 1520 m/s and 1475 m/s.

COMPARISON OF RESONANCE APPROACH AND EXACT COMPUTATIONS

In the vicinity of a resonance the amplitude for a process is described essentially by a Breit-Wigner resonance form plus some slowly varying background [2]. Following this idea the exact expression for the reflection coefficient was expanded in the Taylor series on the powers $(\delta^{`}-\delta_n)$ and $(\eta^{`}-\eta_n)$ around the resonance positions δ_n and η_n:

$$V(\delta^{`},\eta^{`}) \cong V(\delta_n)+V(\eta_n)+V'(\delta_n)(\delta^{`}-\delta_n)+V'(\eta_n)(\eta^{`}-\eta_n)+\ldots \qquad (12)$$

mathematically, it is correspond to retaining only the linear terms in the expansion, the first derivatives are equal to zero due to the Ferma's theorem: $V'(\delta_n) = 0$, $V'(\eta_n) = 0$. So, the resonance expression for the reflection coefficient may be obtained as a sum of the resonance terms, both in the frequency and angular variables. Retaining only linear terms in expansions the resonance expression for the reflection coefficient may be written in the form of the analytical expression in couple pages.

a) b)

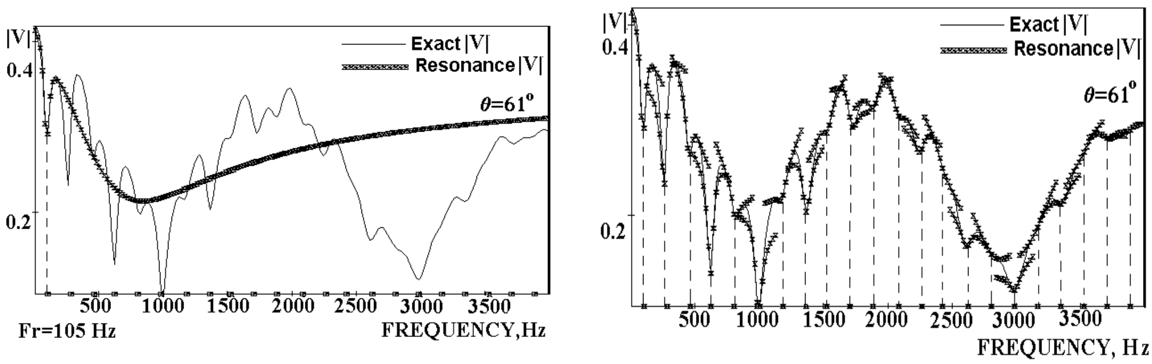

Fig.2. Frequency resonances: a) for the first resonance peak; b) for 22 resonance peaks.

Efficiency of the resonance expression for the elastic layered model, is illustrated in the Figure 2. The Figure 2 a compares the prediction of the frequency resonance approximation with the exact computations for the first frequency resonance of the reflection coefficient, the incidence angle is fixed (θ=61^o). It is seen that the agreement is excellent in the vicinity of the first resonance. The sum over resonances is taken only symbolically since the expansion is assumed to be valid only in the immediate vicinity of each resonance position but is expected to fail at large distances away from the resonance. The solid line corresponds to the exact computation and the markers are present the resonance approach. The Figure 2 b compares the prediction of the resonance approximation with the exact expression of the reflection coefficient for 22 resonance peaks. It is seen, that the approximation fails, as expected, between the resonance peaks and excellent in the vicinity of the resonances.

If the frequency variable is fixed (F=3890 Hz) the positions of the angular resonances may be obtained. The comparison of the resonance approximation and the exact computation of the reflection coefficient are shown for the first angular resonance (Fig.3 a) and for four angular resonances (Fig.3 b). Again, the agreement is excellent in the vicinity of the resonances and fails between the resonance peaks.

a) b)

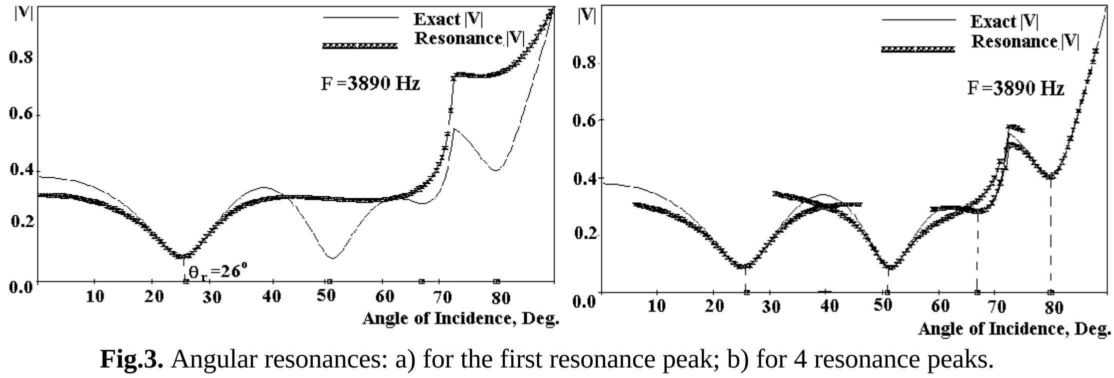

Fig.3. Angular resonances: a) for the first resonance peak; b) for 4 resonance peaks.

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REFERENCES

1. *Thomson W.* Transmission of elastic waves through a stratified solid material// J. Appl. Phys. 1950. V.21. № 1. P.89.
2. *Nagl A.,Überall H., Hoover W.R.* Resonances in acoustic bottom reflection and their relation to the ocean bottom properties// IEEE TGRS. 1982. V.GE-20. №3. P.332.