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THE NATURAL PARABOLIC EQUATIONS FOR THE SEISMOACOUSTICS OF THE RANGE-DEPENDENT OCEAN AND THEIR NUMERICAL SOLUTIONS

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The paper offers an effective algorithm for the computation of the seismoacoustic field. The underlying physics is similar to the pseudodifferential parabolic equation technique. The algorithm deals with stresses and velocities only so it admits an effective numerical implementation for the arbitrary dependence of medium properties on the space coordinates including the discontinuities of the first kind.

Consider the three-dimensional ocean possessing the translational symmetry along the one of the horizontal cartesian axes or the rotational symmetry around the vertical axis. Ðàññìîòðèì òðåõìåðíóþ ìîäåëü îêåàíà, ñâîéñòâà êîòîðîé èíâàðèàíòíû îòíîñèòåëüíî ñäâèãà âäîëü îäíîé èç äåêàðòîâûõ êîîðäèíàò èëè îòíîñèòåëüíî ïîâîðîòà âîêðóã âåðòèêàëüíîé îñè. The isotropic elastic ocean bottom in this case gives way to two types of waves namely the Love waves (torsional, SH) and the Rayleigh waves. In the Love waves the displacement is perpendicular to the wave direction. In the Rayleigh waves the displacement lies in the wave direction and is accompanied by the changes in the specific volume [i]. The point source in the water column over the elastic bottom generates the Rayleigh waves only. The equations governing these waves in the translationally symmetric medium in the absence of sources are:

$$\begin{pmatrix} iw\,r & D_z & 0 & 0 & D_x \\ mD_z & iw & 0 & mD_x & 0 \\ 0 & D_x & D_z & iwr & 0 \\ (l+2m)D_x & 0 & 0 & lD_z & iw \\ lD_x & 0 & iw & (l+2m)D_z & 0 \end{pmatrix} \begin{pmatrix} v_x \\ s_{xz} \\ s_{zz} \\ v_z \\ s_{xx} \end{pmatrix} = 0 \qquad (1)$$

with appropriate linear conditions at the waveguide boundaries. Here v_x and v_z are the cartesian components of the velocity of the medium particles, σ_{xx}, σ_{xz} and σ_{zz} are the cartesian components of stress tensor, D_x and D_z are the derivative operators, r is the medium density, l and m are the Lame coefficients, w is the circular frequency, i is the imaginary unit and the harmonic time dependency $\exp(-iwt)$ is assumed. The first and the third equations are the Newton equations and the rest three represent the Hooke's low. We use the velocities instead the displacements in (1) to match the equations of the acoustic of the compressible liquid without viscosity as $m \to 0$.

We solve now the equations (1) respectively the derivatives D_x and D_z by the multiplying the last two of them by the matrix $l \;\; 2 \quad l \; ^{-1}$ $\; l \;\; 2$

$$\begin{pmatrix} iw\,r & D_z & 0 & 0 & D_x \\ D_z & iw/m & 0 & D_x & 0 \\ 0 & D_x & D_z & iwr & 0 \\ D_x & 0 & -iwl/(4m(l+m)) & 0 & iw(l+2m)/(4m(l+m)) \\ 0 & 0 & iw(l+2m)/(4m(l+m)) & D_z & -iwl/(4m(l+m)) \end{pmatrix} \begin{pmatrix} v_x \\ s_{xz} \\ s_{zz} \\ v_z \\ s_{xx} \end{pmatrix} = 0 \qquad (2)$$

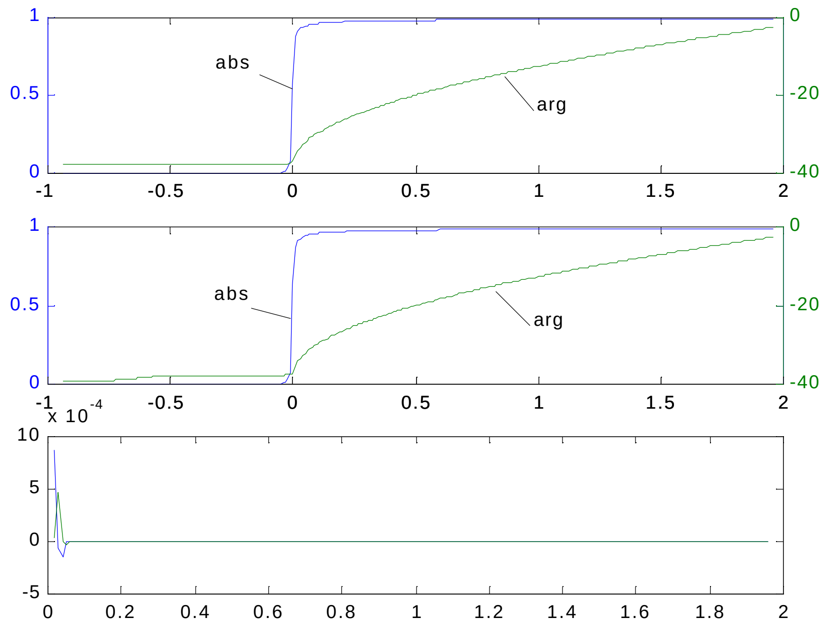
Now one can easily see as $m \to 0$ that the second equation gives $s_{xz} = 0$, the last gives $s_{xx} = s_{zz}$ and the sum of the last two tends to the equation of the continuity of the liquid medium.

Consider now the layered medium. All solutions display then the exponential dependence $\exp(ipx)$ upon the horizontal coordinate x providing the complex p permits the nonzero solution of the system of the ordinary linear differential equations

$$\begin{pmatrix} iw\,r & D_z & 0 & 0 & ip \\ D_z & iw/m & 0 & ip & 0 \\ 0 & ip & D_z & iwr & 0 \\ ip & 0 & -iwl/(4m(l+m)) & 0 & iw(l+2m)/(4m(l+m)) \\ 0 & 0 & iw(l+2m)/(4m(l+m)) & D_z & -iwl/(4m(l+m)) \end{pmatrix} \begin{pmatrix} v_x \\ s_{xz} \\ s_{zz} \\ v_z \\ s_{xx} \end{pmatrix} = 0 \qquad (3)$$

satisfying the boundary conditions. The physical sense tells us that the right and left going waves are equivalent so, if the solution of (3) exists with the factor $\exp(ipx)$, then the same solution with the factor $\exp(-ipx)$ is valid and, therefore, the boundary problem with p must include only its even powers. One can see that the eigenfunctions or eigenvectors $(V_x \quad \Sigma_{xz} \quad s_{zz} \quad v_z \quad s_{xx})^T$ and the eigenvalues q of the boundary problem

$$\begin{pmatrix} iw\,r & D_z & 0 & 0 & -q \\ D_z & iw/m & 0 & -q & 0 \\ 0 & 1 & D_z & iwr & 0 \\ 1 & 0 & -iwl/(4m(l+m)) & 0 & iw(l+2m)/(4m(l+m)) \\ 0 & 0 & iw(l+2m)/(4m(l+m)) & D_z & -iwl/(4m(l+m)) \end{pmatrix} \begin{pmatrix} V_x \\ \Sigma_{xz} \\ s_{zz} \\ v_z \\ s_{xx} \end{pmatrix} = 0 \qquad (4)$$



supply the two eigenvalues $+i\sqrt{q}$ and $-i\sqrt{q}$ and the two eigenvectors $(i\sqrt{q}V_x \quad i\sqrt{q}\Sigma_{xz} \quad s_{zz} \quad v_z \quad s_{xx})^T$ and $(-i\sqrt{q}V_x \quad -i\sqrt{q}\Sigma_{xz} \quad s_{zz} \quad v_z \quad s_{xx})^T$ of the boundary problem (3). Here $\sqrt{q}$ denotes only the one branch of the square root. The physical meaning of such correspondence is that it reveals the different signs of v_x and s_{xz} for the waves of opposite directions. The similar technique was used in [ii]. One can see from (4) that the eigenproblem really contains only s_{xx} and v_z while the other components of the eigenvector are their linear functions. Now one can write for the rightgoing Rayleigh waves propagating through the segment of the layered waveguide of width h the sum of modes:

$$\begin{pmatrix} v_z \\ s_{xx} \end{pmatrix}(x+h) = \begin{pmatrix} V \\ \Sigma \end{pmatrix} diag\,\exp(ih\sqrt{q_l}) \begin{pmatrix} V \\ \Sigma \end{pmatrix}^{-1} \begin{pmatrix} v_z \\ s_{xx} \end{pmatrix}(x) \qquad (5)$$

Here l is the mode number, $(V,\Sigma)^T$ is the matrix with the columns $(v_z, s_{zz})^T$. We assume here the simplicity of the spectrum of the problem (4) and, therefore, the orthonormality of its eigenvectors. Eq. (5) uses the mode basis. This basis diagonalizes the transversal differential operator (3). Denoting the operator (3) as T we can formulate the Eq. (5) in the basis independent way

$$\begin{pmatrix} v_z \\ s_{xx} \end{pmatrix}(x+h) = \exp(ih\sqrt{T})\begin{pmatrix} v_z \\ s_{xx} \end{pmatrix}(x) \qquad \textbf{(6)}$$

For the numerical implementation of the action on the vector by the function of the operator we can use the rational approximation of the function under interest ([iii], [iv]). We succeed to construct such an approximation of the 31 order combining the Pade approximation of the exponent and the square root with the cut on the ray $(0,-i\infty)$. The Figure shows its quality. The upper and the middle parts of the figure show the absolute value and the argument of the superposition of standard mathematical functions and our approximation respectively along the interval $(-1+0.01i, 2+0.01i)$. The lower part shows the absolute value minus unity and the argument of the ratio of the superposition and the approximation along the part of the above interval lying in the right half-plane.

Assuming now the matrices Z and Y to be the components of the finite-dimensional discrete approximation of the problem (4) by the generalized Marchuk's technique [v] we may implement the approximate solution of the Eq. (5) as

$$\begin{pmatrix} v_z \\ s_{xx} \end{pmatrix}(x+h) = \left(\sum_{k=1}^{31}\left(a + \frac{b_k Z}{Y - r_k Z}\right)\right)\begin{pmatrix} v_z \\ s_{xx} \end{pmatrix}(x)$$

Here a, b_k and r_k are the constants of the rational approximation.

Imagine now the range-dependent waveguide consisting of the segments of layered waveguides adjacent along the vertical interfaces. Eq. (6) supplies the algorithm for the computation of the seismoacoustic field in the one-way approximation neglecting the reflection and the difference of the transition operator at the vertical interfaces from unitary one. The papers [vi],[vii] provide the iterative algorithm for the computation of the full field of the waves of both directions in the range dependent waveguide based on the rational approximation.

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