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ELASTIC WAVES DIFRACTION AT THE MULTILAYERED CYLINDRICAL INHOMOGENIUTY WITH UNPERFACTED BOUNDARY BETWEEN LAYERS IN THE SOLID MEDIA.

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Numerical evaluation of reflective properties of the set of cylindrical elastic layers was carried out for the case of incident plane harmonic waves. Boundaries between layers are unperfected which are described by "linear slip" approximate boundary conditions.

Heterogeneities models of cylindrical shapes are widely used for theoretical investigation of unidirectional extended heterogeneities for practical non-destructive testing problems [1]. Widely-distributed one of them are cylindrical cavities and elastic cylinders with perfected boundary that determinates continuity of stress and displacement at the boundary. These models can not correspond to the structure of real flows and because of it they are need to modernize.

Among of early unknown or left out of account factors for modeling heterogeneities in metals one has to emphasize phenomena associated with violations of conditions of waves propagation through boundary "metal-flow". The particularities of these adhesive link violation phenomena appearing because of multiply microcontacts of contiguity crack faces was development in the work [2] in the case of heterogeneities plane models. This approach can be extended on heterogeneities with curved surfaces.

In this work particular case of reflection of the incident normally plane wave on the cylinder is considered. Let plane wave described by potential Ôi in elastic media (($\rho_{ì+1}$ -dencity, $\lambda_{ì+1}$,$\mu_{ì+1}$ -Lame constants) falls on the set of cylindrical layers "M" with radius of each layer - r_m (ρ_m-dencity, λ_m,μ_m - Lame constants, 0≤m≤M) placed as on the figure 1. According to [2] adhesive link violation at the cylindrical surfaces can be taken account by introduction of contact stiffness KGN_m, KGT_m or compliance $KPN_m=1/KGN_m$, $KPT_m=1/KGT_m$ which define elastic displacements in respective to normal and tangential directions at the boundaries. In the case of viscous friction at the boundary and in the media the Lamme constants and stiffness should de considered as complex values[3]. Elastic field in the outer media is described by scalar potential $\Phi = \Phi_i + \Phi_s^{(M+1)}$ and vector potential $\Pi_s^{(M+1)}$. In this problem vector potential has only one component z. because of problem symmetry In the core (m=0) elastic fields are determined by potentials $\Phi_q^{(m)}$ and $\Pi_q^{(m)}$. Submitted factor *exp(-iùt)* one can write:

$$\Phi_i = \Phi_0 \cdot \exp(ik_l^{(M+1)} r\cos(q)) = \Phi_0 \sum_{n=0}^{\infty} e_n (i)^n J_n (k_l^{(M+1)} r)\cos(nq)\ ,$$

$$\Phi_s^{(M+1)} = \sum_{n=0}^{\infty} e_n (i)^n A_n^{(M+1)} H_n^{(1)} (k_l^{(M+1)} r)\cos(nq)\ ,\ \ \Pi_s^{(M+1)} = \sum_{n=0}^{\infty} e_n (i)^n B_n^{(M+1)} H_n^{(1)} (k_t^{(M=1)} r)\sin(nq)\ ,$$

$$\Phi_q^{(0)} = \sum_{n=0}^{\infty} e_n (i)^n C_n^{(0)} J_n(k_l^{(0)} r) \cos(nq)\,, \qquad \Pi_q^{(0)} = \sum_{n=0}^{\infty} e_n (i)^n D_n^{(0)} J_n(k_t^{(0)} r) \sin(nq)\,, \qquad (1)$$

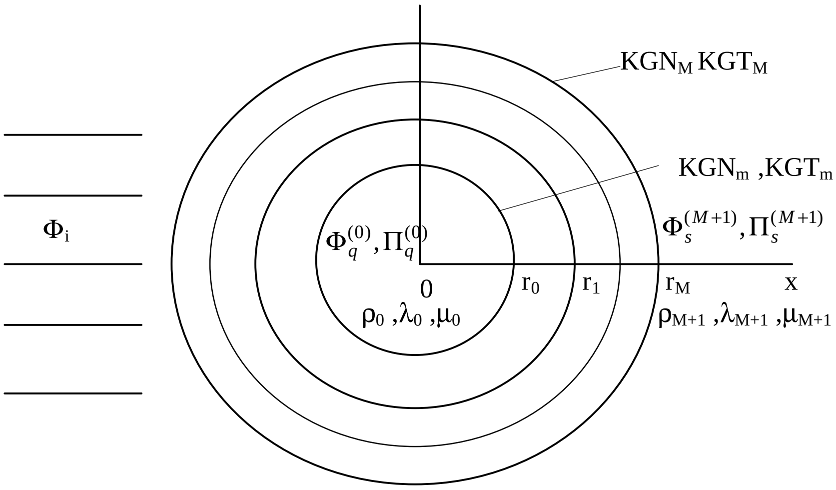

Fig. 1.

were $k_l^{(m)}, k_t^{(m)}$ – longitudinal and shear wave numbers in outer media and layers m, $J_n(kr)$, $H_n^{(1)}(kr)$ – Bessel function and Hankell function of first kind, $e_n = \begin{cases} 1, n = 0 \\ 2, n \succ 0 \end{cases}$ $\grave{e}$ – azimuth angle; $A_n^{(m)}, B_n^{(m)}, C_n^{(m)}, D_n^{(m)}$ - – unknown coefficients in media m and outer media.

"Linear slip" approximate boundary conditions at the each boundary between layers - r$_m$ one can write as:

$$s_{rr}^{(m)} = s_{rr}^{(m+1)}\,, \quad s_{rq}^{(m)} = s_{rq}^{(m+1)}\,, \quad u_r^{(m)} = u_r^{(m+1)} + \frac{s_{rr}^{(m)}}{KGN^{(m)}} \quad , U_q^{(m)} = U_q^{(m+1)} + \frac{s_{rq}^{(m+1)}}{KGT^{(m)}}\,. \qquad (2)$$

Similar with [3] one can shows that equations for determination of unknown coefficients can be rewritten in matrix form:

$$\begin{Bmatrix} U_r^{(0)} \\ U_q^{(0)} \\ s_{rr}^{(0)} \\ s_{rq}^{(0)} \end{Bmatrix} = \{KP^{(0)}\}\{ZS^{(1)}\}\{KP^{(1)}\}....\{KP^{(M-1)}\}\{ZS^{(M)}\}\{KP^{(M)}\}\begin{Bmatrix} U_r^{(M+1)} \\ U_q^{(M+1)} \\ s_{rr}^{(M+1)} \\ s_{rq}^{(M+1)} \end{Bmatrix}, \{KP^{(m)}\} = \begin{Bmatrix} 1 & 0 & KPN^{(m)} & 0 \\ 0 & 1 & 0 & KPT^{(m)} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{Bmatrix} (3)$$

where $\{ZS^{(m)}\}$-propagator matrix for layer of number m [3], $\{KP^{(m)}\}$ propagator matrix for compliance of boundary with number m.

Substituting (1) in expressions for elastic displacement component and expressions for elastic stress component in the cylindrical coordinate system [3] and using (3) the system for determination unknown coefficients was obtained. Solving this system was carried out by numerical methods. In this work to evaluate reflective properties of multilayered heterogeneities the normalized amplitude of stress radial component- RZL, and normalized scattering cross section- QZL were used. These characteristics were calculated according to next expressions:

$$RZL \approx \cdot \left| AL_0^{(M+1)} + 2\sum_{n=1}^{\infty} (-1)^n AL_0^{(M+1)} \right| \quad (4)$$

$$QZL = \frac{2}{k_l^{(M+1)}} \left\{ \left[(\left| AL_0^{(M+1)} \right|)^2 + 2\sum_{n=1}^{\infty} (\left| AL_0^{(M+1)} \right|)^2 \right] + \left[(\left| BL_0^{(M+1)} \right|)^2 + 2\sum_{n=1}^{\infty} (\left| BL_0^{(M+1)} \right|)^2 \right] \right\} \quad (5)$$

Numerical evaluation of expressions (4,5) were carried out for set of the materials combinations having physical parameters and wave sizes of heterogeneities which are typical metallurgical technology under simulation technique and practical conditions of ultrasonic non-destructive testing. Evaluation results for steel carbon matrix (ρ_2=7.8·10^3 kg/m^3,$ñ_{l2}$=5.85·10^3 m/ñ, $ñ_{t2}$=3.23·10^3 m/ñ.) are shown at the figures 2-7. The core of cylinder was metallurgical graphite (ρ_0=2.25·10^3 kg/m^3, λ_0=2.28·10^{10} N/m^2 μ_0=0.15·10^{10} N/m^2) [4]. In particular case under consideration single-layer it was used parameters like epoxy (ρ_1=1.15·10^3 kg/m^3,$ñ_{l1}$=2.5·10^3 m/ñ, $ñ_{t1}$=1.1·10^3 m/ñ).Absolute value of outer cylinder radius was (a=0.5 mm) relative thickness of layer: 0.1à. Numerical results according to expression (4) are shown at the figure 3, according to expression (5) are shown at the figure 5, Solid line present case of elastic cylinder under welded contact ($KGN^{(0)} = KGT^{(0)} = 10^{17} Í\ /ì\ ^3$) and absence of layer. Dotted line corresponds to the cavity of the same size. Broken curve – the layer of the same parameters under welded contact at the both boundaries ($KGN^{(0)} = KGT^{(0)} = KGN^{(1)} = KGN^{(1)} = 10^{17} Í\ /ì\ ^3$). You can see, the intermediate inner layer to be stiff contact at the boundaries of elastic layer, greatly changes resonance phenomena and reflected signal level. At the figures 4,5 broken curve correspond to the case of contact stiffness violations simultaneous at the both boundaries ($KGN^{(0)} = KGT^{(0)} = KGN^{(1)} = KGN^{(1)} = 10^{14} Í\ /ì\ ^3$), dotted line corresponds to taking into account attenuation in the graphite, η_{inc}=0.2. As comparison, solid line shows case of elastic layer under welded contact at the both boundaries. Important that decrease of contact stiffness at the boundary leads to added changing reflected field level and resonance phenomena behavior in comparison with single solid inclusions and solid layer and perfect contact.

At the figures 5 and 6 broken line corresponds to case non-rigid contact at the outer boundary ($KGN^{(1)} = KGT^{(1)} = 10^{14} Í\ /ì\ ^3$). As comparison, solid line shows case of cavity. You can see, adhesion violation at the inner boundary is negative masking phenomenon, which is reason rapid scattered field level increase.

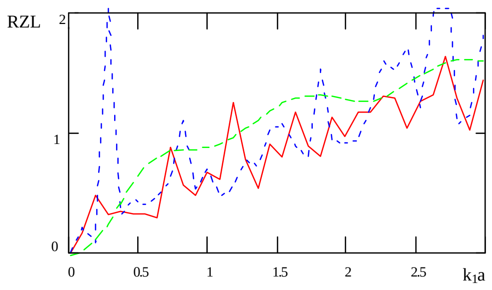

Fig.2.

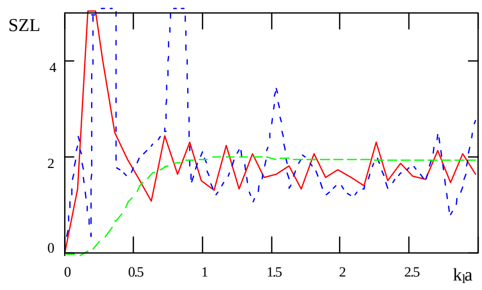

Fig.3.

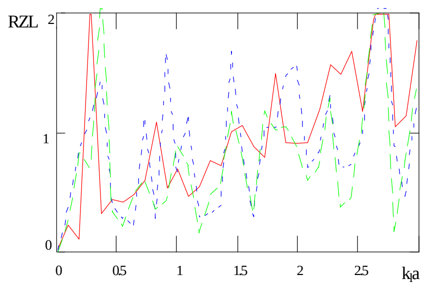

Fig.4.

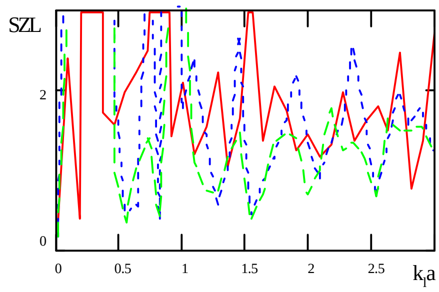

Fig.5.

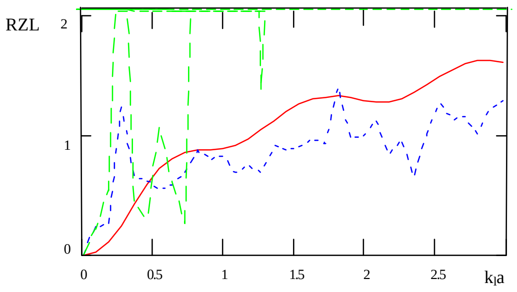

Fig.6.

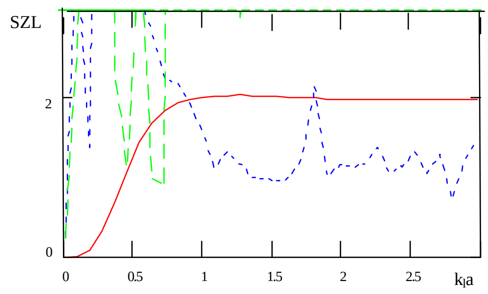

Fig.7.

Obtained behavior proof necessity of taking into consideration adhesion contact violation at the natural origin heterogeneities surface to prepare recommendations to correct of ultrasonic nondestructive testing result interpretation technique.

R E F E R E N C E S

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